Dual Focus on Systemic Risk in Portfolio Management

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Abstract

In this paper, we examine a complex portfolio selection strategy with a dual emphasis on systemic risk. This strategy or only its elements are advisable for both portfolio managers as well as macroprudential regulators. In particular, first, we present the concept of an early warning system (alarm) employing selected entropy measures, which allow us to detect systemic risk in financial markets. Secondly, we apply the two-phase optimization framework to determine the optimal composition of the portfolio. Essentially, the first phase of this strategy includes the reward–risk ratio maximization part and the following phase aims at systematic risk minimization. Furthermore, we approximate the returns using a dynamic set of components obtained from the principal component analysis and the classical ordinary least squares regression. In the empirical analysis using US market data, the wealth paths and statistics of different portfolio strategies are compared with each other. Ex-post results confirm higher profitability of the early warning system with double optimization, even if the transaction costs are taken into account. However, the main benefit lies in the significantly better risk properties of the proposed strategy.

Key Words

early warning system, entropy, systemic risk, portfolio optimization

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Introduction

Since the beginning of financial markets, the prediction and modelling of stock price behaviour have been significant challenges that are examined by both financial analysts and researchers, respectively, see Ahn et al. (2019) and Kouaissah and Hocine (2021). With the increasing fluctuation and volatility of prices in global markets during crisis periods, systemic risk warning is becoming more relevant. According to the general definition, systemic risk has an impact on each institution located on the market and affects the market price of stocks. In other words, systemic risk and the following systemic crisis are related to a common fluctuation that affects the entire economy, while the connection between institutions contributes to an undesirable "domino" effect. Due to the general definition of systemic risk, its measurement is not clearly and uniformly defined, but financial, macroeconomic, and statistical issues are usually used. After several successive crises, e.g., the global financial crisis (2007–2009), the COVID-19 pandemic crisis (2019–2021), or the beginning of the war in Ukraine (2022), the measurement systemic risk often received attention in the literature, see Ahn et al. (2019), Billio et al. (2016), Gradojevic and Caric (2016), Torri et al. (2022), and references therein.

The objective of this work is to study a complex portfolio selection strategy with a dual emphasis on the reduction of systemic risk. This contribution also extends the previous work of Neděla (2022). In particular, we consider the early warning system (hereinafter also referred to as alarm) employing selected entropy measures included in the portfolio selection strategy. This entropy-based alarm should detect systemic risk on the market. For the purposes of this analysis, we use select Shannon and Tsallis entropies (Shannon, 1948; Tsallis, 1988). Furthermore, we apply the two-phase optimization framework, where the first phase is important for obtaining a market benchmark (mean expected return) and the second phase minimizes the systemic risk. The market benchmark is achieved from the reward–risk ratio maximization model, where various reward–risk measures are included. For systemic risk minimization, we use Marginal Expected Shortfall (MES), Value-at-Risk (VaR), and Conditional Value-at-Risk (CoVaR). In addition, we consider optimization based on the approximated return series. Specifically, returns are approximated using a multifactor model with main components obtained by principal component analysis (PCA), see Ortobelli and Tichý (2015). In general, the approximation part is a useful tool for a precise estimation of the expected returns, which improves the decision-making process about the portfolio.

The inclusion of a simple type of alarm in the portfolio selection process was originally proposed by Kouaissah and Hocine (2021). However, the authors used different indicators from technical analysis and their generally applied rules. Thus, contrary to this approach, we decided to replace technical indicators with entropy indicators. In particular, our concept of systemic risk detection is derived from k-day percentage changes of entropy measures. In recent years, the popularity of entropy in the financial and economic areas has increased considerably. In the financial literature, we can find many ways to use this technique. Recently, for example, Billio et al. (2016) introduced an approach to detect systemic risk based on cross-sectional entropy. Later, Post and Potì (2017) introduced a method of stochastic efficiency based on relative entropy. Quantitative analysis of the behavioural characteristics of systemic risks through an entropy-based approach during the financial crisis (2007–2009) was done by Gradojevic and Caric (2016). Regarding the area of portfolio optimization, this issue was examined by Pola (2016), Mercurio et al. (2020), and the literature therein.

The remainder of this paper is structured as follows. This section provides an introduction to the topic. In Section 1, a methodological description of the selected entropy measures and systemic risk indicators used is provided. Section 2 consists of an ex-post empirical analysis using the US market data with a discussion of the results. Finally, the paper is concluded and summarized in the section Conclusion.

1. Methodology of Entropy and Portfolio Selection

This section contains the characterization and formulation of selected entropy indicators, reward–risk measures, and general portfolio optimization frameworks.

Assume a matrix with z risky assets and T observations. We denote a vector of returns $r = [r_1, \dots, r_z]$, where the t-th observation of the i-th return $r_{i,t}$ is calculated as $r_{i,t} = \ln \frac{P_{i,t}}{P_{i,t-1}}$, where $P_{i,t}$ is the price of asset i in time $t = 1, 2, \dots, T$ for $i = 1, \dots, z$. Thus, the portfolio return vector is denoted as $x'r$ while $x = [x_i, \dots, x_z]$ represents a vector of assets weights. If short sales are not considered, the portfolio weights x are from the simplex $S = \{x \in \mathbb{R}^z | x_i = 1; \sum x_i \geq 0; \forall i = 1, \ldots, z\}$.

1.1 Entropy and Systemic Risk Indicators

According to Neděla (2022), in this paper, we consider two types of entropy measures: Shannon's entropy and Tsallis's entropy. In addition, systemic risk indicators CoVaR, VaR, and MES are used for optimization and risk analysis.

Assume a given discrete probability distribution $P = \{p_i, i = 1, \ldots, z\}$, Shannon entropy is defined as a measure of disorder or randomness (Shannon, 1948). Therefore, the Shannon entropy E_S is mathematically formulated as:

$$E_S = -\sum_{i=1}^{z} p_i \log(p_i)\,. \tag{1}$$

We achieve a maximum of E_S when the underlying probability p_i is equal for each i (uniform probability). In the opposite situation, the minimum value of E_S is reached if only one $p_i = 1$ and the rest is equal to zero.

The second widely used entropy measure is the Tsallis entropy E_T (Tsallis, 1988; Billio et al., 2016). The difference from the Shannon entropy in Equation (1) lies in the extension index of the parameter α, which helps us to better identify the relevance rate for the prediction of the crisis by the distribution tails. Thus, E_T is defined as:

$$E_T = -\sum_{i=1}^{z} p_i^{\alpha} \log_{\alpha}(p_i) = \frac{1}{\alpha - 1}\left(1 - \sum_{i=1}^{z} p_i^{\alpha}\right). \tag{2}$$

In relation to the value of α, we take into account more or less tails of the distribution. If the index $\alpha = 1$, the Tsallis entropy is identical to E_S. However, when $\alpha > 1$ ($\alpha < 1$) means that the system is more dominated by usual (unusual) situations.

Alternatively, for systemic risk detection, CoVaR presented by Adrian and Brunnermeier (2016) is possible to use. It is calculated as the VaR of the financial market conditional on a company (stock) being in distress (Billio et al., 2016). First, let us define VaR as follows:

$$Pr\left(r_i \leq VaR_{i,\kappa}\right) = \kappa, \tag{3}$$

where κ is a significant value. $F_X^{-1}(y)$ means the inverse distribution function of the variable X. After that, the CoVaR conditional on the i-th asset return is formulated as:

$$Pr\left(r_b \leq CoVaR_{b,\kappa} \middle| r_i \leq VaR_{i,\kappa}\right) = \kappa, \tag{4}$$

where κ is a significant value. For example, if $\kappa = 0.5$ then $CoVaR_{b,\kappa}$ is the market VaR when the i-th asset returns are below the mean value.

The last measure of systemic risk is the Marginal Expected Shortfall (MES) presented by Acharya et al. (2017). The broad definition of MES represents the expected value of r_i in

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the case of a market decline detected found when a benchmark return r_b is lower than a predefined quantile q_κ. It is formulated as follows:

$$MES_i = E\left(r_i \middle| r_b < q_\kappa\right), \tag{5}$$

where κ is a value from the interval [0,1]. A market index that replicates the whole market or a particular reference asset might serve as the benchmark.

1.2 Reward-Risk Measures and Portfolio Selection Frameworks

In this subsection, several well-known reward-risk (performance) ratios and portfolio models are presented. The most applied one is the Sharpe ratio (SR) involving the portfolio excess return and the standard deviation (Sharpe, 1994), defined as follows:

$$SR = \frac{E\left(x'r - r_f\right)}{(x'Qx)^{\frac{1}{2}}}, \tag{6}$$

where r_f is a risk-free rate (or a benchmark return) and Q represents the covariance matrix. The SR value expresses the return for the unit of risk.

Additionally, the Rachev ratio measures the ratio between the Conditional Value-at-Risk (CVaR) of earnings and the mean of losses beyond Value-at-Risk (VaR), see Rachev et al. (2008). The equation is as follows:

$$RR = \frac{CVaR_\beta\left(r_f - x'r\right)}{CVaR_\alpha\left(x'r - r_f\right)}, \tag{7}$$

where $CVaR_\alpha(x'r) = \frac{1}{\alpha}\int_0^\alpha VaR_y(x'r)dy$.

The next performance measure selected is the STARR, where, in contrast to SR, CVaR with a significance value α replaces the standard deviation. STARR is defined as follows:

$$STARR = \frac{E\left(x'r - r_f\right)}{CVaR_\alpha(x'r)}. \tag{8}$$

The use of CVaR in the denominator captures the downside risk of the portfolio compared to the standard deviation contained in the SR.

According to Ruttiens (2013), an alternative time-dependent risk measurement technique is calculated based on cumulative returns or wealth path $W = \{W_t\}, t \in \{1,2,\dots,T\}$. The Ruttiens´ risk indicator is defined as the standard deviation of the spreads between wealth and its linear alternative leading to the same final W. Vector of spreads is computed as $Y_t = W_t - W_0 - \left(\frac{t}{T}\right)(W_T - W_0)$. The risk measure is formulated as $Ruttiens\ risk = \left(\sum_{t=1}^{T}\frac{1}{T}(Y_t - \bar{Y})^2\right)^{1/2}$, where $\bar{Y}$ is the mean value of vector Y. As defined by Ortobelli et al. (2017), the dynamic performance ratio RuttR can be mathematically formulated as follows:

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$$RuttR = \frac{W_T - 1}{1 + s \cdot Ruttiens\ risk} \tag{9}$$

where s represents the proportional coefficient. If the initial wealth $W_0 \neq 1$, then the numerator of the ratio contains $W_T - W_0$, which represents excess profit of an investment.

In this case, the investor aims to find an optimal portfolio that generates the maximum excess return per unit of expected risk; we can maximize these ratios above in the quadratic optimization framework (Rachev et al., 2008). Generally, the maximization model can be denoted by the following formulation:

$$\begin{gathered} \max_x \rho(x'r) \\ x'e = 1 \\ 0 \leq x_i \leq 0.2\ ;\ i = 1, \dots, z \end{gathered} \tag{10}$$

where $\rho(x'r)$ represents one of the selected performance measures (SR, RR, STARR, or RuttR) and e is a z-column unit vector with all values being equal to 1.

The second rational option is to minimize the risk of the portfolio, as provided in modern portfolio theory (Markowitz, 1952) built on variance minimization. With respect to this theory, the model can be formulated as follows:

$$\begin{gathered} \min_x \omega(x'r) \\ x'e = 1 \\ E(x'r) = M \\ 0 \leq x_i \leq 0.2\ ;\ i = 1, \dots, z \end{gathered} \tag{11}$$

where $\omega(x'r)$ is (systemic) risk measure and M is the required value of the expected return.

2. Empirical Analysis with Results

In this section, we apply the methodology from section above to the US maket data to verify the expected characteristics of our portfolio strategy. In particular, we use daily close prices of stocks, which were active in the S&P 100 index as of 4 November 2021. We chose the length of the time period from 1 January 2006 to 30 June 2021 (approximately 3,900 daily observations), during which two major crisis periods occurred. Because some of the price series have incomplete data for selected periods, these stocks are not further considered. Furthermore, as the risk-free rate, we use the return of the 3-month US Treasury bill, which is included in the computation of performance indicators, as well as an alternative investment when the early warning system is triggered. The data was downloaded from the Bloomberg database and the Investing.com website.

2.1 Ex-post portfolio analysis with results

To perform the analysis, the entire computational algorithm, including the portfolio optimization process, can be divided into several steps, similar to Neděla (2022):

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Step 1: Compute the entropy measures based on historical return data of one year (252 days) of the S&P 500 index using Equations (1) and (2). The parameter α for the Tsallis entropy is set as 0.6. According to these daily values, the 1-day and 5-days differences vectors are computed given by $\phi = \frac{(E_t - E_{t-k})}{E_{t-k}}$ for Shannon entropy and Tsallis entropy as well. If the 1-day decline is greater than 0.03 (i.e., 3%), the alarm is triggered, indicating a shift of investment to a risk-free asset until the 5-days differences of entropy increase more than 0.01 (i.e., 1%). If the alarm rule is not considered, skip to Step 2.

Step 2: Apply the PCA approach to the Pearson correlation matrix of the returns to obtain the main s factors (components) explaining at least 85% of the total portfolio variability. Then use a common OLS estimator for the approximation of returns assuming that r_i is a linear function of factors f_j formulated as $r_i = a_i + \sum_{j=1}^{s} b_{i,j} f_j + \varepsilon_i$, where a_i is constant of the i-th return, $b_{i,j}$ is coefficient for factor f_j and ε_i is residual part of the i-th return (Ortobelli and Tichý, 2015; Kouaissah and Hocine, 2021).

Step 3: Apply the two-phase optimization framework to find the asset's weights. In the first phase, the maximization performance ratio model (10) is applied to obtain the market potential (the expected portfolio return M). Then, we proceed with the minimization model formulated in Equation (11) using systemic risk measures (3), (4), or (5). The model has the condition that the expected portfolio return is identical to M acquired from the first optimization. In both cases, the upper limit of the portfolio weight is set at 0.2 to better diversify the portfolio.

Step 4: Compute portfolio statistics and the final wealth W_T while taking into account transaction costs tc_{t_d} set as 20 basis points. W_t is calculated by the following formulation:

$$W_{t_{d+1}} = \begin{cases} \left(W_{t_d} - tc_{t_d}\right)(1 + r_{b,d+1}) & \textit{if alarm is performed} \\ \left(W_{t_d} - tc_{t_d}\right)(x_M)' r_{t_{d+1}}^{ex-post} & \textit{otherwise,} \end{cases} \quad (13)$$

where $r_{t_{d+1}}^{ex-post}$ is the return between the period t_d and $t_{(d+1)}$. The time $t_{(d+1)} = t_d + \tau$, where $\tau = 21$.

The whole algorithm (steps 1 to 4) is applied again until daily observations are available. For an easier comparison of strategies, we consider that the initial investment W_0 is set to 1 (unit). All results of this ex-post analysis are presented in Tab. 1-4 and Fig. 1-2. The comparison of compiled portfolios is made according to daily statistics, i.e., mean (%), standard deviation (%), VaR5% (%), CVaR5% (%), MES (%), SR (%), RR and final wealth.

Tab. 1 shows the statistics of the max SR portfolio strategies with the statistics of the S&P 100 index selected as a benchmark (buy & hold principle). First, if portfolios without the alarm are compared, we can see that the two-phase optimization strategy with CoVaR minimization generates the best performance. In more detail, SR-CoVaR strategy has higher profitability than the simple one-phase, but surprisingly slightly higher risk. However, by incorporating alarm into the portfolio strategy, we are able to rapidly reduce the risk and increase the overall performance. In particular, MES decreases significantly, which confirms the success of limiting exposure to systemic risk. It is also evident that the Tsalis entropy with A-SR-MES strategy generates the best results.

Tab. 1: Ex-post portfolio statistics of different strategies with max SR model

Strategy	*mean*	*SD*	*VaR5%*	*CVaR5%*	*MES*	*SR*	*RR*	*final W*
SR	0.0299	1.3475	2.1705	3.4286	-0.3422	2.1624	0.8602	2.9743
SR-CoVaR	0.0358	1.5309	2.5003	3.8791	-0.4899	2.2911	0.8404	3.6931
SR-MES	0.0276	1.6142	2.5890	4.0514	-0.4084	1.6637	0.8642	2.7368
SR-VaR	0.0287	1.3632	2.2127	3.4838	-0.3384	2.0508	0.8473	2.8490
Shannon entropy								
A-SR	0.0336	1.1461	1.9240	2.9652	-0.2744	2.8641	0.8699	3.4024
A-SR-CoVaR	0.0320	1.3207	2.3774	3.3772	-0.2922	2.3677	0.8545	3.2147
A-SR-MES	0.0348	1.3815	2.3889	3.5098	-0.2676	2.4663	0.8599	3.5606
A-SR-VaR	0.0347	1.1631	1.9977	2.9933	-0.3050	2.9214	0.8842	3.5483
Tsallis entropy								
A-SR	0.0382	1.1918	1.9498	3.0308	-0.2718	3.1431	0.8665	4.0286
A-SR-CoVaR	0.0447	1.3593	2.4524	3.4121	-0.2897	3.2311	0.8576	5.0984
A-SR-MES	0.0535	1.4414	2.4402	3.5643	-0.2690	3.6588	0.8742	7.0317
A-SR-VaR	0.0344	1.2103	2.0488	3.0747	-0.2816	2.7778	0.8743	3.5021
S&P 100	0.0299	1.2726	1.9284	3.2268	3.2296	2.3354	0.8836	2.2112

Source: authors' calculations in Matlab

Tab. 2: Ex-post portfolio statistics of different strategies with max RR model

Strategy	*mean*	*SD*	*VaR5%*	*CVaR5%*	*MES*	*SR*	*RR*	*final W*
RR	0.0071	1.3473	2.0627	3.3891	-0.3639	0.4730	0.8702	1.2969
RR-CoVaR	0.0173	1.5708	2.5645	3.9880	-0.5085	1.0533	0.8462	1.8792
RR-MES	0.0190	1.5856	2.5393	3.9152	-0.3740	1.1506	0.8910	1.9993
RR-VaR	0.0209	1.1654	1.7967	2.9457	-0.2296	1.7296	0.8583	2.1437
Shannon entropy								
A-RR	0.0036	1.1739	1.8843	3.0355	-0.1956	0.2409	0.8613	1.1397
A-RR-CoVaR	0.0109	1.4138	2.4426	3.6708	-0.2551	0.7221	0.8243	1.4916
A-RR-MES	0.0231	1.3906	2.4047	3.5810	-0.2526	1.6074	0.8495	2.3224
A-RR-VaR	0.0257	0.9923	1.6155	2.5369	-0.1700	2.5220	0.8730	2.5600
Tsallis entropy								
A-RR	0.0082	1.2374	1.9657	3.1216	-0.1643	0.6009	0.8671	1.3481
A-RR-CoVaR	0.0217	1.4604	2.4580	3.7336	-0.2641	1.4351	0.8271	2.2073
A-RR-MES	0.0464	1.4627	2.4091	3.6292	-0.2524	3.1181	0.8814	5.4225
A-RR-VaR	0.0298	1.0416	1.6940	2.6032	-0.1666	2.7850	0.8819	2.9603

Source: authors' calculations in Matlab

Tab. 3: Ex-post portfolio statistics of different strategies with max STARR model

Strategy	*mean*	*SD*	*VaR5%*	*CVaR5%*	*MES*	*SR*	*RR*	*final W*
STARR	0.0255	1.3466	2.1353	3.4288	-0.3723	1.8341	0.8618	2.5298
STARR- CoVaR	0.0438	1.5446	2.4927	3.8947	-0.5397	2.7867	0.8553	4.9385
STARR-MES	0.0243	1.6074	2.6021	4.0321	-0.4069	1.4670	0.8636	2.4288
STARR-VaR	0.0255	1.3638	2.1746	3.4855	-0.3817	1.8153	0.8519	2.5352
Shannon entropy								
A-STARR	0.0347	1.1391	1.9588	2.9429	-0.2908	2.9808	0.8578	3.5451
A-STARR-CoVaR	0.0365	1.3193	2.2780	3.3762	-0.2581	2.7127	0.8522	3.7906
A-STARR-MES	0.0304	1.3655	2.4312	3.4796	-0.2898	2.1730	0.8441	3.0329
A-STARR-VaR	0.0386	1.1578	1.9385	2.9609	-0.2735	3.2701	0.8765	4.0881
Tsallis entropy								
A-STARR	0.0401	1.1774	1.9631	2.9573	-0.2879	3.3423	0.8752	4.3161
A-STARR- CoVaR	0.0411	1.3649	2.3778	3.4388	-0.2685	2.9579	0.8551	4.4797
A-STARR-MES	0.0498	1.4183	2.4817	3.4920	-0.3036	3.4586	0.8827	6.1479
A-STARR-VaR	0.0394	1.1966	1.9850	3.0093	-0.2543	3.2283	0.8773	4.2039

Source: authors' calculations in Matlab

Tab. 4: Ex-post portfolio statistics of different strategies with max RuttR model

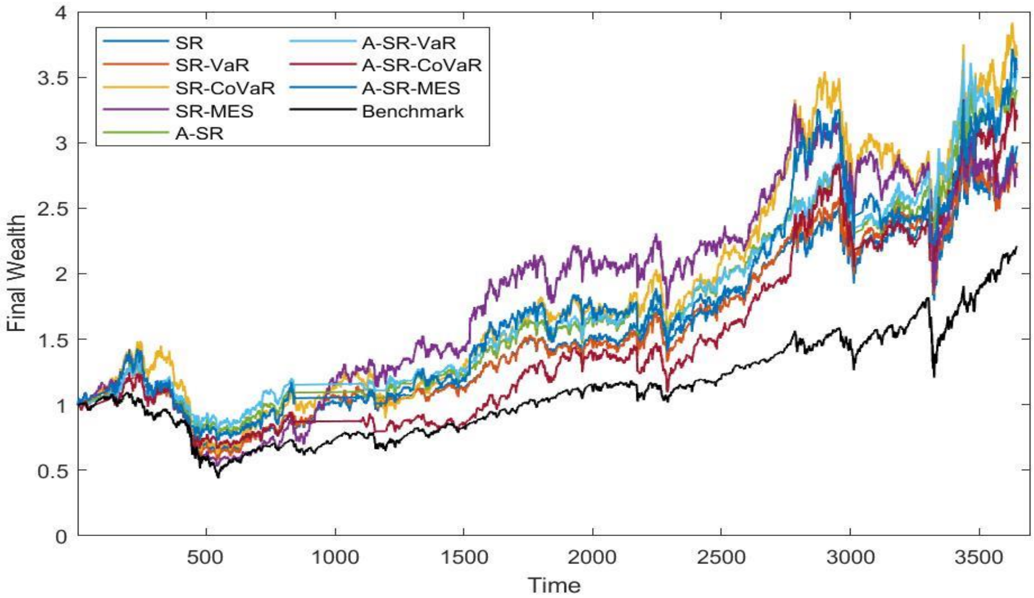
Strategy	*mean*	*SD*	*VaR5%*	*CVaR5%*	*MES*	*SR*	*RR*	*final W*
RuttR	0.0446	1.6580	2.8057	4.1833	-0.5274	2.6436	0.8367	5.0822
RuttR-CoVaR	0.0437	1.6692	2.8376	4.2236	-0.5130	2.5729	0.8361	4.9206
RuttR-MES	0.0429	1.6738	2.8526	4.2323	-0.5027	2.5178	0.8354	4.7788
RuttR-VaR	0.0462	1.6556	2.8375	4.1773	-0.5287	2.7442	0.8364	5.3876
Shannon entropy								
A-RuttR	0.0350	1.4377	2.5477	3.7336	-0.3052	2.3831	0.8356	3.5850
A-RuttR-CoVaR	0.0395	1.4476	2.6302	3.7404	-0.3370	2.6796	0.8455	4.2286
A-RuttR-MES	0.0385	1.4353	2.5711	3.7041	-0.2929	2.6271	0.8447	4.0651
A-RuttR-VaR	0.0368	1.4326	2.5961	3.7156	-0.3027	2.5133	0.8398	3.8204
Tsallis entropy								
A-RuttR	0.0498	1.4498	2.4527	3.6392	-0.3477	3.3798	0.8576	6.1361
A-RuttR-CoVaR	0.0501	1.4887	2.6467	3.7459	-0.3507	3.3174	0.8477	6.2231
A-RuttR-MES	0.0435	1.4962	2.6319	3.7777	-0.3005	2.8546	0.8514	4.8784
A-RuttR-VaR	0.0468	1.4479	2.4447	3.6421	-0.3428	3.1802	0.8564	5.5096

Source: authors' calculations in Matlab

In Tabs. 2–4, the results of the strategies using the other performance measures are shown. Generally, they basically confirm the findings obtained for the strategies with the SR. It is obvious that the profitability of portfolio strategies with the alarm is in most cases higher than the strategies without it. Note that complex strategies reduce even classical risk measures as well as systemic risk measure (MES), due to the involvement of the alarm. Obviously, the highest profitability is achieved by maximizing the RuttR in Table 4. Finally, it is evident that only a few strategies maximizing RR do not outperform the benchmark (S&P 100 index) in the sense of final W and some strategies have lower risk than the benchmark.

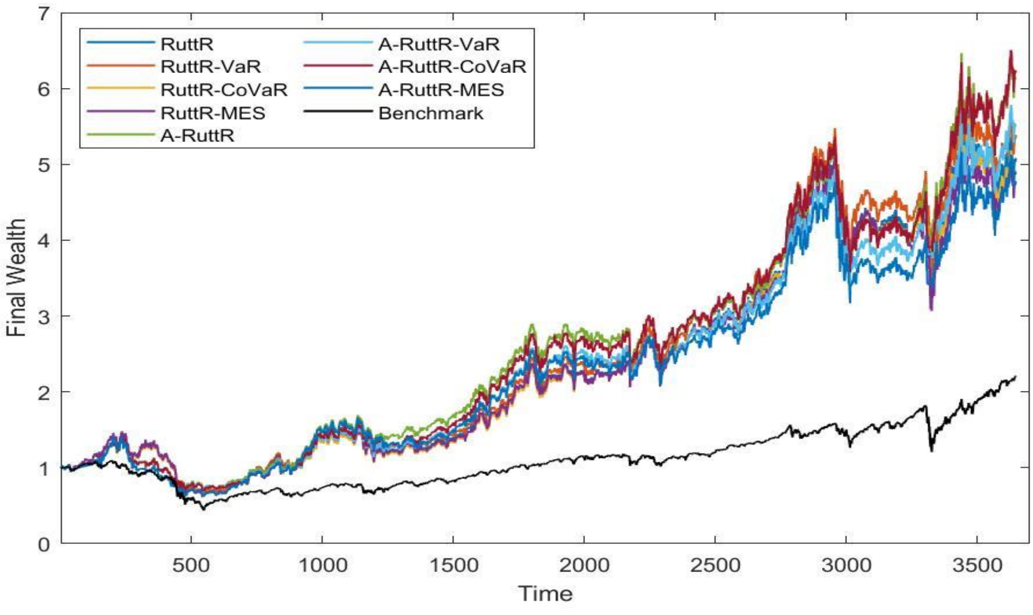
To better capture the evolution of wealth paths for various portfolio strategies and the effect of the alarm, we show Figs. 1 and 2. We select only the comparison based on two performance measures (SR and RuttR) because of the saving of space. Note that for both entropy measures, the alarm was triggered 12 times with a different durations.

Fig. 1: Ex-post wealth paths for max SR portfolio strategies with and without the Shannon entropy alarm compared to a benchmark S&P 100



Source: authors' own calculations in Matlab

Fig. 2: Ex-post wealth paths for max RuttR portfolio strategies with and without the Tsallis entropy alarm compared to a benchmark S&P 100



Source: authors' own calculations in Matlab

The benefit of the alarm tool was already noticeable during the financial crisis that was portrayed at the start of the selected period. Additionally, it effectively illustrates how a portfolio behaves during an economic expansion when a partial market shock causes investment to be interrupted for longer. An upward tendency is clearly visible before the interruption and is linked to earlier growth following the transition to risky assets. However, all strategies still experience significant drawdowns during the COVID-19 crisis period, when daily declines in financial markets were more pronounced.

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Conclusions

This paper examined a complex portfolio selection strategy focused on the minimization of systemic risk from two perspectives. First, we use the early warning system using Shannon and Tsallis entropy measures to detect the threat of systemic risk. Furthermore, we applied the two-phase optimization portfolio selection approach on approximated returns. This strategy consisted of maximizing reward-risk ratios in the first phase and minimizing systemic risk in the second phase while maintaining the expected return obtained from the first phase. In the empirical part, we analyzed the portfolio statistics and wealth paths for different portfolio strategies with and without alarm. By comparing obtained results based on the US market data, we confirmed better properties of alarm strategies with two-phase optimization provided even the transaction costs are incorporated in the wealth computation. To sum it up, the two-phase optimization helped us to improve the performance of portfolios compared to the simple strategy while the alarm reduced the total risk of generated portfolios. In further research, we can compare the effectiveness of the entropy early warning system with the technical analysis one.

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