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SYMMETRY PROPERTIES OF DOMAIN WALLS ASSOCIATED WITH COMPLETELY TRANSPOSABLE DOMAIN PAIRS

THESIS FOR HABILITATION

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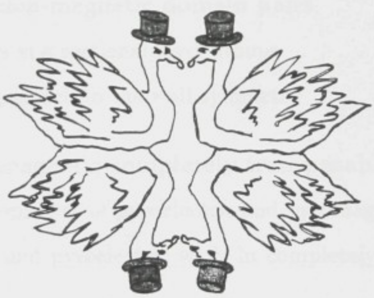
Фактов всегда достаточно — не хватает фантазии.

Д. Блохинцев



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Contents

List of the most important symbols

$\mathbf{E_n}$	n-dimensional Euclidean space
$\mathcal{G} = (G, T_G)$	space group
G	point group
T_G	translational group
$G_1^0 \equiv Gv$	pointlike line group
$G_2^0 \equiv G\mathrm{v}$	pointlike plane group
$G_3^0 \equiv G\mathrm{V}$	pointlike space group
${}^2G_3 \equiv Gv$	pointlike layer group
${}^2G_3^0$	point group of the pointlike layer group ${}^2G_3$
(hkl)	plane determined by Miller indices
$(hkil)$	plane determined by Bravis-Miller indices
$\overline{G}(hkl) \equiv \overline{G}, \overline{J}_{12}$	sectional layer group
$\{(hkl), G\}_f \equiv \{hkl\}_f$	
$\{(hkil), G\}_f \equiv \{hkil\}_f$	face orbit
$\{hkl\}, \{hkil\}$	plane orbits
$\overline{L}_H$	holohedral sectional layer group
D_i	i-th domain
S_i	i-th domain state
F_i	symmetry group of the domain state S_i
(S_i, S_j)	ordered domain pair
$\{S_i, S_j\}$	unordered domain pair
F_{ij}	symmetry group of the domain pair (S_i, S_j)
J_{ij}	symmetry group of the domain pair $\{S_i, S_j\}$
$[S_1(hkl)S_2] \equiv [S_1(\mathbf{n})S_2]$	domain wall between domains with domain states S_1 and S_2

$T[S_1(hkl)S_2] \equiv T_{12}(hkl)$	
$T_{12}(\mathbf{n}) \equiv T_{12}$	symmetry group of a domain wall
$\underline{u}, \underline{s}_{12}$	side-exchanging operations
$\underline{u}^*, \underline{t}^*_{12}$	side&state-exchanging operations
$\widehat{F}_1, \widehat{J}_{12}$	one-sided sectional layer groups
$\mathbf{sf}(G, \rho)$	simple form generated by the plane ρ and the group G
$J_{12}[F_1]$	explicit notation for a domain pair symmetry group
$1'$	the time inversion
$\bar{1}$	the space inversion
$\bar{1}'$	the space and time inversion
q	magnetic group not containing the operations $1', \bar{1}, \bar{1}'$
i	magnetic group not containing the operations $1', \bar{1}'$
i'	magnetic group not containing the operations $1', \bar{1}$
$\mathbf{M}$	magnetization
$\mathbf{P}$	polarization
MP	pyromagnetic and pyroelectric groups
M	pyromagnetic and non-pyroelectric groups
P	non-pyromagnetic and pyroelectric groups
0	non-pyromagnetic and non-pyroelectric groups
$\mathcal{MP}$	type of a domain pair where domain walls can be pyromagnetic and pyroelectric
$\mathcal{M}$	type of a domain pair where domain walls cannot be pyroelectric
$\mathcal{P}$	type of a domain pair where domain walls cannot be pyromagnetic
$\mathcal{O}$	type of a domain pair where domain walls are non-pyromagnetic and non-pyroelectric
A	asymmetric
S	symmetric
I	irreversible
R	reversible

Chapter 1

Introduction

Non-homogeneity can induce effects that are forbidden in a homogeneous systems, e.g. a non-homogeneous temperature or non-homogeneous deformation give rise to an electric polarization in solid or liquid crystals. The existence of these effects follows from the fact that a non-homogeneity usually decreases the symmetry and thus allows the existence of some properties that cannot be found in the homogeneous system because of its higher symmetry.

Domain walls represent a special kind of a non-homogenity. The lowering of the translational symmetry to two dimensions confines the appearance of a new effects to a certain layer the symmetry of which is described by so called *pointlike layer groups* [8,19,31]. These groups further exclude some symmetry elements that may exist in domain bulks, e.g. rotation and inversion axes that are not perpendicular or parallel to the wall. On the other hand, a planar domain wall may be invariant under operations that interexchanges domain states on two opposite sides of the wall. Since these operations cannot exist in the bulk of domains we encounter an enhancement of symmetry. Thus the symmetry difference between the domain bulks (domain states) and the wall is, in general, not a simple symmetry lowering and can thus induce not only an appearance of new effects in the domain wall that do not exist in the domain states of adhearing domains but also a disappearance of some properties existing in domain states.

In some papers a special interest was attended to situations where domain walls acquire properties that do not exist in domain bulks. Baryakhatar *at al* [1,2] have studied theoretically the appearance of electric polarization in domain walls in magnetically ordered crystals. Walker and Gooding [32] predicated theoretically the existence of a spontaneous polarization of a wall joining two non-polar domains of quartz. Similar results for quartz were obtained by Saint-Grégoire and Janovec [27] from symmetry analysis.

In this report a question of possible appearance of spontaneously polarized (pyroelectric) and spontaneously magnetized (pyromagnetic) domain walls joining antiferroelectric and antiferromagnetic domains with zero average polarization and magnetization is discussed. *The general analysis will be further restricted to non-ferroelastic domain walls which bridge domains with the same spontaneous deformation.* A complete tables of layer groups which describe symmetry of a domain wall will be determine for such domain pairs. The same method will be applied on 380 magnetic completely transposable domain pairs in order to determine tables of layer groups describing symmetry of domain walls between them. These domain pairs will be classified according to possible existence of spontaneous polarization and magnetization in their domain walls.

Chapter 2

Basic notions

2.1 Mathematical tools

A *group* G is a set of distinct elements $g_1, g_2, ...$ such that for any two elements g_i and g_j, an operation called the group multiplication ($\circ$) is defined which satisfies the following four requirements :

1. The set G is closed under multiplication: for any two elements g_i and g_j of G, their unique product $g_i \circ g_j$ also belongs to G.

2. The associative law holds:
$$g_k \circ (g_j \circ g_i) = (g_k \circ g_j) \circ g_i. \tag{2.1}$$

3. There exists in G an element g° which satisfies
$$g^\circ \circ g = g \circ g^\circ = g \tag{2.2}$$
for any element $g \in G$. Such an element g° is called the *unit element* or the *identity element*; it will be denoted by e:
$e \circ g = g \circ e = g$.

4. For any element $g \in G$, there exists an element g^{-1} (the inverse element) which satisfies
$$g^{-1} \circ g = g \circ g^{-1} = e. \tag{2.3}$$

The elements g_i are sometimes called *group elements*, particularly when one wishes to emphasize that they are members of the group G. Groups having an infinite number of elements are called *infinite groups*, while groups having a finite number of elements are *finite groups.* The total number of elements in a finite group is the *order of the group.*

If any two elements g_i and g_j of a given group G commute, i.e., if

$$g_i \circ g_j = g_j \circ g_i \tag{2.4}$$

(commutative law) holds, then such a group G is said to be a *commutative group* or an *Abelian group.*

If a group G contains an element g such that its powers g^i ($g^2 = g \circ g$, $g^i = g \circ g \circ ... \circ g$ $i-$times) exhaust all the elements of the group, i.e.,

$$G = \{g, g^2, ..., g^i, ..., g^n = e\}, \tag{2.5}$$

such a group is called *cyclic*, and its order is equal to n. Such an element g_i, whose powers are the other elements of the group, is called a generating element, or a *generator.*

In the next text the product symbol $\circ$ will be omitted.

A *subgroup* H of a group G ($H \subset G$) is a subset of G that is itself a group under the multiplication defined in the mother group G. Both the single element $\{e\}$ and the group G itself are *trivial* subgroups of G. The other subgroups, if any, are *proper* subgroups.

Let H be a subgroup of G, $H \subset G$. If element g_i in G is not contained in the subgroup H, we can form *left coset* g_iH, or *right coset* Hg_i which consists of all the products g_ih_j (or h_jg_i), where h_j runs through the elements of H. Two cosets g_iH and g_jH ($i \neq j$) have no elements in common or are identical. If G is a finite group then the group G can be expanded with respect to the subgroup H by representing G in the form of the union (symbol $+$ is used instead of $\cup$) of the cosets of H

$$G = eH + g_1H + ... + g_pH, \tag{2.6}$$

or

$$G = He + Hg_1 + ... + Hg_p. \tag{2.7}$$

The numbers of cosets appearing in two preceding equalities are equal, although their contents may be different.

Let the group G of order n contain the subgroup H of order k. Since every coset $g_i H$ consists of k distinct elements, the equality $n = kp$ must hold. Hence, the order of the mother group G is divisible by the order of the subgroup H. The integer $p = n/k$ is called the *index* of H in G. When n is a prime number, k must be equal to n or to unity, so, the group whose order is a prime number has no proper subgroups.

An element b of the group G is said to be *conjugate* to a if there exists a group element g such that

$$b = gag^{-1}. \tag{2.8}$$

Sometimes, we say that b is the *transform* of a by g.If b is conjugate to a, then a is conjugate to b. If b is conjugate to a and c is conjugate to b, then c is conjugate to a, because from $b = gag^{-1}$ and $c = hbh^{-1}$ it follows that $c = (hg)a(hg)^{-1}$. The set of all elements that are conjugate to each other is called a *conjugate class* or simply a *class*. By this definition, different classes have no elements in common.

Let H be a subgroup of the group G. If we transform the elements of H with an element g of G, the set of those elements gHg^{-1} is a subgroup of G. It is called a *conjugate subgroup* of H.

If the subgroup H satisfies the relation $gHg^{-1} = H$ for all elements $g \in G$, then the subgroup H is called an *invariant subgroup* of G. An invariant subgroup is also called a *normal subgroup* or *normal divisor*.

An invariant subgroup satisfies the equality

$$gH = Hg, \tag{2.9}$$

which means that the left coset is identical with the right coset as a set, i.e., an invariant subgroup is a subgroup whose right and left cosets are identical.

2.2 Pointlike groups in $\mathbf{E}_1, \mathbf{E}_2$ and $\mathbf{E}_3$

In a continuum description of a medium in $\mathbf{E_n}$ (n-dimensional Euclidean space) special types of continuous space groups $\mathcal{G} = (G, T_G)$, so called *pointlike* space groups [19], are used in order to indicate that each point P of a material with such symmetry has the *site-point* symmetry G_P [7,19] (G_P is a point group G located at the point P, i.e. G_P is a point group of symmetry operations which leave at least the point P unmoved) and continuous *translational subgroup* T_G spans the whole vector space $\mathbf{V_n}$ underlying the corresponding Euclidean space $\mathbf{E_n}$. It means that it is the point group G of the space group $\mathcal{G}$ (in general the group $\mathcal{G}$ is called a space group if T_G spans the whole space V_n, [19]) which completely determines the properties of the medium from the group-theory point of view.

In this report only crystallographic point groups in $\mathbf{E_1}, \mathbf{E_2}$ and $\mathbf{E_3}$ will be used and therefore the term "crystallographic" will be later sometimes omitted. Their generators are restricted to the crystallographic cases, axes of rotation 1, 2, 3, 4, 6 and axes of rotoinversion $\bar{1}, \bar{2} = m, \bar{3}, \bar{4}, \bar{6}$. The numbers of crystallographic point groups are finite.

In $\mathbf{E}_1$ *pointlike line groups* are denoted as Gv, where v indicate one-dimensional continuous vector space of translations. There are only 2 point line groups G_1^0 (the lower index represents 1-dimensional Euclidean space, the upper index represents a point group [14,20,24]).

$$G_1^0 = 1,\ \bar{1}.$$

In $\mathbf{E}_2$ *pointlike plane groups* are denoted as Gv, where v indicate two-dimensional continuous vector space of translations. There are 10 point plane groups G_2^0 (the lower index represents 2-dimensional Euclidean space, the upper index represents a point group).

$$G_2^0 = 1,\ 21,\ 1m,\ 2mm,\ 4,\ 4mm,\ 3,\ 3m,\ 6,\ 6mm.$$

In $\mathbf{E_3}$ *pointlike space groups* are denoted as GV, where V indicate three-dimensional continuous vector space of translations. There are 32 point groups $G = G_3^0$ (the lower index represents 3-dimensional Euclidean space, the upper index represents a point group).

$$G = G_3^0 = 1,\ \bar{1};\quad 2,\ m,\ 2/m;\quad 222,\ 2mm,\ 2/m2/m2/m;$$

$$4,\ \bar{4},\ 4/m,\ 422,\ 4mm,\ \bar{4}2m,\ 4/m2/m2/m;$$

$$3,\ \bar{3},\ 32,\ 3m,\ \bar{3}2/m;\quad 6,\ \bar{6},\ 622,\ 6mm,\ \bar{6}2m,\ 6/m2/m2/m;$$

$$23,\ 2/m\bar{3},\ 432,\ \bar{4}3m,\ 4/m\bar{3}2/m.$$

They form 7 crystal systems. All the groups belonging to a given crystal system are subgroups of the maximal supergroup (*holohedry*) characterizing the crystal system.

An extension of classical groups describing geometrical symmetry is possible by a group of non-geometrical properties which assign to each point of geometrical space $p > 1$ different values. In case $p = 2$ so called *two-colour* or *Shubnikov* (resp. *antisymmetric*) groups are generated, assigned values are *"black/white"* or "$+1/-1$", [3,4,23,29,30]. When the operation "change of colours" (or "change of signs") is replaced by the time inversion $1'$ then two-colour groups can be reinterpreted as *magnetic groups*.

Magnetic point groups are often divided into three types [2]:

Type I: *ordinary (trivial or white (or black)) point groups* (32)

Point groups G are identical with 32 crystallographic point groups mentioned in the beginning of this section.

Type II: *non-magnetic (gray) point groups* (32)

Point group of this type is expressed as direct product of ordinary point group G and the group of time inversion {1,1'}

$$M = G \otimes \{1, 1'\}. \tag{2.10}$$

Type III: *non-trivial magnetic (black and white) point groups* (58)

Point group of this type is given as

$$M = H + 1'(G - H), \tag{2.11}$$

where H is a halving subgroup of the ordinary point group G.

The numbers in parentheses give the number of point groups of each type.

All three types of magnetic point groups are derived from 32 crystallographic point groups and thus they can be divided into 32 magnetic families, in accordance with the initial ordinary group G.

2.3 Pointlike layer and sectional layer groups

A layer is a three-dimensional diperiodic object. For the layer (or two-sided pane which can be considered as a layer of zero thickness) bisecting space two outer normals can be distinguished, each of them is directed towards one halfspace. See Fig. 1.1.

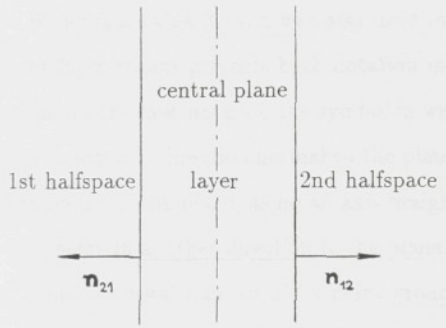


Figure 2.1: Outer normals of a layer

The unit vector $\mathbf{n}_{12}$ represents the outer normal directed towards the second halfspace, $\mathbf{n}_{21}$ the outer normal towards the first one. Because $\mathbf{n}_{12} = -\mathbf{n}_{21}$, it is possible to use simpler notation $\mathbf{n}_{12} = \mathbf{n}$ and $\mathbf{n}_{21} = -\mathbf{n}$.

The *pointlike layer groups* (or *net groups* [23]) denoted as Gv or ${}^2G_3$ (v or left upper index indicates dimension of continuous translational group T_G) are created by all symmetry operations leaving the given layer (or two-sided plane) invariant (unchanged). The point group ${}^2G_3^0$ of a pointlike layer group contains two types of such operations [4,8,30].

1. The symmetry operation(s) transforming points on one side (face) of the layer to the equivalent points on the same side (face). These *one-sided* symmetry operations keep also unchanged outer normal. It is evident that 2, 3, 4 and 6-folded axes and mirror planes are of this type only if they are perpendicular to the layer, and rotoinversions must be excluded.

2. The symmetry operations transforming points lying on one side (face or upper side) of the layer into equivalent points lying on the opposite side (back face or lower side). These operations invert outer normal $\mathbf{n}$ into $-\mathbf{n}$ and vice versa.

In this case 3, 4 and 6-folded axes must be completely excluded. Mirror planes and 2-folded axes satisfy the given condition only if they lie in the central plane of the layer, and axes of inversion rotation $(\bar{1}, \bar{3}, \bar{4}, \bar{6})$ must be perpendicular to the layer.

In order to distinguish these two types of operations the second one will be *underlined* in this report (e.g. $\underline{2}$, $\underline{m}$, etc.). This auxiliary notation was introduced in consideration of domain twins [6] and was also used in the recent papers [25,24]. Next list of point layer groups presents both notation introduced by Holser [8] and auxiliary notation. In the first notation the symbol is written in the following sequence:
(1) symmetry in a direction normal to the plane, arbitrarily taken as the z axis,
(2) symmetry in the plane, along an axis designed x, and
(3) symmetry in another direction in the plane.

From the total number of 32 point groups 31 point layer groups ${}^2G_3^0$ can be determined [8,23,29,31]. (In [8] they are presented as subgroups of the point group in a two-sided plane.) These point layer groups can be divided into two families.

A) 10 *one-sided point layer groups* include only operations leaving side invariant ("one-sided" operations not reversing the outer normal or side-preserving operations), none of generators of these groups is underlined :

Holser's notation	1	21	$1m$	$2mm$	4	$4mm$	3	$3m$	6	$6mm$
Auxiliary notation	1	2	m	$mm2$	4	$4mm$	3	$3m$	6	$6mm$

(all axes and mirror planes are perpendicular to the layer).
In fact, they describe symmetry of a plane and therefore they are analogous to the plane groups operating in $\mathbf{E_2}$.

B) 21 two-sided point layer groups contain also an operation exchanging half-spaces ("two-sided" operation reversing the outer normals or side-reversing operations). It is possible to subdivide them into 2 parts:

a) 10 groups with side-exchanging operation(s), containing the reflection through the central plane of the layer.

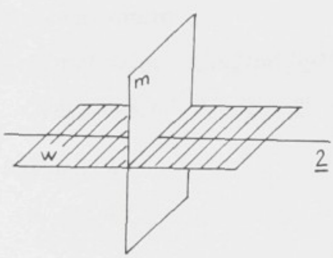
Holser's n.	$m1$	$2/m1$	$m2m$	mmm	$4/m$	$4/mmm$	$\bar{6}$	$6/m$	$\bar{6}m2$	$6/mmm$
Auxiliary n.	$\underline{m}$	$2/\underline{m}$	$\underline{m}m2$	$\underline{m}mm$	$4/\underline{m}$	$4/\underline{m}mm$	$\underline{\bar{6}}$	$6/\underline{m}$	$\underline{\bar{6}}m\underline{2}$	$6/\underline{m}mm$

b) 11 groups with side-exchanging operation(s), not containing the reflection through the central plane of the layer.

Holser's n.	$\bar{1}$	12	$12/m$	222	$\bar{4}$	422	$\bar{4}2m$	$\bar{3}$	32	$\bar{3}m$	622
Auxiliary n.	$\underline{\bar{1}}$	$\underline{2}$	$\underline{2}/m$	$\underline{22}2$	$\underline{\bar{4}}$	$4\underline{22}$	$\underline{\bar{4}2}m$	$\underline{\bar{3}}$	$3\underline{2}$	$\underline{\bar{3}2}/m$	$6\underline{22}$

(axes $2, 4, 3, 6, \underline{\bar{3}}, \underline{\bar{4}}, \underline{\bar{6}}$ and plane m are perpendicular to the layer, axis $\underline{2}$ and plane $\underline{m}$ are parallel to the layer).

Fig. 1.2 illustrates different position of axes and mirror planes with respect to the central layer plane w for the layer groups $\underline{2}/m$ and $2/\underline{m}$.



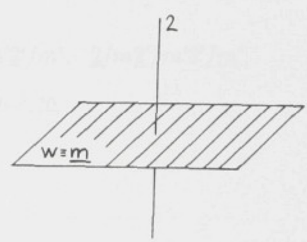


Figure 2.2:

The total number of 122 magnetic point groups (including all three types) [1,4,23,30] determine following 125 *magnetic layer groups.* Operations reversing normal will be again underlined in auxiliary notation. These groups are subdivided into 27 sublists with respect to which magnetic family the point group belongs. A magnetic family includes all three types of magnetic point groups derived from one ordinary point group. (27 families = 32 ordinary point groups minus 5 cubic ones)

1. Family **1**

   $1,\ 1'$

2. Family $\bar{\mathbf{1}}$

   $\underline{\bar{1}}$, $\underline{\bar{1}}1'$, $\underline{\bar{1}}'$

3. Family **2**

   2, $\underline{2}$, $21'$, $\underline{2}1'$, $2'$, $\underline{2}'$

4. Family **m**

   m, $\underline{m}$, $m1'$, $\underline{m}1'$, m', $\underline{m}'$

5. Family **2/m**

   $\underline{2}/m$, $2/\underline{m}$, $\underline{2}/m1'$, $2/\underline{m}1'$, $\underline{2}'/m'$,

   $2'/\underline{m}'$, $\underline{2}/m'$, $2/\underline{m}'$, $\underline{2}'/m$, $2'/\underline{m}$

6. Family **222**

   $\underline{2}\underline{2}2$, $\underline{2}\underline{2}21'$, $\underline{2}\underline{2}'2'$, $2\underline{2}'\underline{2}'$

7. Family **mm2**

   $mm2$, $m\underline{m}\underline{2}$, $mm21'$, $m\underline{m}\underline{2}1'$ $m'm'2$, $m'\underline{m}'\underline{2}$, $m'm2'$, $m'\underline{m}\underline{2}'$, $\underline{m}'m\underline{2}'$

8. Family **mmm**

   $\underline{2}/m\underline{2}/m2/\underline{m}$, $\underline{2}/m\underline{2}/m2/\underline{m}1'$, $2/\underline{m}\underline{2}'/m'\underline{2}'/m'$, $\underline{2}/m\underline{2}'/m'2'/\underline{m}'$,

   $2/\underline{m}'\underline{2}/m'\underline{2}/m'$, $2/\underline{m}'\underline{2}'/m\underline{2}'/m$, $\underline{2}/m'\underline{2}'/m2'/\underline{m}$

9. Family **4**

   4, $41'$, $4'$

10. Family $\bar{\mathbf{4}}$

    $\underline{\bar{4}}$, $\underline{\bar{4}}1'$, $\underline{\bar{4}}'$

11. Family **4/m**

    $4/\underline{m}$, $4/\underline{m}1'$, $4'/\underline{m}$, $4/\underline{m}'$, $4'/\underline{m}'$

12. Family **422**

    $4\underline{2}\underline{2}$, $4\underline{2}\underline{2}1'$ $4'\underline{2}\underline{2}'$, $4\underline{2}'\underline{2}'$

13. Family **4mm**

    $4mm$, $4mm1'$, $4'mm'$, $4m'm'$

14. Family $\mathbf{\bar{4}2m}$

    $\underline{\bar{4}2}m$, $\underline{\bar{4}2}m1'$, $\underline{\bar{4}}'\underline{2}m'$, $\underline{\bar{4}}'\underline{2}'m$, $\underline{\bar{4}2}'m'$

15. Family $\mathbf{4/mmm}$

    $4/\underline{m2}/m\underline{2}/m$, $4/\underline{m2}/m\underline{2}/m1'$, $4'/\underline{m2}/m\underline{2}'/m'$, $4/\underline{m2'}/m'\underline{2}'m'$,
    $4/\underline{m}'\underline{2}/m'\underline{2}/m'$, $4/\underline{m}'\underline{2}'/m\underline{2}'/m$, $4'/\underline{m}'\underline{2}'/m\underline{2}/m'$

16. Family $\mathbf{3}$

    3, $31'$

17. Family $\mathbf{\bar{3}}$

    $\underline{\bar{3}}$, $\underline{\bar{3}}1'$, $\underline{\bar{3}}'$

18. Family $\mathbf{32}$

    $3\underline{2}$, $3\underline{2}1'$, $3\underline{2}'$

19. Family $\mathbf{3m}$

    $3m$, $3m1'$, $3m'$

20. Family $\mathbf{\bar{3}m}$

    $\underline{\bar{3}}m$, $\underline{\bar{3}}m1'$, $\underline{\bar{3}}m'$, $\underline{\bar{3}}'m'$, $\underline{\bar{3}}'m$

21. Family $\mathbf{6}$

    6, $61'$, $6'$

22. Family $\mathbf{\bar{6}}$

    $\underline{\bar{6}}$, $\underline{\bar{6}}1'$, $\underline{\bar{6}}'$

23. Family $\mathbf{6/m}$

    $6/\underline{m}$, $6/\underline{m}1'$, $6'/\underline{m}'$, $6/\underline{m}'$, $6'/\underline{m}$

24. Family $\mathbf{622}$

    $6\underline{22}$, $6\underline{22}1'$, $6'\underline{22}'$, $6\underline{2}'\underline{2}'$

25. Family $\mathbf{6mm}$

    $6mm$, $6mm1'$, $6'mm'$, $6m'm'$

26. Family $\mathbf{\bar{6}m2}$

    $\underline{\bar{6}}m\underline{2}$, $\underline{\bar{6}}m\underline{2}1'$, $\underline{\bar{6}}'m'\underline{2}$, $\underline{\bar{6}}'m\underline{2}'$, $\underline{\bar{6}}m'\underline{2}'$

27. Family $\mathbf{6/mmm}$

    $6/\underline{m2}/m\underline{2}/m$, $6/\underline{m2}/m\underline{2}/m1'$, $6'/\underline{m}'\underline{2}/m\underline{2}'/m'$, $6/\underline{m2}'/m'\underline{2}'/m'$,
    $6/\underline{m}'\underline{2}/m'\underline{2}/m'$, $6/\underline{m}'\underline{2}'/m\underline{2}'/m$, $6'/\underline{m2}'/m\underline{2}/m'$

If the points on the two sides of the plane are coloured white (upper side) and black (lower side) then the one-sided operations (side-preserving) and two-sided operations (side-exchanging) are colour-preserving and colour-exchanging, respectively. The layer groups are isomorphous to black-white groups [7,29]. According to the previous division, the one-sided layer groups are called white, two-sided layer groups with reflection $\underline{m}$ are called gray (a white point on the upper side always occurs with a black point on the lower side at the same location) and two-sided layer groups without mirror reflection are the "proper black-and-white" groups.

The *sectional layer group* for a given point group G and for a given bisecting plane is the group (subgroup of G) of all those elements of the given group which leave the given plane invariant. In accordance with this definition it is a layer group depending on the given group G and on the sectional plane. When a sectional plane belongs to the set of planes determined by Miller (or Bravis-Miller) indices (hkl) (or $(hkil)$) then the sectional layer group will be denoted as $\overline{G}(hkl)$, or only $\overline{G}$ when the sectional plane (hkl) is clear from context.

Appendix B presents tables of generators of 7 crystallographic holohedral point groups showing to which type they belong (reversing "$\downarrow$" or not reversing "$\uparrow$" outer normal) with respect to the sectional plane (hkl). These tables cover all non-equivalent *plane orbits* $\{hkl\}$, (or *orientational orbit* [19]), each of them is represented by one sectional plane (hkl). A plane orbit $\{hkl\}$ contains all planes which are generated by the point group G acting on the plane (hkl). Non-equivalent plane orbits correspond to symmetrically non-equivalent crystal faces used in crystal morphology [6,7].

Then it is possible to determine holohedral sectional layer groups $\overline{L}_H$ for each crystal system and non-equivalent sectional planes. Results are presented in Tables 2.1, 2.2, 2.3, 2.4, 2.5, 2.6 and 2.7 where each plane orbit is again represented only by one sectional plane (hkl).

The sectional layer group $\overline{G}(hkl)$ of the point group G for the given sectional plane (hkl) is then the intersection of the concrete point group G and the holohedral sectional layer group $\overline{L}_H(hkl)$ of the same crystal system

$$\overline{G}(hkl) = G \cap \overline{L}_H(hkl). \tag{2.12}$$

Example:

a) $G = 4_z 2_x 2_{xy}, (h0l) \;\Rightarrow\; \overline{G}(h0l) = \{4_z 2_x 2_{xy}\} \cap \{\underline{2}_y/m_y\} = \underline{2}_y$,

b) $G = 4_z/m_z, (h0l) \;\Rightarrow\; \overline{G}(h0l) = \{4_z/m_z\} \cap \{\underline{2}_y/m_y\} = \underline{\bar{1}}$.

The complete list of the sectional layer groups for all 32 crystallographic point groups can be found in Appendix E as a special part of the sectional layer groups of 122 magnetic point groups.

Table 2.1: **Triclinic system**

(hkl)	$\overline{L}_H$
(hkl)	$\underline{\bar{1}}$

Table 2.2: **Monoclinic system**

(hkl)	$\overline{L}_H$
(001)	$2_z/\underline{m}_z$
$(hk0)$	$\underline{2}_z/m_z$
(hkl)	$\underline{\bar{1}}$

Table 2.3: **Orthorhombic system**

(hkl)	$\overline{L}_H$
(001)	$\underline{2}_x/m_x\underline{2}_y/m_y2_z/\underline{m}_z$
(010)	$\underline{2}_x/m_x2_y/\underline{m}_y\underline{2}_z/m_z$
(100)	$2_x/\underline{m}_x\underline{2}_y/m_y\underline{2}_z/m_z$
$(hk0)$	$\underline{2}_z/m_z$
$(h0l)$	$\underline{2}_y/m_y$
$(0kl)$	$\underline{2}_x/m_x$
(hkl)	$\underline{\bar{1}}$

Table 2.4: **Tetragonal system**

(hkl)	$\overline{L}_H$
(001)	$4_z/\underline{m}_z\underline{2}_x/m_x\underline{2}_{xy}/m_{xy}$
(100)	$2_x/\underline{m}_x\underline{2}_y/m_y\underline{2}_z/m_z$
(110)	$2_{xy}/\underline{m}_{xy}\underline{2}_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m_z$
$(hk0)$	$\underline{2}_z/m_z$
(hhl)	$\underline{2}_{x\bar{y}}/m_{x\bar{y}}$
$(h0l)$	$\underline{2}_y/m_y$
(hkl)	$\underline{\bar{1}}$

Table 2.5: **Trigonal system**

(hkl)	$\overline{L}_H$
(0001)	$\underline{\bar{3}}_z\underline{2}_{10}/m_{2\bar{1}}$
$(2\bar{1}\bar{1}0)$	$2_{10}/\underline{m}_{2\bar{1}}$
$(0\bar{1}0)$	$\underline{2}_{10}/m_{2\bar{1}}$
$(2h\bar{h}\bar{h})$	$\underline{\bar{1}}$
$(0h\bar{h}l)$	$\underline{2}_{10}/m_{2\bar{1}}$
$(hki0)$	$\underline{\bar{1}}$
$(hkil)$	$\underline{\bar{1}}$

Table 2.6: **Hexagonal system**

(hkl)	$\overline{L}_H$
(0001)	$6_z/\underline{m}_z\underline{2}_{10}/m_{2\bar{1}}\underline{2}_{12}/m_{01}$
$(2\bar{1}\bar{1}0)$	$2_{10}/\underline{m}_{2\bar{1}}\underline{2}_{12}/m_{01}\underline{2}_z/m_z$
$(0\bar{1}0)$	$\underline{2}_{10}/m_{2\bar{1}}2_{12}/\underline{m}_{01}\underline{2}_z/m_z$
$(2h\bar{h}\bar{h})$	$\underline{2}_{12}/m_{01}$
$(0h\bar{h}l)$	$\underline{2}_{10}/m_{2\bar{1}}$
$(hki0)$	$\underline{2}_z/m_z$
$(hkil)$	$\underline{\bar{1}}$

Table 2.7: **Cubic system**

(hkl)	$\overline{L}_H$
(001)	$4_z/\underline{m}_z\underline{2}_x/m_x\underline{2}_{xy}/m_{xy}$
(110)	$2_{xy}/\underline{m}_{xy}\underline{2}_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m_z$
$(hk0)$	$\underline{2}_z/m_z$
(hhl)	$\underline{2}_{x\bar{y}}/m_{x\bar{y}}$
(hkl)	$\underline{\bar{1}}$
(111)	$\underline{\bar{3}}_p\underline{2}_{x\bar{y}}/m_{x\bar{y}}$

The *sectional magnetic layer groups* for the magnetic point groups of the II and III type can be derived by the assistance of crystallographic sectional layer groups as the time inversion $1'$ has no influence on the orientation of the outer normal. For the given section plane and within the given magnetic family the sectional layer groups of the second type differ from that of the first type only by redouble of generators as a result of the direct product with $\{1, 1'\}$, and in the sectional layer groups of the third type some classical symmetry operations are formally replaced by primed operations in dependence on concrete initial magnetic point group.

The tables of the magnetic sectional layer groups are presented in Appendix E, each plane orbit is again represented by one sectional plane (hkl).

2.4 Domain states and domain pairs

A *domain* D_i is a region with homogeneous distorted structure. A crystalline domains can arise in phase transition from a *high-symmetry (parent, disordered) phase* of symmetry G to a *low-symmetry (distorted, ordered) phase* of symmetry F which is a subgroup of G.

A *domain state* S_i is the bulk structure (extended into the entire space) of a possible domain in a domain structure. A domain D_i is then defined by the domain state S_i and by the connected region Ω_i to which the structure of S_i is confined in a real domain structure.

The term *single domain state* S_i is used for a domain state in a special case when the crystal in the distorted phase consists of one domain only (the region Ω_i covers the whole crystal). The basic symmetry properties of single domain states follow from symmetry groups G and F of the parent and distorted phases,resp. The number n of single domain states can be calculated from a formula

$$n = |G| : |F|, \tag{2.13}$$

where $|G|$ and $|F|$ are the orders of the point groups G and F [10,11,12,15].

A *domain pair* is a set of two domain states S_i and S_j that are treated irrespectively of their coexistence [8,30].

In an *ordered domain pair* (S_i, S_j) the *transposed domain pair* (S_j, S_i) is not identical with the original domain pair (S_i, S_j) unless $i = j$ (trivial domain pair).

In an *unordered domain pair* $\{S_i, S_j\}$ the identity $\{S_i, S_j\} = \{S_j, S_i\}$ holds for all i and j.

If F_i and F_j are the symmetry groups of S_i and S_j, respectively,
(i.e. $F_iS_i = S_i, F_jS_j = S_j$) then any operation f that belongs both to F_i and to F_j, $f \in F_i \cap F_j \equiv F_{ij}$, is a symmetry operation of the unordered domain pair $\{S_i, S_j\}$, $f\{S_i, S_j\} = \{fS_i, fS_j\} = \{S_i, S_j\}$.

If, moreover, there exists such $g^*_{ij} \in G$ which transposes (interexchanges) S_i and S_j, i.e. $g^*_{ij}S_i = S_j, g^*_{ij}S_j = S_i$, then all operations from the left coset $g^*_{ij}F_{ij}$ do so as well. ($g^*_{ij}F_{ij}S_i = g^*_{ij}S_i = S_j$ and $g^*_{ij}F_{ij}S_j = g^*_{ij}S_j = S_i$.) These operations are also symmetry operations of $\{S_i, S_j\}$ since for unordered domain pair $\{S_i, S_j\} = \{S_j, S_i\}$. Thus *symmetry group* J_{ij} *of the unordered domain pair* $\{S_i, S_j\}$ can be, in a general case, expressed in the following way:

$$J_{ij} = F_{ij} + g^*_{ij}F_{ij}. \tag{2.14}$$

Taking into account this equation, the symmetry group J_{ij} of the domain pair $\{S_i, S_j\}$ can be formally treated as a *dichromatic* (black and white) group. This colouring changes the unordered domain pair $\{S_i, S_j\}$ into an ordered domain state (S_i, S_j) in which the first domain state is taken as black and the second one as white. The change of order of S_i and S_j changes the ordered domain pair (S_i, S_j) into another (S_j, S_i) , $((S_i, S_j) \neq (S_j, S_i), i \neq j)$, it means that a "black and white " domain pair is changed into a "white and black" one identical with a transposed domain pair.

Operations $f \in F_{ij}$ leaving both domain states unchanged can be treated as *colour-preserving* (or *state-preserving*) operations and the group F_{ij} as the symmetry group of the ordered domain pair (S_i, S_j).

Operations g^*_{ij} reversig S_i into S_j and S_j into S_i can be treated as *colour-changing* (or*state-changing*) operations which transform the ordered domain pair (S_i, S_j) into the transposed domain pair (S_j, S_i). For such operations the auxiliary notation (with asterisk) will be used in this report. In [12,14,16] these operations were "primed".

Domain pairs can be classified according to their symmetry J_{ij}. Pairs for which g^*_{ij} exists are called *transposable* (or *ambivalent*) *domain pairs*, pairs for which an interexchanging operation g^*_{ij} cannot be found are called *non-transposable* (or *polar*) *domain pairs*. The symmetry of a non-transposable pair is reduced to F_{ij}.

It was mentioned that all operations that transforms S_i into S_j are contained in the left coset $g^*_{ij}F_i$ since $g^*_{ij}F_iS_i = g^*_{ij}S_i = S_j$. If $\{S_i, S_j\}$ is a transposable pair and, moreover, $F_i = F_j = F_{ij}$ then all operations of the left coset $g^*_{ij}F_i$ transform simultaneously S_j into S_i. Such pairs are called *completely transposable domain* *pairs.* The symmetry group J_{ij} of a completely transposable pair $\{S_i, S_j\}$ is then

$$J_{ij} = F_i + g^*_{ij} F_i. \tag{2.15}$$

If $F_i \neq F_j$ then $F_{ij} < F_i$ and the number of transposing operations of a transposable domain pair is smaller than the number of operations transforming S_i into S_j. These domain pairs are, therefore, called *partially transposable domain pairs.*

Depending on the spontaneous deformation $e^{(i)}$ and $e^{(j)}$ of S_i and S_j, resp., domain pairs can be divided into two types:

(1) A *non-ferroelastic* domain pairs for which $e^{(i)} = e^{(j)}$, i.e. the domain states S_i and S_j have the same spontaneous deformation, and

(2) a *ferroelastic* domain pairs for which $e^{(i)} \neq e^{(j)}$, i.e. the domain states S_i and S_j have different spontaneous deformation .

For completely transposable domain pairs a simple criterion holds: $\{S_i, S_j\}$ is ferroelastic iff $\mathrm{Fam}F_i \in \mathrm{Fam}J_{ij}$, where the symbol Fam denotes the crystal family of a point group [4].(Crystal families represent six categories as both trigonal and haxagonal crystal systems belong to the hexagonal family.)

There are 15 groups of the form that satisfy $\mathrm{Fam}F_i \in \mathrm{Fam}J_{ij}$ and $F_i = F_j$, i.e. 15 ferroelastic completely transposable non-magnetic domain pairs [16].

Table 2.8: **Symmetries of 15 ferroelastic completely transposable domain pairs**

$F_i = F_j$	1	1	$\bar{1}$	2	2	m	$2/m$
J_{ij}	2^*	m^*	$2^*/m^*$	2^*2^*2	m^*m^*2	m^*2^*m	m^*m^*m

$F_i = F_j$	2	2	$2/m$	222	$mm2$	$mm2$	222	mmm
J_{ij}	4^*	$\bar{4}^*$	$4^*/m$	4^*22^*	4^*mm^*	$\bar{4}^*2^*m$	$\bar{4}^*m^*2$	$4^*/mmm^*$

There exist 48 groups J_{ij} describing symmetries of non-magnetic non-ferroelastic domain pairs [14]. All non-ferroelastic domain pairs are completely transposable.

Table 2.9: **Symmetries of 48 non-ferroelastic non-magnetic domain pairs**

F	J_{ij}	F	J_{ij}	F	J_{ij}	F	J_{ij}
1	$\bar{1}^*$	422	$4/m^*m^*m^*$	3	6^*	$\bar{6}$	$\bar{6}m^*2^*$
2	$2/m^*$	$4mm$	$4/m^*mm$	3	$\bar{6}^*$	$\bar{3}$	$6^*/m^*mm^*$
m	$2^*/m$	$\bar{4}2m$	$4/m^*m^*m$	$\bar{3}$	$6^*/m^*$	$6/m$	$6/mm^*m^*$
222	$m^*m^*m^*$	3	$\bar{3}^*$	6	$6/m^*$	622	$6/m^*m^*m^*$
$mm2$	mmm^*	3	32^*1	$\bar{6}$	$6^*/m$	$6mm$	$6/m^*mm$
4	$4/m^*$	3	312^*	32	6^*22^*	$\bar{6}2m$	$6^*/m^*mm$
$\bar{4}$	$4^*/m^*$	3	$3m^*1$	6	62^*2^*	23	$m^*\bar{3}$
4	42^*2^*	3	$31m^*$	$3m$	6^*mm^*	23	4^*32^*
4	$4m^*m^*$	$\bar{3}$	$\bar{3}m^*1$	6	$6m^*m^*$	23	$\bar{4}^*3m^*$
$\bar{4}$	$\bar{4}2^*m^*$	$\bar{3}$	$\bar{3}1m^*$	32	$\bar{6}^*2m^*$	$m\bar{3}$	$m\bar{3}m^*$
$\bar{4}$	$\bar{4}m^*2^*$	32	$\bar{3}^*m^*$	$\bar{6}$	$\bar{6}^*2^*m$	432	$m^*\bar{3}m^*$
$4/m$	$4/mm^*m^*$	$3m$	$\bar{3}^*m$	$3m$	$\bar{6}^*m2^*$	$\bar{4}3m$	$m^*\bar{3}m$

Complete tables of 380 completely transposable magnetic domain pairs are presented in [21].

Chapter 3

Domain walls in non-magnetic domain pairs

3.1 Wall symmetry and sectional layer groups

A planar domain wall, as a thin transient region connecting two domains with domain states S_1 and S_2, can be considered as a layer lying between them. If the wall is parralel to the plane determined by Miller indices (hkl), the symbol $[S_1(hkl)S_2]$ will be used for such a wall. Because the planar wall can be also determined by its normal $\mathbf{n}$, the wall can be also represented by an equivalent symbol $[S_1(\mathbf{n})S_2]$. In case the direction of $\mathbf{n}$ of the wall is not essential or it is known from the context, both preceding symbols will be shortened to $[S_1|S_2]$.

The symmetry properties of the wall $[S_1(\mathbf{n})S_2]$ are described by a layer group. Let it be $T[S_1(hkl)S_2] = T[S_1|S_2] = T_{12}(hkl) = T_{12}(\mathbf{n}) = T_{12}$, expressing that the planar wall connects domain states S_1 and S_2 and is parallel to the plane (hkl) or that it is perpendicular to the vector $\mathbf{n}$ (i.e. $\mathbf{n}$ is its normal).

An operation $u \in T_{12}(\mathbf{n})$ must fulfill two necessary conditions:

1. Being an operation of a layer group, u must either keep normal to the layer $\mathbf{n}$ unchanged or invert it into the opposite one $-\mathbf{n}$. Operations of the latter type are underlined according to the convention introduced in Section 2.2 .

2. Being an operation of the symmetry group J_{12} of the domain pair $\{S_1, S_2\}$, u must either keep both states S_1 and S_2 unchanged or exchange them, i.e. $uS_1 = S_2$ and $uS_2 = S_1$. The latter one, *state exchanging operations*, will be

denoted by an asterisk (u^*) according to the convention introduced in Section 2.3.

These two conditions can be fulfilled in four different ways each of which specifies how an operation u transforms both the normal $\mathbf{n}$ and domain states S_1 and S_2. Thus we can distinguish:

1. *Side-preserving operations*:

   (a) An operation $u = f_{12}$ that neither changes the normal $\mathbf{n}$ nor the domain states S_1 and S_2. It obviously keeps the wall $[S_1|S_2]$ unchanged. Such operations are called *trivial symmetry operations of a domain wall* [25].

   (b) An operation $u = r_{12}^*$ that exchanges domain states S_1 and S_2 but does not invert the normal $\mathbf{n}$, i.e. it keeps the half-spaces on both sides of the wall in their initial positions. It transforms the initial domain wall into a *reversed wall* $[S_2|S_1]$ with respect to the initial wall $[S_1|S_2]$ with S_1 and S_2 on the opposite sides.

2. *Side-reversing operations*:

   (a) An operation $u = \underline{s}_{12}$ that inverts the normal $\mathbf{n}$ and thus exchanges the half-spaces on the left and right sides of the wall. Since these half-spaces are occupied by domain states S_1 and S_2 this exchange of half-spaces is accompanied by an exchange of domain states on both sides of the wall. The operation $\underline{s}_{12}$ thus transforms the initial wall into a *reversed wall* $[S_2|S_1]$.

   (b) An operation $u = \underline{t}_{12}^*$ that exchanges the half-spaces (inverts the normal $\mathbf{n}$) and independently exchanges domain states S_1 and S_2. It means it transforms the wall into itself. This operation $\underline{t}_{12}^*$ is, therefore, a symmetry operation of the wall. Such operations are called *non-trivial symmetry operations of a domain wall.*

Table 3.1 summarizes the action of these four types of the layer operations on the normal $\mathbf{n}$, on both domain states S_1 and S_2, and on the domain wall $[S_1(\mathbf{n})S_2]$.

Table 3.1: **Action of the four types of an operation u on a domain wall**

u	$u\mathbf{n}$	uS_1	uS_2	$u[S_1(\mathbf{n})S_2]$	*wall*
f_{12}	$\mathbf{n}$	S_1	S_2	$[S_1(\mathbf{n})S_2] \equiv [S_2(-\mathbf{n})S_1]$	*initial wall*
$r^{\star}_{12}$	$\mathbf{n}$	S_2	S_1	$[S_2(\mathbf{n})S_1] \equiv [S_1(-\mathbf{n})S_2]$	*reversed wall*
$\underline{s}_{12}$	$-\mathbf{n}$	S_1	S_2	$[S_1(-\mathbf{n})S_2] \equiv [S_2(\mathbf{n})S_1]$	*reversed wall*
$\underline{t}^{\star}_{12}$	$-\mathbf{n}$	S_2	S_1	$[S_2(-\mathbf{n})S_1] \equiv [S_1(\mathbf{n})S_2]$	*initial wall*

Then the layer group $T_{12}(hkl)$ that describes the symmetry of the wall $[S_1(hkl)S_2]$ consists of all trivial and non-trivial symmetry operations of the wall.

Further we shall confine our consideration to domain walls that arise from completely transposable domain pairs for which $F_1 = F_2$ holds, as mentioned in Chapter 3. This covers all non-ferroelastic domain pairs and only some, but not all, ferroelastic domain pairs.

The procedure that enables one to find the group $T_{12}(hkl)$, for a given completely transposable domain pair $\{S_1, S_2\}$ and the wall determinated by a plane (hkl), consists of the following steps [10,11,12,24,33]:

A) Find the sectional layer group of $F_1 = F_2$ along the plane (hkl). (This sectional layer group can be considered as the symmetry grup of a trivial domain wall $[S_1|S_1]$ with the same domain states S_1 on both sides of the wall.) Its symmetry consists of all operations of the group F_1 that leave invariant the plane (hkl) bisecting the domain state S_1 with symmetry F_1. This sectional layer group will be denoted $\overline{F}_1(hkl)$, or symply $\overline{F}_1$.

The sectional layer group F_1 must contain all trivial symmetry operations f_{12} of the domain wall. The set of all these operations forms a one-sided layer group $\widehat{F}_1$ which is a subgroup of the general sectional layer group $\overline{F}_1$, $\widehat{F}_1 \leq \overline{F}_1$ [11].

If $\underline{s}_{12}$ is a side-reversing operation of $\overline{F}_1$ then the left coset $\underline{s}_{12}\widehat{F}_1$ comprises all such side-reversing operations and the sectional layer group $\overline{F}_1$ can be expressed in the form

$$\overline{F}_1 = \widehat{F}_1 + \underline{s}_{12}\widehat{F}_1. \tag{3.1}$$

Thus the trivial part $\widehat{F}_1$ of the wall symmetry can be deduced from the sectional layer group $\overline{F}_1$ as its halving one-sided subgroup.

B) Determine the symmetry group J_{12} of the non-ordered domain pair $\{S_1, S_2\}$ as

$$J_{12} = F_1 + j_{12}^{\star} F_1, \tag{3.2}$$

where F_1 is the symmetry of the domain states S_1 and S_2, and $j_{12}^{\star}$ an operation exchanging these states, $j_{12}^{\star} S_1 = S_2$ and $j_{12}^{\star} S_2 = S_1$. It was shown that the sectional layer group $\overline{J}_{12}$ of J_{12} along the plane (hkl) has the form [11,12]

$$\overline{J}_{12} = \widehat{F}_1 + \underline{t}_{12}^{\star} \widehat{F}_1 + r_{12}^{\star} \widehat{F}_1 + \underline{s}_{12} \widehat{F}_1, \tag{3.3}$$

where the operations $\underline{t}_{12}^{\star}, r_{12}^{\star}$ and $\underline{s}_{12}$ are defined in Table 3.1., and the subgroup $\widehat{F}_1$ is the one-sided sectional layer group of F_1 determined in the preceding step. In this left cosets decomposition $\underline{t}_{12}^{\star} \widehat{F}_1$ contains all side&state reversing operations and $r_{12}^{\star} \widehat{F}_1$ all state reversing operations. Then the symmetry of the wall T_{12} can be derived from the sectional layer group $\overline{J}_{12}$ of the symmetry group J_{12} of the domain pair $\{S_1, S_2\}$ as its first two left cosets

$$T_{12} = \widehat{F}_1 + \underline{t}_{12}^{\star} \widehat{F}_1. \tag{3.4}$$

Thus the task of finding the symmetry group $T_{12}(hkl)$ of the domain wall $[S_1(hkl)S_2]$ is based on determination of the two sectional layer groups $\overline{F}_1(hkl)$ and $\overline{J}_{12}(hkl)$, and their halving subgroups $\widehat{F}_1(hkl)$ and $T_{12}(hkl)$, resp.

The first two left cosets in equation 3.3 assemble all operations that leave the domain wall invariant in two ways, not exchanging domain states or exchanging them. Accordingly, one can distinguish

- *symmetrical walls* for which $T_{12} > \widehat{F}_1$
- *asymmetrical walls* for which $T_{12} = \widehat{F}_1$.

The last two left cosets in the eq.(3.4) assemble all operations that transform the domain wall $[S_1(hkl)S_2]$ into the reversed wall $[S_2(hkl)S_1]$ with opposite order of the domain states. They enable to distinguish

- *reversible wall* for which $\overline{J}_{12} > T_{12}$

- *irreversible wall* for which $\overline{J}_{12} = T_{12}$.

Appendix C presents tables of sectional layer groups $\overline{J}_{12}$ of the symmetry group J_{12} for all 48 non-ferroelastic domain pairs listed in the Section 2.3, and a concrete sectional plane (hkl) as a representative of each crystallographical plane orbit $\{hkl\}$ associated with the point group J_{12}. (A plane orbit unlike a face orbit does not distinguish between two different orientations of the outer normal to the plane, e.g. in trigonal system there are two eguivalent face orbits $\{11\bar{2}0\}_f = [(11\bar{2}0), (\bar{2}110), (1\bar{2}10)]$ and $\{\bar{1}\bar{1}20\}_f = [(\bar{1}\bar{1}20), (2\bar{1}\bar{1}0), (\bar{1}2\bar{1}0)]$ which belong to the same plane orbit $\{11\bar{2}0\}$.) These tables presents:

1. Symmetry group F_1 of the domain state S_1.
2. Symmetry group J_{12} of the unordered domain pair $\{S_1, S_2\}$.
3. Side, state and side&state reversing operations $\underline{s}_{12}, r^*_{12}$ and $\underline{t}^*_{12}$.
4. One-sided sectional layer groups $\widehat{F}_1$ containing all trivial symmetry operations of the domain wall.
5. Sectional layer groups $\overline{F}_{12} = \widehat{F}_1 + \underline{s}_{12}\widehat{F}_1$ of the groupF_1.
6. One-sided sectional layer groups $\widehat{J}_{12} = \widehat{F}_1 + r^*_{12}\widehat{F}_1$ of the group J_{12}.
7. Wall symmetry groups $T_{12} = \widehat{F}_1 + \underline{t}^*_{12}\widehat{F}_1$.
8. The complete sectional layer group $\overline{J}_{12}$ defined by the equation (3.3).

These tables enable one to determine (a)symmetrical and (ir)reversible walls. Explicite results are presented in Appendix A.

Table 3.2 summarizes number (given in parentheses) of the non-equivalent walls according to their classification and number of the non-empty left cosets in the groups T_{12} and J_{12}.

Table 3.2: **Number of the left cosets (l.c.) and corresponding types of domain walls.**
R = reversible I = irreversible S = symmetrical A = asymmetrical
(n) = number of non-equivalent walls of the given type

$$
\begin{tikzcd}[column sep=small]
\overline{J}_{12}: & 4\ l.c. \arrow[dr, dash] & & & & 2\ l.c. \arrow[dl, dash] \arrow[dr, dash] & & & & 1\ l.c. \arrow[dl, dash] \\
\text{wall:} & & S, R(112) \arrow[dr, dash] & & S, I(84) \arrow[dl, dash] & & A, R(56) \arrow[dr, dash] & & A, I(32) \arrow[dl, dash] & \\
T_{12}: & & & 2\ l.c. \arrow[dr, dash] & & & & 1\ l.c. \arrow[dl, dash] & & \\
\text{wall:} & & & & S(196) \arrow[dr, dash] & & A(88) \arrow[dl, dash] & & & \\
\hat{F}_1: & & & & & 1\ l.c. & & & &
\end{tikzcd}
$$

3.2 Geometrical approach to the wall symmetry

The property *"symmetry-asymmetry"* and *"reversibility-irreversibility"* of a domain wall is closely related to a geometrical representation of groups describing a domain pair, F_1 and J_{12} .

By application of all the symmetry operations of a group G on assigned plane $\rho = (hkl)$ we get a set of equivalent planes $\{\rho, G\}_f = \{\rho\}_f$, a *face orbit.* (If A is a subgroup of B, then $\{\rho, A\}_f \subseteq \{\rho, B\}_f$). These planes generate a convex polyhedron (open or closed) which is called *simple form*, $\mathbf{sf}(G, \rho)$ [5,30,31].

Because of the structure, the high symmetry group $J_{12} = F_1 + g^*_{12}F_1$ can be treated as a dichromatic group (e.g. black and white). Then a simple form associated with a group J_{12} can be decomposed into two geometrically equal polyhedra which represent a simple form $\mathbf{sf}(F_1, \rho)$ associated with the left coset F_1 (faces are hatched horizontally $\mapsto$ white colour) and a simple form-like polyhedron $\mathbf{sf}^\star(g^*_{12}F_1, \rho)$ associated with the second left coset $g^*_{12}F_1$ (faces are hatched vertically $\mapsto$ black colour), see Fig. 3.1. The notation $\mathbf{sf}^\star(g^*_{12}F_1, \rho)$ was used in order to express the fact that $g^*_{12}F_1$ is not a group.

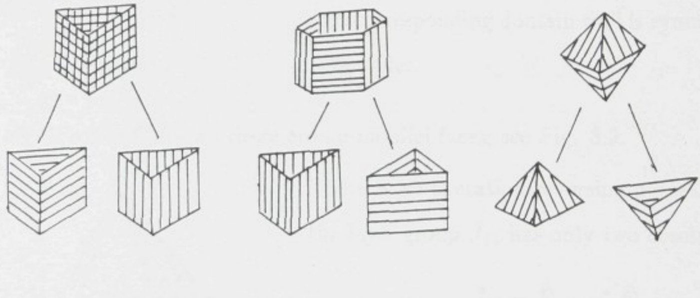
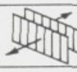


Figure 3.1: Decomposition of $\mathbf{sf}(J_{12}, \rho)$ into $\mathbf{sf}(F_1, \rho)$ and $\mathbf{sf}^\star(g^*_{12}F_1, \rho)$.

If a plane which determines the sectional layer group $\overline{J}_{12}$ is parallel to one face of the simple form associated with J_{12}, then using geometrical symmetry of the corresponding polyhedron and colour symmetry of its faces we can determine very easily which left coset of $\overline{J}_{12}$ is (non)empty, see Table 3.3.

Table 3.3: **Left cosets of $\overline{J}_{12}$ and corresponding types of faces.**
Arrow-heads represent outer normals.

left coset	corresponding faces	
$\widehat{F}_1$	single, one-colour face	 or 
$r^{*}_{12}\widehat{F}_1$	single, two-colour face	
$\underline{s}_{12}\widehat{F}_1$	two parallel faces having the same colour	 or 
$\underline{t}^{*}_{12}\widehat{F}_1$	two parallel faces with different colours	

Comparing simple forms associated with symmetry groups F_1 and J_{1j}, we can distinguish five different cases.

(1) $\mathbf{sf}(F_1,\rho) = \mathbf{sf}(J_{12},\rho)$, there are parallel faces; see Fig. 3.2.

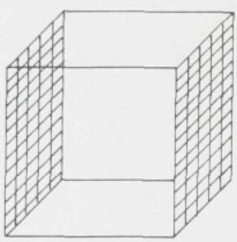


Figure 3.2:

We can see that there must be present operations reversing normals and keeping colours ($\underline{s}$), reversing normals and changing colours ($\underline{t}^{*}$), and changing colours and keeping normals (r^{*}). The layer group $\overline{J}_{12}$ has four nonempty left cosets and therefore corresponding domain wall is symmetrical and reversible.

(2) $\mathbf{sf}(F_1,\rho) = \mathbf{sf}(J_{12},\rho)$, there are no parallel faces; see Fig. 3.3.

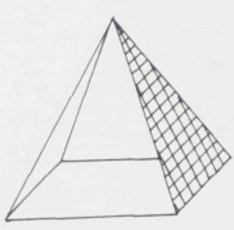


Figure 3.3:

There is no operation reversing normals, it means the layer group $\overline{J}_{12}$ has only two cosets

$$\overline{J}_{12} = \widehat{F}_1 + r^{*}_{12}\widehat{F}_1, \tag{3.5}$$

corresponding domain wall is asymmetrical but reversible.

(3) $\mathbf{sf}(F_1, \rho) \subset \mathbf{sf}(J_{12}, \rho)$, $\mathbf{sf}(J_{12}, \rho)$ has parallel faces; see Fig. 3.4.

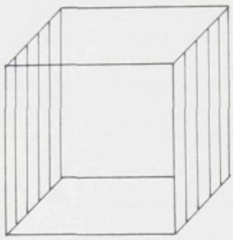


Figure 3.4:

There are only operations reversing normals and leaving colours unchanged ($\underline{s}$). The layer group $\overline{J}_{12}$ has only two left cosets,

$$\overline{J}_{12} = \widehat{F}_1 + \underline{s}_{12}\widehat{F}_1. \tag{3.6}$$

Corresponding domain wall is asymmetrical and reversible.

(4) $\mathbf{sf}(F_1, \rho) \subset \mathbf{sf}(J_{12}, \rho)$, $\mathbf{sf}(J_{12}, \rho)$ has parallel faces, $\mathbf{sf}(F_1, \rho)$ has not; see Fig. 3.5.

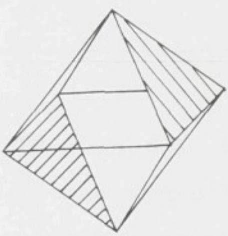


Figure 3.5:

We can find only operations reversing both normals and colours ($\underline{t}^*$). The layer group has again only two left cosets,

$$\overline{J}_{1j} = \widehat{F}_1 + \underline{t}^*_{1j}\widehat{F}_1, \tag{3.7}$$

and corresponding domain wall is symmetrical and irreversible.

(5) $\mathbf{sf}(F_1, \rho) \subset \mathbf{sf}(J_{12}, \rho)$, $\mathbf{sf}(J_{12}, \rho)$ has no parallel faces; see Fig. 3.6.

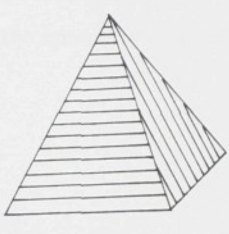


Figure 3.6:

In this case there are no $\underline{s}, r^*$ and $\underline{t}^*$ operations, the layer group $\overline{J}_{12}$ can not be decomposed into left cosets,

$$\overline{J}_{1j} = \widehat{F}_1, \tag{3.8}$$

and corresponding domain wall is asymmetrical and irreversible.

These results are summarized in Table 3.4.

Table 3.4: **Dependence of domain wall symmetry on transition $\mathbf{sf}(F_1, \rho) \to \mathbf{sf}(J_{12}, \rho)$.** $\#$ - ***sf*** has parallel faces.

number of faces in $\mathbf{sf}(F_1, \rho)$	number of faces in $\mathbf{sf}(J_{12}, \rho)$	left cosets in $\overline{J}_{12}$	symmetry of a domain wall
n	$2n$	$\widehat{F}_1$	asymmetrical irreversible
	n	$\widehat{F}_1 + r^*_{12}\widehat{F}_1$	asymmetrical reversible
	$2n\#$	$\widehat{F}_1 + \underline{t}^*_{12}\widehat{F}_1$	symmetrical irreversible
$n\#$	$2n\#$	$\widehat{F}_1 + \underline{s}_{12}\widehat{F}_1$	asymmetrical reversible
	$n\#$	$\widehat{F}_1 + \underline{s}_{12}\widehat{F}_1 + r^*_{12}\widehat{F}_1 + \underline{t}^*_{12}\widehat{F}_1$	symmetrical reversible

Tables 3.5, 3.6, 3.7, 3.8, 3.9, 3.10 and 3.11 present number of equivalent faces on a polyhedron representing a simple form. Tables cover all 32 point groups and all plane orbits $\{hkl\}$ associated with a group G. Domain walls are parallel to individual face of a simple form representing the given face orbit [14]. The symbol "#" is used again when a simple form has parallel faces.

Results for all 48 non-ferroelastic domain pairs are presented in Appendix D. They are identical with results obtained by the analysis of a sectional layer group $\overline{J}_{12}$ of the symmetry group J_{12} describing a domain pair $\{S_1, S_2\}$.

Table 3.5: **Triclinic system**

$\{hkl\}$	1	$\bar{1}$
$\{hkl\}$	1	1

Table 3.6: **Monoclinic system**

$\{hkl\}$	2	m	$2/m$
$\{001\}$	1	2#	2#
$\{hk0\}$	2#	1	2#
$\{hkl\}$	2	2	4#

Table 3.7: **Orthorhombic system**

$\{hkl\}$	222	$mm2$	mmm
$\{001\}$	2#	1	2#
$\{010\}$	2#	2#	2#
$\{100\}$	2#	2#	2#
$\{hk0\}$	4#	4#	4#
$\{h0l\}$	4#	2	4#
$\{0kl\}$	4#	2	4#
$\{hkl\}$	4	4	8#

Table 3.8: **Tetragonal system**

$\{hkl\}$	4	$\bar{4}$	$4/m$	422	$4mm$	$\bar{4}2m$	$\bar{4}m2$	$4/mmm$
$\{001\}$	1	2#	2#	2#	1	2#	2#	2#
$\{100\}$	4#	4#	4#	4#	4#	4#	4#	4#
$\{110\}$	4#	4#	4#	4#	4#	4#	4#	4#
$\{hk0\}$	4#	4#	4#	8#	8#	8#	8#	8#
$\{h0l\}$	4	4	8#	8#	4	8#	4	8#
$\{hhl\}$	4	4	8#	8#	4	4	8#	8#
$\{hkl\}$	4	4	8#	8	8	8	8	16#

Table 3.9: **Trigonal system**

$\{hkl\}$	3	$\bar{3}$	321	312	$31m$	$3m1$	$\bar{3}m1$	$\bar{3}1m$
$\{0001\}$	1	2#	2#	2#	1	1	2#	2#
$\{2\bar{1}\bar{1}0\}$	3	6#	3	6#	3	6#	6#	6#
$\{01\bar{1}0\}$	3	6#	6#	3	6#	3	6#	6#
$\{2h\bar{h}\bar{h}l\}$	3	6#	6	6#	3	6	12#	6#
$\{0h\bar{h}l\}$	3	6#	6#	6	6	3	6#	12#
$\{hki0\}$	3	6#	6	6	6	6	12#	12#
$\{hkil\}$	3	6#	6	6	6	6	12#	12#

Table 3.10: **Hexagonal system**

$\{hkl\}$	6	$\bar{6}$	$6/m$	622	$6mm$	$\bar{6}m2$	$\bar{6}2m$	$6/mmm$
$\{0001\}$	1	2#	2#	2#	1	2#	2#	2#
$\{2\bar{1}\bar{1}0\}$	6#	3	6#	6#	6#	6#	3	6#
$\{01\bar{1}0\}$	6#	3	6#	6#	6#	3	6#	6#
$\{2h\bar{h}\bar{h}l\}$	6	6	12#	12#	6	12#	6	12#
$\{0h\bar{h}l\}$	6	6	12#	12#	6	6	12#	12#
$\{hki0\}$	6#	3	6#	12#	12#	6	6	12#
$\{hkil\}$	6	6	12#	12	12	12	12	24#

Table 3.11: **Cubic system**

$\{hkl\}$	23	$m\bar{3}$	432	$\bar{4}3m$	$m\bar{3}m$
$\{001\}$	6#	6#	6#	6#	6#
$\{110\}$	12#	12#	12#	12#	12#
$\{hk0\}$	12#	12#	24#	24#	24#
$\{hhl\}$	12	24#	24#	12	24#
$\{hkl\}$	12	24#	24	24	48#
$\{111\}$	4	8#	8#	4	8#

Chapter 4

Domain walls in magnetic completely transposable domain pairs

4.1 Point group symmetry of pyroelectric and pyromagnetic layers

In Section 1.3 existence of 122 magnetic point groups was mentioned. A complete list of 125 magnetic layer groups is presented there, where they are divided into 27 families. Another criterion of division takes into account presence or absence of the time inversion $1'$ in the layer group. In analogy with division of the magnetic point groups, it is possible to find:

- ⋆ 31 trivial magnetic layer groups, i.e. classical layer groups without the time inversion $1'$.
- ⋆ 31 non-magnetic layer groups which are direct product of classical point groups and the group consisting of identity and time inversion $\{1, 1'\}$.
- ⋆ 63 non-trivial magnetic layer groups containing some primed symmetry operations but not the time inversion $1'$.

For the next part of this report a special interest attract magnetoelectric groups (58 cases) [3,21] and those where exhibition of spontaneous magnetization and polarization is possible.

a) pyromagnetic groups (31 cases):

$1, 2, 2', m, m', m'm2', m'm'2, 3, 3m', 4, 4m'm', 6, 6m'm'$,

$2'2'2, 32', \bar{4}, 42'2', \bar{4}2'm', 62'2'$,

$\bar{1}, 2/m, 2'/m', m'm'm, \bar{3}, \bar{3}m', 4/m, 4/mm'm', \bar{6}, 6/m, 6/mm'm', \bar{6}m'2'$,

b) pyroelectric groups (31 cases):

$1, 2, 2', m, m', m'm2', m'm'2, 3, 3m', 4, 4m'm', 6, 6m'm'$,

$mm2, 3m, 4', 4mm, 4'm'm, 6mm$,

$1', 21', m1', mm21', 31', 3m1', 41', 4mm1', 61', 6', 6mm1', 6'm'm$.

From this total number of 125 magnetic layer groups only

- 65 are *magnetoelectric*
- 42 are *pyromagnetic*
- 42 are *pyroelectric.*

The intersection of the last two types is not empty, it covers 20 cases with simultaneous appearance of non-zero spontaneous polarization and magnetization.

Direction of the magnetization **M** and polarization **P** is, except the trivial triclinic symmetry $1, 1'$ and $\bar{1}$, partly or completely determined by a layer group. Next arrangement of 64 pyroelectric and pyromagnetic layer groups respects this predetermination [25]. There are *five* different positions of the normal to the layer and magnetization for pyromagnetic non-pyroelectric layers so as *five* different positions of the normal to the layer and polarization for pyroelectric non-pyromagnetic layers. In case of pyroelectric and pyromagnetic layers *eleven* different positions of magnetization **M**, polarization **P** and normal **n** to the layer can be distinguished. To make this division more transparent each possibility (except the trivial triclinic symmetry $1, 1'$ and $\bar{1}$) is accompanied by an illustrative figure. In all figures the central plane of the layer "w" is horizontal (hatched) and the outer normal is oriented upward.

1. Pyromagnetic ($\mathbf{M} \neq 0$) non-pyroelectric ($\mathbf{P} = 0$) layers.

1.1 Magnetization is completely determined by symmetry.

1.1.1 Magnetization is perpendicular to the layer ($\mathbf{M} \parallel \mathbf{n}$) for the layer groups:
$L = 2/\underline{m},\ \underline{2}'\underline{2}'2(me),\ m'm'\underline{m},\ \underline{\bar{4}}(me),\ 4\underline{2}'\underline{2}'(me),\ \underline{\bar{4}2}'m'(me),\ 4/\underline{m},$
$4/\underline{m}m'm', \underline{\bar{3}},\ 3\underline{2}'(me),\ \underline{\bar{3}}m',\ \underline{\bar{6}},\ 6/\underline{m},\ 6\underline{2}'\underline{2}'(me),\ \underline{\bar{6}}m'\underline{2}',\ 6/\underline{m}m'm'.$
See Fig. 4.1.

1.1.2 Magnetization is parallel to the layer ($\mathbf{M} \perp \mathbf{n}$) for the layer groups:
$L = \underline{2}/m,\ \underline{m}'m'm,\ \ (\mathbf{M} \parallel \underline{2}),\ \underline{22}'2'(me),\ \ (\underline{2} \parallel \mathbf{M} \perp\ m).$ See Fig. 4.2.

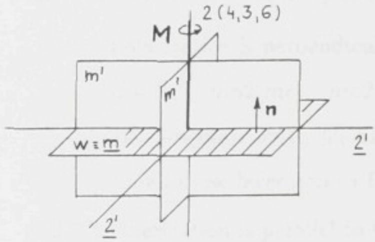


Figure 4.1:

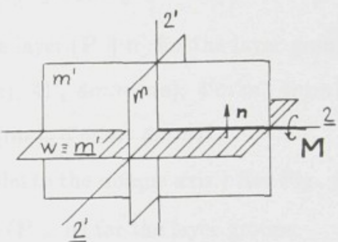


Figure 4.2:

1.2 Magnetization is partly determined by symmetry.

1.2.1 Magnetization is parallel to the layer ($\mathbf{M} \perp \mathbf{n}$) for the layer group:
$L = 2'/\underline{m}',\ (2' \perp \mathbf{M} \parallel \underline{m}').$ See Fig. 4.3.

1.2.2. Magnetization is confined to a plane perpendicular to the layer for the layer group:
$L = \underline{2}'/m'\ \ (\underline{2}' \perp \mathbf{M} \parallel m').$ See Fig. 4.4.

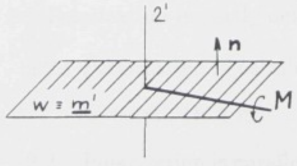


Figure 4.3:

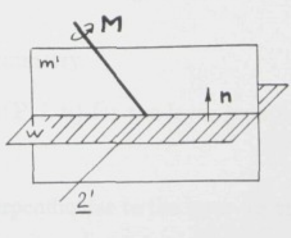


Figure 4.4:

1.3 Magnetization is not restricted by symmetry for the non-trivial triclinic layer group:

$L = \underline{\bar{1}}$.

The symbol (me) indicates that the group L is magnetoelectric, nevertheless, form of the magnetoelectric tensor is such that the polarization $\mathbf{P}$ equals zero.

2. Pyroelectric ($\mathbf{P} \neq 0$) non-pyromagnetic ($\mathbf{M} = 0$) layers.

2.1 Polarization is completely determined by symmetry.

2.1.1 Polarization is perpendicular to the layer ($\mathbf{P} \parallel \mathbf{n}$) for the layer groups:

$L = 21'$, $mm2(me)$, $mm21'$, $4'(me)$, $41'$, $4mm(me)$, $4'm'm$, $4mm1'$, $31'$, $3m(me)$, $3m1'$, $6'$, $61'$, $6mm(me)$, $6'm'm$, $6mm1'$.

(In all these layer groups $\mathbf{P}$ is parallel to the unique axis.) See Fig. 4.5.

2.1.2 Polarization is parallel to the layer ($\mathbf{P} \perp \mathbf{n}$) for the layer groups:

$L = \underline{2}1'$, $\underline{m}m\underline{2}(me)$, $\underline{m}m\underline{2}1'$, ($\mathbf{P} \parallel \underline{2}$). See Fig. 4.6.

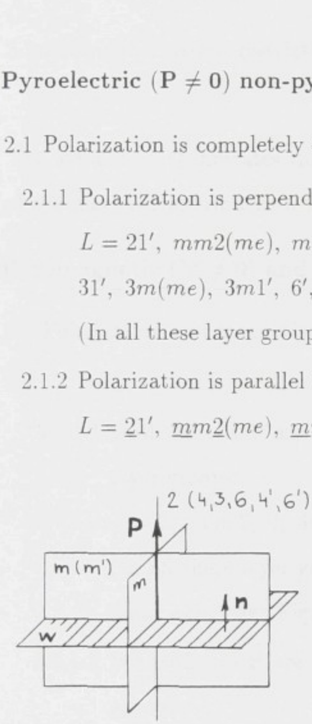


Figure 4.5:

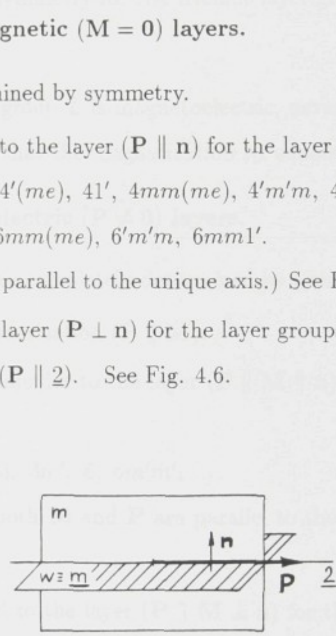


Figure 4.6:

2.2 Polarization is partly determined by symmetry.

2.2.1 Polarization is parallel to the layer ($\mathbf{P} \perp \mathbf{n}$) for the layer group:

$L = \underline{m}1'$, ($\mathbf{P} \parallel \underline{m}$). See Fig. 4.7.

2.2.2 Polarization is parallel to a plane perpendicular to the layer for the layer group:

$L = m1'$, ($\mathbf{P} \parallel m$). See Fig. 4.8.

Figure 4.7:

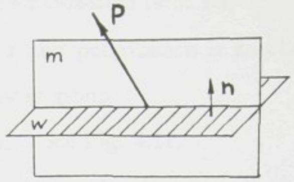


Figure 4.8:

2.3 Polarization is not restricted by symmetry for the triclinic layer group: $L = 1'$.

The symbol (me) indicates that the group L is magnetoelectric, nevertheless, form of the magnetoelectric tensor is such that the magnetization $\mathbf{M}$ equals zero.

3. Pyromagnetic $(\mathbf{M} \neq \mathbf{0})$ and pyroelectric $(\mathbf{P} \neq \mathbf{0})$ layers.

3.1 Polarization and magnetization are completely determined by symmetry.

3.1.1 Polarization is parallel to magnetization $(\mathbf{P} \parallel \mathbf{M})$.

3.1.1.1 Both $\mathbf{M}$ and $\mathbf{P}$ are perpendicular to the layer $(\mathbf{P} \parallel \mathbf{M} \parallel \mathbf{n})$ for the layer groups:

$L = 2,\ m'm'2,\ 4,\ 4m'm',\ 3,\ 3m',\ 6,\ 6m'm'.$

(In all these layer groups both $\mathbf{M}$ and $\mathbf{P}$ are parallel to the unique polar axis.) See Fig. 4.9.

3.1.1.2 Both $\mathbf{M}$ and $\mathbf{P}$ are parallel to the layer $(\mathbf{P} \parallel \mathbf{M} \perp \mathbf{n})$ for the layer groups:

$L = \underline{2},\ \underline{2}m'\underline{m'},\ (\mathbf{M} \parallel \mathbf{P} \parallel \underline{2})$. See Fig. 4.10.

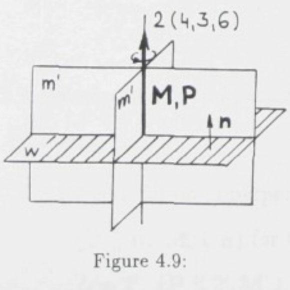


Figure 4.9:

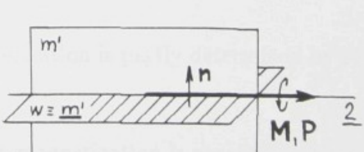


Figure 4.10:

3.1.2 Polarization is perpendicular to magnetization ($\mathbf{P} \perp \mathbf{M}$).

3.1.2.1 Magnetization is perpendicular and polarization is parallel to the layer ($\mathbf{M} \parallel \mathbf{n}, \mathbf{P} \perp \mathbf{n}$) for the layer group:

$L = \underline{2}'\underline{m}m'$, ($\mathbf{P} \parallel \underline{2}'$, $\mathbf{M} \perp \underline{m}$). See Fig. 4.11.

3.1.2.2 Magnetization is parallel and polarization is perpendicular to the layer ($\mathbf{M} \perp \mathbf{n}, \mathbf{P} \parallel \mathbf{n}$) for the layer group:

$L = m'm2'$, ($\mathbf{M} \perp m$, $\mathbf{P} \parallel 2'$). See Fig. 4.12.

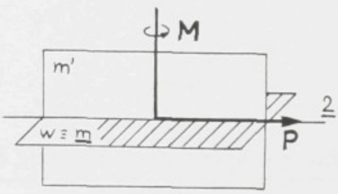


Figure 4.11:

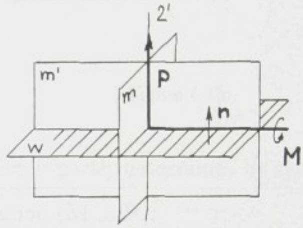


Figure 4.12:

3.1.2.3 Polarization and magnetization are parallel to the layer ($\mathbf{P}, \mathbf{M} \perp \mathbf{n}$) for the layer group:

$L = \underline{2}'m\underline{m}'$, ($\mathbf{M} \perp m$, $\mathbf{P} \parallel \underline{2}'$). See Fig. 4.13.

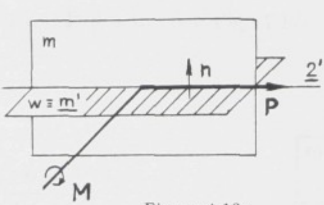


Figure 4.13:

3.2 Polarization is completely and magnetization is partly determined by symmetry.

3.2.1 Polarization is perpendicular and magnetization is parallel to the layer ($\mathbf{P} \parallel \mathbf{n}$, $\mathbf{M} \perp \mathbf{n}$) for the layer group :

$L = 2'$, ($\mathbf{P} \parallel 2', \mathbf{M} \perp 2'$). See Fig. 4.14.

3.2.2 Polarization is parallel to the layer ($\mathbf{P} \perp \mathbf{n}$) and magnetization is parallel to a plane perpendicular to the layer for the layer group:

$L = \underline{2}'$, ($\mathbf{P} \parallel \underline{2}', \mathbf{M} \perp \underline{2}'$). See Fig.2.15.

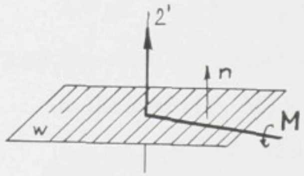


Figure 4.14:

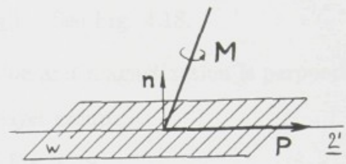


Figure 4.15:

3.3 Magnetization is completely and polarization is partly determined by symmetry, *magnetization is perpendicular to polarization* ($\mathbf{M} \perp \mathbf{P}$).

3.3.1 Magnetization is perpendicular and polarization is parallel to the layer ($\mathbf{M} \parallel \mathbf{n}$, $\mathbf{P} \perp \mathbf{n}$) for the layer group:

$L = \underline{m}$, ($\mathbf{M} \perp \underline{m}$, $\mathbf{P} \parallel \underline{m}$). See Fig.2.16.

3.3.2 Magnetization is parallel to the layer ($\mathbf{M} \perp \mathbf{n}$) and polarization is parallel to a plane perpendicular to the layer for the layer group:

$L = m$, ($\mathbf{M} \perp m$, $\mathbf{P} \parallel m$). See Fig. 4.17.

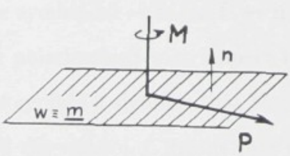


Figure 4.16:

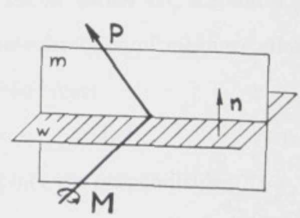


Figure 4.17:

3.4 Polarization and magnetization are parallel to the same plane but there is no specific relation between directions of **P** and **M**.

3.4.1 The plane determined by polarization and magnetization is parallel to the layer ($\{\mathbf{P}, \mathbf{M}\} \perp \mathbf{n}$) for the layer group:

$L = \underline{m'}$, ($\mathbf{n} \perp \mathbf{P} \parallel \underline{m'}$, $\mathbf{n} \perp \mathbf{M} \parallel \underline{m'}$). See Fig. 4.18.

3.4.2 The plane determined by polarization and magnetization is perpendicular to the layer ($\{\mathbf{P}, \mathbf{M}\} \parallel \mathbf{n}$) for the layer group:

$L = m'$, ($\mathbf{P} \parallel m'$, $\mathbf{M} \parallel m'$). See Fig. 4.19.

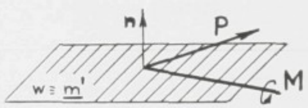


Figure 4.18:

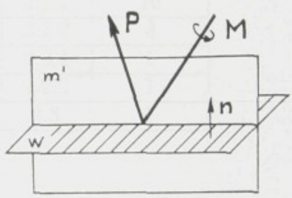


Figure 4.19:

3.5 Direction of neither polarization **P** nor magnetization **M** is determined by symmetry for the layer group of trivial triclinic symmetry:

$L = 1$.

Recapitulation of this division is summarized in Tables 4.1, 4.2 and 4.3 where the symbol $\mathbf{M} \leftrightarrow \mathbf{n}$ (or $\mathbf{P} \leftrightarrow \mathbf{n}$) is used to indicate direction of magnetization (or polarization) with respect to the normal of the layer:

the symbol $\parallel$ signifies that the given two vectors are parallel,

and the symbol $\perp$ means that the given two vectors are perpendicular.

Table 4.1: **Pyromagnetic ($\mathbf{M} \neq 0$) non-pyroelectric ($\mathbf{P} = 0$) layers.**

determination of $\mathbf{M}$ by symmetry	$\mathbf{M} \leftrightarrow \mathrm{n}$	layer groups	number of groups
total	$\parallel$	1.1.1	16
	$\perp$	1.1.2	3
partial	$\perp$	1.2.1	1
		1.2.2	1
no		1.3	1

Table 4.2: **Pyroelectric ($\mathbf{P} \neq 0$) non-pyromagnetic ($\mathbf{M} = 0$) layers.**

determination of $\mathbf{P}$ by symmetry	$\mathbf{P} \leftrightarrow \mathrm{n}$	layer groups	number of groups
total	$\parallel$	2.1.1	16
	$\perp$	2.1.2	3
partial	$\perp$	2.2.1	1
		2.2.2	1
no		2.3	1

Table 4.3: **Pyromagnetic ($\mathbf{M} \neq 0$) and pyroelectric ($\mathbf{P} \neq 0$) layers.**

determination of $\mathbf{P}$ and $\mathbf{M}$ by symmetry	$\mathbf{P} \leftrightarrow \mathbf{M}$	$\mathbf{P} \leftrightarrow \mathrm{n}$	$\mathbf{M} \leftrightarrow \mathrm{n}$	layer groups	number of groups
total	$\parallel$	$\parallel$	$\parallel$	3.1.1.1	8
		$\perp$	$\perp$	3.1.1.2	2
	$\perp$	$\perp$	$\parallel$	3.1.2.1	1
		$\parallel$	$\perp$	3.1.2.2	1
		$\perp$		3.1.2.3	1
$\mathbf{P}$ - total $\mathbf{M}$ - partial	$\perp$	$\parallel$	$\perp$	3.2.1	1
		$\perp$		3.2.2	1
$\mathbf{P}$ - partial $\mathbf{M}$ - total	$\perp$	$\perp$	$\parallel$	3.3.1	1
			$\perp$	3.3.2	1
partial		$\perp$	$\perp$	3.4.1	1
				3.4.2	1
no				3.5	1

In these tables only two limiting orientations are taken into account, $\mathbf{M}$ or $\mathbf{P}$ are either perpendicular or parallel to the normal (i.e. parallel or perpendicular to the layer). An empty "box" indicates general position when the layer group does not prefer any direction of magnetization or polarization.

These tables show that the total predetermination appears in 51 cases, in 30 of them polarization and magnetization are perpendicular to the layer, partial in 10 cases and only 3 symmetries do not restrict mutual direction of **M** and **P**.

4.2 Pyromagnetic and pyroelectric walls in completely transposable domain pairs

It was shown [21] that 122 electromagnetic point groups create 380 classes of completely transposable twin laws. In these cases, like for non-magnetic transposable twin laws [13,15,20], the symmetry magnetic group J_{12} expressing symmetry of a domain pair $\{S_1, S_2\}$ can be written as [17]

$$J_{12} = F_1 + g^{\star}_{12} F_1, \tag{4.1}$$

where F_1 is the symmetry group of both domain states S_1 and S_2 and $g^{\star}_{12}$ represents an operation interexchanging (transposing) the two domain states, i.e. $g^{\star}_{12} S_1 = S_2$ and $g^{\star}_{12} S_2 = S_1$.

The analogy of completely transposable twin laws for non-magnetic and magnetic twin laws admits to use the same algorithm introduced in Section 3.1 for determination of a symmetry group of a non-magnetic domain wall. It means it is necessary to determine a sectional layer group $\overline{J}_{12}$ of the group J_{12} and its subgroup

$$T_{12} = \widehat{F}_1 + \underline{t}^{\star}_{12} \widehat{F}_1, \tag{4.2}$$

where $\widehat{F}_1$ is a group containing all symmetry operations f_{12} leaving the sectional (one-sided) plane invariant and $\underline{t}^{\star}_{12}$ is a state&side exchanging operation.

According to symmetry of the layer group T_{12} a domain wall can belong to one of four types.

1. *Pyromagnetic and pyroelectric*, i.e. symmetry allows existence of non-zero spontaneous magnetization and polarization, $\mathbf{M} \neq \mathbf{0}, \mathbf{P} \neq \mathbf{0}$.
2. *Pyromagnetic and non-pyroelectric*, i.e. symmetry allows existence of non-zero spontaneous magnetization but only zero polarization, $\mathbf{M} \neq \mathbf{0}, \mathbf{P} = \mathbf{0}$.
3. *Non-pyromagnetic and pyroelectric*, i.e. symmetry allows existence of non-zero spontaneous polarization but only zero magnetization, $\mathbf{M} = \mathbf{0}, \mathbf{P} \neq \mathbf{0}$.
4. *Non-pyromagnetic and non-pyroelectric*, i.e. symmetry allows existence of spontaneous non-zero neither magnetization nor polarization, $\mathbf{M} = \mathbf{0}, \mathbf{P} = \mathbf{0}$.

Symmetry of layer groups $\overline{J}_{12}$ and T_1 which determine the type of a domain wall depends not only on the group J_{12} of the domain pair but also on the sectional plane (hkl). Nevertheless it is possible to distinguish among 380 classes of magnetic completely transposable twin laws $J_{12}[F_1]$ *four* types analogous to types of domain walls. (In this explicite notation the first part of the expression $J_{12}[F_1]$ represents the symmetry group of a domain pair and the second part (in brackets "[ ]") is the symmetry group of both domain states.) They will be denoted as:

- $\mathcal{MP}$ - groups $J_{12}[F_1]$ describing such domain pairs that some domain walls can be simultaneously pyromagnetic and pyroelectric.
- $\mathcal{M}$ - groups $J_{12}[F_1]$ describing such domain pairs that some domain walls can be pyromagnetic but *all* walls are non-pyroelectric.
- $\mathcal{P}$ - groups $J_{12}[F_1]$ describing such domain pairs that some domain walls can be pyroelectric but *all* walls are non-pyromagnetic.
- $\mathcal{O}$ - groups $J_{12}[F_1]$ describing such domain pairs that *all* domain walls are both non-pyromagnetic and non-pyroelectric.

This division is determined by inner structure of the groups $J_{12}[F_1]$, in particular if they contain the time inversion $1'$, or the space $\bar{1}$ and space-time $\bar{1}'$ inversions as state exchanging operations. If $1', \underline{\bar{1}}^*$ and $\underline{\bar{1}}'^*$ are generators of the layer group T_{12} then the type of the domain wall is predetermined, considering the fact that
groups containing the time inversion $1'$ cannot be pyromagnetic,
groups containing the space inversion $\bar{1}$ cannot be pyroelectric and
groups containing the space-time inversion $\bar{1}'$ are neither pyromagnetic nor pyroelectric.

In the following symbolic notation

- q - a magnetic group containing primed or unprimed operations except $1', \bar{1}$ and $\bar{1}'$,
- i - a magnetic group containing primed or unprimed operations except $1'$ and $\bar{1}'$, i.e. $\bar{1}$ is its generator,

- i' - a magnetic group containing primed or unprimed operations except $1'$ and $\bar{1}$, i.e. $\bar{1}'$ is its generator,

the structure of 380 $J[F]$ groups can be briefly expressed as

$$q[q],\ i[q],\ i'[q],\ q1'[q],\ q1'[q1'],\ i[i],\ i1'[i],\ i1'[q1'],\ i1'[i1'],\ i'[i'],\text{ and }\ i1'[i'].$$

All groups with structure $i'[q], i1'[i]$ and $i1'[q1']$ have among their generators the space-time inversion $\bar{1}'$ as a state exchanging operation. Therefore $\underline{\bar{1}}'^{*} \in T_{12}$ holds for all domain walls and these groups belong to the type $\mathcal{O}$. This situation appears in 90 cases.

All groups with structure $i1'[i1']$ and $q1'[q1']$ have among their generators the time inversion $1'$ as state&side-not-exchanging operation. Therefore $1' \in T_{12}$ holds for all domain walls and these groups belong to the type $\mathcal{P}$. This situation appears in 37 cases.

All groups with structure $i[q]$ and $i1'[i']$ have among their generators the space inversion $\bar{1}$ as side exchanging operation. Therefore $\underline{\bar{1}} \in T_{12}$ holds for all domain walls and these groups belong to the type $\mathcal{M}$. This situation appears in 69 cases.

In the last four types, i.e. $q[q], q1'[q], i[i]$ and $i'[i']$, the concrete form of layer groups [26] shows that it is always possible to find such an orientation for which the wall is simultaneously pyromagnetic and pyroelectric. These groups then belong to the type $\mathcal{MP}$. This situation appears in 184 cases.

These results are summarized in the following two tables, Tab.4.4 and Tab.4.5. The symbols MP, M, P and 0 are used in the explicite notation to indicate groups allowing:

MP - non-zero magnetization and polarization,

M - non-zero magnetization and zero polarization,

P - zero magnetization and non-zero polarization,

0 - the case when both magnetization and polarization are zero.

The second table, copying the first one, contains additional information about the number of non-ferroelastic magnetoelectric and ferroelastic domain pairs.

Complete list of 380 $J[F]$ groups, describing symmetry of domain pairs, divided into four sections in accordance with type of their walls is presented in Appendix F.

Table 4.4: **Number and type of the domain pairs with pyromagnetic and pyroelectric domain walls**

	J_{12}	MP	M	P	0	M	0	P	0	0	
wall	$[F_1]$	$[MP]$	$[MP]$	$[MP]$	$[MP]$	$[M]$	$[M]$	$[P]$	$[P]$	$[0]$	$\sum$
$\mathcal{PM}$	$q[q]$	16	9	9	8	6	6	6	6	16	82
	$q1'[q]$			13			8	8		19	48
	$i[i]$					11	8			8	27
	$i'[i']$									27	27
$\mathcal{M}$	$i[q]$		13			8			8	19	48
	$i1'[i']$									21	21
$\mathcal{P}$	$q1'[q1']$							11	8	8	27
	$i1'[i1']$									10	10
$\mathcal{O}$	$i'[q]$				13		8		8	19	48
	$i1'[q1']$								10	11	21
	$i1'[i]$						10			11	21
	$\sum$	16	22	22	21	25	40	25	40	169	380

Table 4.6 presents distribution of individual types of F_1 groups and domain walls belonging to 32 magnetic *J-families*. This table shows that:

- Symmetry of $J[F]$-groups in $\mathbf{\bar{1}}, \mathbf{\bar{3}}$ and $\mathbf{m\bar{3}}$ *J-families* completely excludes $\mathcal{MP}$ and $\mathcal{P}$ types of domain walls. (All domain walls in these 15 domain pairs are non-pyroelectric.)

- Symmetry of $J[F]$-groups in $\mathbf{1}, \mathbf{3}$ and $\mathbf{23}$ *J-families* establishes domain walls only of $\mathcal{MP}$ type. (In fact, in these 3 cases all domain walls can be simultaneously pyromagnetic and pyroelectric, except the case when the wall is parallel to (001), then $\mathbf{P} = \mathbf{0}$.)

- Domain walls of $\mathcal{M}$ type are present only in 11 *J-families*
$\mathbf{\bar{1}, 2/m, mmm, 4/m, 4/mmm, \bar{3}, \bar{3}m, 6/m, 6/mmm, m\bar{3}, m\bar{3}m}$.

(This covers 69 $J[F]$ groups.)

- Domain walls of $\mathcal{O}$ type are present in the same 11 *J-families.*

  (This covers 90 $J[F]$ groups.)

- Non-ferroelastic magnetoelectric domain pairs have walls only of $\mathcal{MP}$ and $\mathcal{M}$ type.

  (It appears in 83 and 58 cases.)

- Ferroelastic domain pairs have walls only of $\mathcal{MP}$ and $\mathcal{P}$ type.

  (This appears in 56 and 15 cases.)

- Non-pyromagnetic and non-pyroelectric domains do not exist in *10* magnetic *J-families*

  $\mathbf{1, 2, m, mm2, 4, 4mm, 3, 3m, 6, 6mm}$.

- Between non-pyromagnetic and non-pyroelectric domains we can meet

  - walls of only the $\mathcal{M}$ type in 2 *J-families*

    $\mathbf{\bar{1}, \bar{3}}$,

    both groups are non-ferroelastic magnetoelectric,

  - walls of only the $\mathcal{MP}$ type in 5 *J-families*

    $\mathbf{222, \bar{4}, 32, \bar{6}, 23}$,

    all five groups are non-ferroelastic magnetoelectric,

  - walls of the $\mathcal{O}$ type only in 8 *J-families*

    $\mathbf{mmm, 4/m, 4/mmm, \bar{3}m, 6/m, 6/mmm, m\bar{3}, m\bar{3}m}$.

    (41 cases)

Table 4.5: **Number of domain pairs with pyromagnetic and pyroelectric domain walls** (total/non-ferroelastic magnetoelectric/ferroelastic)

wall	$J_{12}[F_1]$	$MP[MP]$	$M[MP]$	$P[MP]$	$0[MP]$	$M[M]$	$0[M]$	$P[P]$	$0[P]$	$0[0]$	$\sum$
$\mathcal{PM}$	$q[q]$	16/3/11	9/5/4	9/5/4	8/3/3	6/2/2	6/2/2	6/2/2	6/2/2	16/8/4	82/32/34
	$q1'[q]$			13/13/0			8/6/0	8/6/0		19/15/0	48/40/0
	$i[i]$					11/0/6	8/0/3			8/0/2	27/0/11
	$i'[i']$									27/11/11	27/11/11
$\mathcal{M}$	$i[q]$		13/13/0			8/6/0			8/6/0	19/15/0	48/40/0
	$i1'[i']$									21/18/0	21/18/0
$\mathcal{P}$	$q1'[q1']$							11/0/6	8/0/3	8/0/2	27/0/11
	$i1'[i1']$									10/0/4	10/0/4
$\mathcal{O}$	$i'[q]$				13/0/0		8/0/0		8/0/0	19/0/0	48/0/0
	$i1'[q1']$								10/0/0	11/0/0	21/0/0
	$i1'[i]$						10/0/0			11/0/0	21/0/0
	$\sum$	16/3/11	22/18/4	22/18/4	21/3/3	25/8/8	40/8/5	25/8/8	40/8/5	169/67/23	380/141/71

Table 4.6: **Type and number of domain pairs and their walls in J-family classes**
(total /non-ferroelastic magnetoelectric [ame] / ferroelastic[fa])

J-family	F_1				wall, type of F_1 is 0				wall, general case			
	MP	M	P	0	$\mathcal{MP}$	$\mathcal{M}$	$\mathcal{P}$	$\mathcal{O}$	$\mathcal{MP}$	$\mathcal{M}$	$\mathcal{P}$	$\mathcal{O}$
1	1/1/0								1/1/0			
$\bar{\mathbf{1}}$	2/1/0	1/0/0	1/0/0	1/1/0		1/1/0				2/2/0		3/0/0
2	4/2/2		1/0/1						4/2/2		1/0/1	
m	4/2/2		1/0/1						4/2/2		1/0/1	
2/m	8/4/0	4/0/2	2/0/0	5/2/3	2/0/2	2/2/0	1/0/1		4/0/4	6/6/0	1/0/1	8/0/0
222	3/0/3	1/1/0	1/0/1	1/1/0	1/1/0				5/2/3		1/0/1	
mm2	9/2/7		3/1/2						10/3/7		2/0/2	
mmm	4/2/0	6/1/3	3/1/0	10/3/4	3/0/3	3/3/0	1/0/1	3/0/0	6/0/6	7/7/0	1/0/1	9/0/0
4	3/1/2		2/1/1						4/2/2		1/0/1	
$\bar{\mathbf{4}}$	2/0/2	1/1/0	1/0/1	1/1/0	1/1/0				4/2/2		1/0/1	
4/m	2/1/0	5/1/2	3/1/0	9/3/3	2/0/2	3/3/0	1/0/1	3/0/0	4/0/4	6/6/0	1/0/1	8/0/0
422	2/2/0	3/1/2	2/1/0	5/2/3	4/2/2		1/0/1		10/6/4		2/0/1	
4mm	5/3/2		7/3/3						10/6/4		2/0/1	
$\bar{4}$2m	2/0/2	5/3/2	3/0/3	9/5/3	7/5/2		2/0/1		16/8/8		3/0/2	
4/mmm	2/1/0	9/2/2	5/2/0	29/11/7	10/3/6	8/8/0	2/0/1	9/0/0	14/3/8	13/13/0	2/0/1	16/0/0

Table 4.6 (continued)

J-family	F_1				wall, type of F_1 is 0				wall, general case			
	MP	M	P	0	$\mathcal{MP}$	$\mathcal{M}$	$\mathcal{P}$	$\mathcal{O}$	$\mathcal{MP}$	$\mathcal{M}$	$\mathcal{P}$	$\mathcal{O}$
3	1/1/0								1/1/0			
$\bar{3}$	2/1/0	1/0/0	1/0/0	1/1/0		1/1/0				2/2/0		3/0/0
32	2/2/0	1/1/0	1/0/0	1/1/0	1/1/0				4/4/0		1/0/0	
3m	3/3/0		2/1/0						4/4/0		1/0/0	
$\bar{3}$m	2/1/0	5/1/0	3/1/0	9/5/0	2/2/0	3/3/0	1/0/0	3/0/0	4/2/0	6/6/0	1/0/0	8/0/0
6	3/2/0		2/0/0						4/2/0		1/0/0	
$\bar{6}$	2/1/0	1/0/0	1/0/0	1/1/0	1/1/0				4/2/0		1/0/0	
6/m	2/1/0	5/0/0	3/0/0	9/3/0	2/1/0	3/2/0	1/0/0	3/0/0	4/1/0	6/3/0	1/0/0	8/0/0
622	2/2/0	3/2/0	2/0/0	5/2/0	4/2/0		1/0/0		10/6/0		2/0/0	
6mm	5/4/0		7/2/0						10/6/0		2/0/0	
$\bar{6}$m2	2/1/0	5/1/0	3/1/0	9/5/0	7/5/0		2/0/0		16/8/0		3/0/0	
6/mmm	2/1/0	9/1/0	5/1/0	29/9/0	10/4/0	8/5/0	2/0/0	9/0/0	14/4/0	13/8/0	2/0/0	16/0/0
23				1/1/0	1/1/0				1/1/0			
m$\bar{3}$				5/2/0		2/2/0		3/0/0		2/2/0		3/0/0
432				5/2/0	4/2/0		1/0/0		4/2/0		1/0/0	
$\bar{4}$3m				5/2/0	4/2/0		1/0/0		4/2/0		1/0/0	
m$\bar{3}$m				19/4/0	4/1/0	6/3/0	1/0/0	8/0/0	4/1/0	6/3/0	1/0/0	8/0/0
$\sum_{total}$	81	65	65	169	70	40	18	41	184	69	37	90
$\sum_{ame}$	42	16	16	67	34	33	0	0	83	58	0	0
$\sum_{fa}$	22	13	13	23	17	0	6	0	56	0	15	0

4.3 Pyromagnetic and pyroelectric domain walls joining non-pyromagnetic and non-pyroelectric domains

The Table 4.6 in the preceding section shows that among 380 domain pairs described by magnetic completely transposable twin laws there exist 169 cases with non-pyromagnetic and non-pyroelectric domains. Nevertheless, *all* four types of domain walls can be found between them.

1. $\mathcal{O}$ - type of the domain wall appears in 41 cases.

   The domain pair symmetry groups $J_{12}[F]$ are of $i1'[i], i1'[q1']$ and $i'[q]$ types.

   - $\star$ $mmm1'[mmm]$, $mmm1'[2221']$, $m'm'm'[222]$,
   - $\star$ $4/m1'[\bar{4}1']$, $4/m1'[4'/m]$, $4/m'[\bar{4}']$,
   - $\star$ $4/mmm1'[4/mmm]$, $4/mmm1'[4221']$, $4/mmm1'[\bar{4}2m1']$, $4/mmm1'[4'/mmm']$, $4/m'm'm'[422]$, $4/m'm'm'[\bar{4}'2m']$, $4/m'mm[\bar{4}'2'm]$, $4'/m'm'm[\bar{4}'2m]$, $4'/m'm'm[4'22']$,
   - $\star$ $\bar{3}m1'[\bar{3}m]$, $\bar{3}m1'[321']$, $\bar{3}'m'[32]$,
   - $\star$ $6/m1'[\bar{6}1']$, $6m1'[6'/m']$, $6/m'[\bar{6}']$,
   - $\star$ $6/mmm1'[6/mmm]$, $6/mmm1'[6221']$, $6/mmm1'[\bar{6}m21']$, $6/mmm1'[6'/m'mm']$, $6/m'm'm'[622]$, $6/m'm'm'[\bar{6}'m'2]$, $6/m'mm[\bar{6}'m2']$, $6'/mmm'[\bar{6}m2]$, $6'/mmm'[6'2'2]$,
   - $\star$ $m\bar{3}1'[m\bar{3}]$, $m\bar{3}1'[231']$, $m'\bar{3}'[23]$,
   - $\star$ $m\bar{3}m1'[m\bar{3}m]$, $m\bar{3}m1'[\bar{4}3m1']$, $m\bar{3}m1'[4321']$, $m\bar{3}m1'[m\bar{3}m']$, $m'\bar{3}m'[432]$, $m'\bar{3}'m'[\bar{4}'3m']$, $m'\bar{3}'m[\bar{4}3m]$, $m'\bar{3}m[4'32']$.

   None of these groups describes either ferroelastic or non-ferroelastic magnetoelectric domain pair.

2. $\mathcal{P}$ - type of the domain wall appears in 18 cases.

   The domain pair symmetry groups $J_{12}[F]$ are of $q1'[q1']$ and $i1'[i1']$ types,

   6 of them are ferroelastic (a) and

   12 non-ferroelastic magnetoelectric (b) domain pairs.

- $\star$ $2/m1'[\bar{1}1'](a)$,
- $\star$ $mmm1'[2/m1'](a)$,
- $\star$ $4/m1'[2/m1'](a)$,
- $\star$ $4221'[2221'](a)$,
- $\star$ $\bar{4}2m1'[2221'](a)$, $\bar{4}2m1'[\bar{4}1']$,
- $\star$ $4/mmm1'[mmm1'](a)$, $4/mmm1'[4/m1']$,
- $\star$ $\bar{3}m1'[\bar{3}1']$,
- $\star$ $6/m1'[\bar{3}1']$,
- $\star$ $6221'[321']$,
- $\star$ $\bar{6}m21'[321']$, $\bar{6}m21'[\bar{6}1']$,
- $\star$ $6/mmm1'[\bar{3}m1']$, $6/mmm1'[6/m1']$,
- $\star$ $4321'[231']$,
- $\star$ $\bar{4}3m1'[231']$, $m\bar{3}m1'[m\bar{3}1']$.

From the total number of 22 pyroelectric non-pyromagnetic layer groups (see Section 4.1) only 8 are present here as the symmetry (layer) group T_{12} of the domain wall, in particular

$11'$, $21'$, $\underline{2}^{*}1'$, $m1'$, $\underline{m}^{*}1'$, $mm21'$, $\underline{m}^{*}m\underline{2}^{*}1'$, $31'$.

3. $\mathcal{M}$ - type of the domain wall appears in 40 cases.

   The domain pair symmetry groups $J_{12}[F]$ are of $i[q]$ and $i1'[i']$ types, 33 of them are non-ferroelastic magnetoelectric (b) and none of them ferroelastic domain pair.

   - $\star$ $\bar{1}1'[\bar{1}'](b)$,
   - $\star$ $2/m1'[2/m'](b)$, $2/m1'[2'/m](b)$,
   - $\star$ $mmm[222](b)$, $mmm1'[m'mm](b)$, $mmm1'[m'm'm'](b)$,
   - $\star$ $4/m1'[4/m'](b)$, $4/m1'[4'/m'](b)$, $4'/m[\bar{4}'](b)$,

- ⋆ $4/mmm[422](b)$, $4/mmm[\bar{4}2m](b)$, $4/mmm1'[4/m'm'm'](b)$, $4/mmm1'[4/m'mm](b)$, $4/mmm1'[4'/m'm'm](b)$, $4'/mmm'[4'22'](b)$, $4'/mmm'[\bar{4}'2m'](b)$, $4'/mmm'[\bar{4}'m2'](b)$,
- ⋆ $\bar{3}1'[\bar{3}'](b)$,
- ⋆ $\bar{3}m[32](b)$, $\bar{3}m1'[\bar{3}'m](b)$, $\bar{3}m1'[\bar{3}'m'](b)$,
- ⋆ $6/m1'[6/m'](b)$, $6/m1'[6'/m]$, $6'/m'[\bar{6}'](b)$,
- ⋆ $6/mmm[622](b)$, $6/mmm[\bar{6}m2]$, $6/mmm1'[6/m'm'm'](b)$, $6/mmm1'[6/m'mm](b)$, $6/mmm1'[6'/mmm']$, $6'/m'mm'[6'22']$, $6'/m'mm'[\bar{6}'m2'](b)$, $6'/m'mm'[\bar{6}'2m'](b)$,
- ⋆ $m\bar{3}[23](b)$, $m\bar{3}1'[m'\bar{3}'](b)$,
- ⋆ $m\bar{3}m[\bar{4}3m]$, $m\bar{3}m[432](b)$, $m\bar{3}m1'[m'\bar{3}'m]$, $m\bar{3}m1'[m'\bar{3}'m'](b)$, $m\bar{3}m'[4'32']$, $m\bar{3}m'[\bar{4}'3m'](b)$.

From the total number of 22 pyromagnetic non-pyroelectric layer groups (Section 4.1) only 13 are present here as the symmetry (layer) group T_{12} of the domain wall, in particular

$\underline{\bar{1}}^*$, $\underline{2}^*/m$, $2'/\underline{m}'^*$, $\underline{2}'^*/m'$, $\underline{m}'^*m'm$, $m'm'\underline{m}^*$, $4/\underline{m}^*$, $4/\underline{m}^*m'm'$,
$\underline{\bar{3}}^*$, $\underline{\bar{3}}^*m'$, $6/\underline{m}^*$, $6/\underline{m}^*m'm'$.

4. $\mathcal{MP}$ - type of the domain wall appears in 70 cases.

   The domain pair symmetry groups $J_{12}[F]$ are of $q[q], q1'[q], i[i]$ and $i'[i']$ types, 34 of them determine non-ferroelastic magnetoelectric (b) and 17 ferroelastic (a) domin pairs.

   - ⋆ $2'/m[\bar{1}'](a)$, $2/m'[\bar{1}'](a)$,
   - ⋆ $2221'[222](b)$,
   - ⋆ $m'mm[2'/m](a)$, $m'mm[2/m'](a)$, $m'm'm'[2/m'](a)$,
   - ⋆ $\bar{4}1'[\bar{4}'](b)$,
   - ⋆ $4/m'[2/m'](a)$, $4'/m'[2/m'](a)$,
   - ⋆ $422[222](a)$, $4221'[422](b)$, $4221'[4'22'](b)$, $4'22'[222](a)$,

- $\bar{4}2m[222](a)$, $\bar{4}2m1'[\bar{4}2m](b)$, $\bar{4}2m1'[\bar{4}'2m'](b)$,
  $\bar{4}2m1'[\bar{4}'2'm](b)$, $\bar{4}'2m'[222](a)$,
  $\bar{4}'2m'[\bar{4}'](b)$, $\bar{4}'2'm[\bar{4}'](b)$,
- $4/mmm[mmm](a)$, $4'/mmm'[mmm](a)$, $4'/mmm'[4'/m]$,
  $4/m'm'm'[4/m'](b)$, $4/m'm'm'[m'm'm'](a)$, $4/m'mm[4/m'](b)$,
  $4/m'mm[mmm'](a)$, $4'/m'm'm[4'/m'](b)$,
  $4'/m'm'm[m'm'm'](a)$, $4'/m'm'm[mmm'](a)$,
- $321'[32](b)$,
- $\bar{3}'m[\bar{3}'](b)$, $\bar{3}'m'[\bar{3}'](b)$,
- $\bar{6}1'[\bar{6}'](b)$,
- $6/m'[\bar{3}']$, $6'/m[\bar{3}'](b)$,
- $622[32]$, $6221'[622](b)$,
  $6221'[6'22']$, $6'22'[32](b)$,
- $\bar{6}m2[32](b)$, $\bar{6}m21'[\bar{6}m2]$, $\bar{6}m21'[\bar{6}'m'2](b)$,
- $\bar{6}m21'[\bar{6}'m2'](b)$, $\bar{6}'m'2[32]$, $\bar{6}'m2'[\bar{6}'](b)$, $\bar{6}'m'2[\bar{6}'](b)$,
- $6/mmm[\bar{3}m]$, $6'/m'mm'[\bar{3}m]$, $6'/m'mm'[6'/m']$,
  $6/m'm'm'[6/m'](b)$, $6/m'm'm'[\bar{3}'m']$, $6/m'mm[6/m'](b)$,
  $6/m'mm[\bar{3}'m]$, $6'/mmm'[6'/m]$,
  $6'/mmm'[\bar{3}'m](b)$, $6'/mmm'[\bar{3}'m'](b)$,
- $231'[23](b)$,
- $432[23]$, $4321'[432](b)$, $4321'[4'32']$, $4'32'[23](b)$,
- $\bar{4}3m[23](b)$, $\bar{4}3m1'[\bar{4}3m]$, $\bar{4}'3m1'[\bar{4}'3m'](b)$, $\bar{4}3m'[23]$,
- $m\bar{3}m[m\bar{3}]$, $m\bar{3}m'[m\bar{3}]$, $m'\bar{3}'m'[m'\bar{3}']$, $m'\bar{3}'m[m'\bar{3}'](b)$.

From the total number of 20 pyromagnetic pyroelectric layer groups (see Section 4.1) only 15 appear here as the symmetry (layer) group T_{12} of the domain wall, in particular

$1, 2, \underline{2}^*, \underline{2}'^*, m, \underline{m}^*, m', m'm'2, \underline{m}'^*, m'm2', m'\underline{m}^*\underline{2}'^*, m\underline{m}'^*\underline{2}'^*, m'\underline{m}'^*\underline{2}^*, 3, 3m'$.
Missing five groups are: $2', 4, 4m'm', 6, 6m'm'$.

Tables, presented in Appendices G, H and I cover 128 domain pairs with $\mathcal{P}, \mathcal{M}$ and $\mathcal{MP}$ type of the wall. They contain

- F_1 - symmetry group of the domain state S_1,
- J_{12} - symmetry group of the domain pair $\{S_1, S_2\}$,
- (hkl) - plane parallel to the domain wall
- $\widehat{F}_1$ - one-sided sectional layer group of F_1,
- T_{12} - symmetry (layer) group of the domain wall,
- $\overline{J}_{12}$ - sectional layer group of J_{12},
- T - type of the group T_{12},
- $\overline{J}$ - type of the sectional layer group $\overline{J}_{12}$, i.e. MP, M, P or 0, (see Section 4.2)
- number specifying the twin law $J[F]$. The numbering is taken from the numbering of the 380 transposable magnetic twin laws [21].

Analysis of these tables enables one to separate all existing combinations of layer groups T_{12} and $\overline{J}_{12}$, and to study relations between them independently on the central plane (hkl) of the domain wall . Tables 4.7, 4.8 and 4.9 present following information about the domain walls of $\mathcal{P}, \mathcal{M}$ and $\mathcal{MP}$ types.

- Symmetry group of the domain wall T_{12}.
- For each group T_{12} all sectional layer groups $\overline{J}_{12}$ which appear in combination $T_{12} \in \overline{J}_{12}$ for the given type of the domain wall.
- Type of these groups with respect to predetermination of the nonzero or zero magnetization and polarization, i.e. MP, M, P and 0.
- Direction of the polarization and magnetization predeterminated by the wall symmetry group (see Section 4.1).
- Information about symmetry properties of the given domain wall, concretely if it is (**A**)Symmetrical or (**I**) **R**eversible (definitions are in Section 3.1).

- Presence of the given combination $T_{12} \in J_{12}$, i.e. the given type of wall, among walls connecting some special type of the domains, as ferroelastic, non-ferroelastic magnetoelectric, etc.

- Direction of the spontaneous polarization $\mathbf{P}(W_{12})$ and the spontaneous magnetization $\mathbf{M}(W_{12})$ in the domain wall $W_{12} = [S_1(hkl)S_2]$ and $\mathbf{M}(W_{21})$ in the reversed domain wall $W_{21} = [S_2(hkl)S_1]$.

  When the determination is not unique, only two prominent directions are mentioned, saving or reverting orientation. A possible change of the magnitude is expressed by indices.

Table 4.7: **Pyroelectric non-pyromagnetic walls**
(fa ... ferroelastic domain pair ,
na ... non-ferroelastic domain pair)

T_{12}	$\mathbf{P}$	$\overline{J}_{12}$	fa, na	type of $\overline{J}_{12}$	sym. of wall	$\mathbf{P}(W_{12})$	$\mathbf{P}(W_{21})$
$11'$		$11'$	fa, na	P	AI	$\mathbf{P}_1$	$\mathbf{P}_2$
		$\underline{1}1'$	na	0	AR	$\mathbf{P}$	$-\mathbf{P}$
		$\underline{2}1'$	fa, na	P	AR	$\mathbf{P}_{\perp\underline{2}}$	$-\mathbf{P}_{\perp\underline{2}}$
						$\mathbf{P}_{\parallel\underline{2}}$	$\mathbf{P}_{\parallel\underline{2}}$
		m^*1'	fa, na	P	AR	$\mathbf{P}_{\perp m^*}$	$-\mathbf{P}_{\perp m^*}$
						$\mathbf{P}_{\parallel m^*}$	$\mathbf{P}_{\parallel m^*}$
$21'$	$\mathbf{P}\parallel 2$	$2m^*m^*1'$	na	P	AR	$\mathbf{P}$	$\mathbf{P}$
		$2\underline{22}1'$	fa	0	AR	$\mathbf{P}$	$-\mathbf{P}$
$\underline{2}^*1'$	$\mathbf{P}\parallel\underline{2}^*$	$\underline{2}^*1'$	fa, na	P	SI	$\mathbf{P}_1$	$\mathbf{P}_2$
		$\underline{2}^*/m^*1'$	fa, na	0	SR	$\mathbf{P}$	$-\mathbf{P}$
		$\underline{2}^*\underline{2}2^*1'$	fa, na	0	SR	$\mathbf{P}$	$-\mathbf{P}$
$m1'$	$\mathbf{P}\parallel m$	$m1'$	fa, na	P	AI	$\mathbf{P}_1$	$\mathbf{P}_2$
		$\underline{2}/m1'$	fa, na	0	AR	$\mathbf{P}$	$-\mathbf{P}$
		$2^*m^*m1'$	na	P	AR	$\mathbf{P}_{\perp 2^*}$	$-\mathbf{P}_{\perp 2^*}$
						$\mathbf{P}_{\parallel 2^*}$	$\mathbf{P}_{\parallel 2^*}$
$\underline{m}^*1'$	$\mathbf{P}\parallel\underline{m}^*$	$2^*/\underline{m}^*1'$	fa, na	0	SR	$\mathbf{P}$	$-\mathbf{P}$
		$\underline{m}^*m^*\underline{2}1'$	fa, na	P	SR	$\mathbf{P}_{\perp\underline{2}}$	$-\mathbf{P}_{\perp\underline{2}}$
						$\mathbf{P}_{\parallel\underline{2}}$	$\mathbf{P}_{\parallel\underline{2}}$
$mm21'$	$\mathbf{P}\parallel 2$	$\underline{m}mm1'$	fa	0	AR	$\mathbf{P}$	$-\mathbf{P}$
$\underline{m}^*m\underline{2}^*1'$	$\mathbf{P}\parallel\underline{2}^*$	$\underline{m}^*m\underline{2}^*1'$	na	P	SI	$\mathbf{P}_1$	$\mathbf{P}_2$
		$\underline{m}^*m^*m1'$	fa, na	0	SR	$\mathbf{P}$	$-\mathbf{P}$
$31'$	$\mathbf{P}\parallel 3$	$3m^*1'$	na	P	AR	$\mathbf{P}$	$\mathbf{P}$

Table 4.8: **Pyromagnetic non-pyroelectric walls**
(nme ... non-magnetoelectric domain pairs,
ame ... non-ferroelastic magnetoelectric domain pairs)

T_{12}	M	$\overline{J}_{12}$	*nme, ame*	*type of* $\overline{J}_{12}$	*wall sym.*	$\mathbf{M}(W_{12})$	$\mathbf{M}(W_{21})$
$\underline{\bar{1}}^{*}$		$\underline{\bar{1}}^{*}$	*nme, ame*	M	SI	$\mathbf{M}_1$	$\mathbf{M}_2$
		$\underline{\bar{1}}'1'^{*}$	*nme, ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$\underline{2}/m^{*}$	*nme, ame*	M	SR	$\mathbf{M}_{\|\underline{2}}$	$\mathbf{M}_{\|\underline{2}}$
						$\mathbf{M}_{\perp\underline{2}}$	$-\mathbf{M}_{\perp\underline{2}}$
		$\underline{2}'/m'^{*}$	*nme, ame*	M	SR	$\mathbf{M}_{\|\underline{2}'}$	$-\mathbf{M}_{\|\underline{2}'}$
						$\mathbf{M}_{\perp\underline{2}'}$	$\mathbf{M}_{\perp\underline{2}'}$
$\underline{2}^{*}/m$	$\mathbf{M}\|\underline{2}^{*}$	$\underline{2}/m$	*nme, ame*	M	SI	$\mathbf{M}_1$	$\mathbf{M}_2$
		$\underline{2}'/m1'^{*}$	*nme, ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$mm^{*}\underline{m}$	*nme, ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
$2/\underline{m}^{*}$	$\mathbf{M}\|2$	$2/\underline{m}^{*}$	*ame*	M	SI	$\mathbf{M}_1$	$\mathbf{M}_2$
		$2/\underline{m}'1'^{*}$	*ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$m^{*}m^{*}\underline{m}^{*}$	*ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$m'^{*}m'^{*}\underline{m}^{*}$	*nme*	M	SR	$\mathbf{M}$	$\mathbf{M}$
		$4'^{*}/\underline{m}^{*}$	*ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
$2'/\underline{m}'^{*}$	$\mathbf{M}\|\underline{m}'^{*}$	$2'/\underline{m}1'^{*}$	*ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$\underline{m}'^{*}m'^{*}m^{*}$	*nme, ame*	M	SR	$\mathbf{M}_{\|\underline{2}}$	$-\mathbf{M}_{\|\underline{2}}$
						$\mathbf{M}_{\|\underline{2}'}$	$\mathbf{M}_{\|\underline{2}'}$
$\underline{2}'^{*}/m'$	$\mathbf{M}\|m'$	$\underline{2}'^{*}/m'$	*ame*	M	SI	$\mathbf{M}_1$	$\mathbf{M}_2$
		$\underline{2}/m'1'^{*}$	*nme, ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$m^{*}m'\underline{m}'$	*ame*	M	SR	$\mathbf{M}_{\|\underline{2}}$	$\mathbf{M}_{\|\underline{2}}$
						$\mathbf{M}_{\|2'^{*}}$	$-\mathbf{M}_{\|2'^{*}}$
		$m'm'^{*}\underline{m}$	*nme, ame*	M	SR	$\mathbf{M}_{\|2'^{*}}$	$\mathbf{M}_{\|2'^{*}}$
						$\mathbf{M}_{\|\underline{2}'}$	$-\mathbf{M}_{\|\underline{2}'}$
$\underline{m}'^{*}m'm$	$\mathbf{M}\|\underline{2}$	$\underline{m}'^{*}m'm$	*ame*	M	SI	$\mathbf{M}_1$	$\mathbf{M}_2$
		$\underline{m}mm'1'^{*}$	*nme, ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
$m'm'\underline{m}^{*}$	$\mathbf{M}\|2$	$m'm'\underline{m}^{*}$	*ame*	M	SI	$\mathbf{M}_1$	$\mathbf{M}_2$
		$m'm'\underline{m}'1'^{*}$	*ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$4'^{*}/\underline{m}^{*}m^{*}m'$	*ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
$4/\underline{m}^{*}$	$\mathbf{M}\|4$	$4/\underline{m}'1'^{*}$	*ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$4/\underline{m}^{*}m^{*}m^{*}$	*ame*	0	SR	$\mathbf{M}$	$-\mathbf{M}$
$4/\underline{m}^{*}m'm'$	$\mathbf{M}\|4$	$4/\underline{m}'m'm'1'^{*}$	*amne*	0	SR	$\mathbf{M}$	$-\mathbf{M}$

Table 4.8 (continued)

T_{12}	M	$\overline{J}_{12}$	nme, ame	type of $\overline{J}_{12}$	wall sym.	$\mathbf{M}(W_{12})$	$\mathbf{M}(W_{21})$
$\underline{\bar{3}}^{*}$	$\mathbf{M}\|\|3$	$\underline{\bar{3}}^{*}$	ame	M	SI	$\mathbf{M}_1$	$\mathbf{M}_2$
		$\underline{\bar{3}}'1'^{*}$	ame	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$\underline{\bar{3}}^{*}m^{*}$	ame	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$\underline{\bar{3}}^{*}m'^{*}$	nme	M	SR	$\mathbf{M}$	$\mathbf{M}$
		$6'^{*}/\underline{m}'$	ame	0	SR	$\mathbf{M}$	$-\mathbf{M}$
$\underline{\bar{3}}^{*}m'$	$\mathbf{M}\|\|3$	$\underline{\bar{3}}^{*}m'$	ame	M	SI	$\mathbf{M}_1$	$\mathbf{M}_2$
		$\underline{\bar{3}}'m'1'^{*}$	ame	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$6'/\underline{m}'m^{*}m'$	ame	0	SR	$\mathbf{M}$	$-\mathbf{M}$
$6/\underline{m}^{*}$	$\mathbf{M}\|\|6$	$6/\underline{m}'1'^{*}$	ame	0	SR	$\mathbf{M}$	$-\mathbf{M}$
		$6/\underline{m}^{*}m^{*}m^{*}$	ame	0	SR	$\mathbf{M}$	$-\mathbf{M}$
$6/\underline{m}^{*}m'm'$	$\mathbf{M}\|\|6$	$6/\underline{m}'m'm'1'^{*}$	ame	0	SR	$\mathbf{M}$	$-\mathbf{M}$

Table 4.9: **Pyroelectric and pyromagnetic walls**
(fa ... ferroelastic domain pairs ,
ame ... non-ferroelastic magnetoelectric domain pairs
me ... magnetoelectric domain pairs,
nme ... non-magnetoelectric domain pairs)

T_{12}	M	$\overline{J}_{12}$	fa, ame me, nme	$type$ $of\ \overline{J}_{12}$	$wall$ $sym.$	$\mathbf{P}, \mathbf{M}(W_{12})$	$\mathbf{P}, \mathbf{M}(W_{21})$
1		1	fa, nme, ame	MP	AI	$\mathbf{P}_1, \mathbf{M}_1$	$\mathbf{P}_2, \mathbf{M}_2$
		$1'^*$	nme, ame	P	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
		$\underline{\bar{1}}$	fa, nme	M	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, \mathbf{M}$
		$\underline{\bar{1}}'$	fa, ame, me, nme	0	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
		$\underline{2}$	fa, ame, me, nme	MP	AR	$\mathbf{P}_{\parallel\underline{2}}, \mathbf{M}_{\parallel\underline{2}}$	$\mathbf{P}_{\parallel\underline{2}}, \mathbf{M}_{\parallel\underline{2}}$
						$\mathbf{P}_{\perp\underline{2}}, \mathbf{M}_{\perp\underline{2}}$	$-\mathbf{P}_{\perp\underline{2}}, -\mathbf{M}_{\perp\underline{2}}$
		m^*	fa, ame	MP	AR	$\mathbf{P}_{\parallel m^*}, \mathbf{M}_{\parallel m^*}$	$\mathbf{P}_{\parallel m^*}, -\mathbf{M}_{\parallel m^*}$
						$\mathbf{P}_{\perp m^*}, \mathbf{M}_{\perp m^*}$	$-\mathbf{P}_{\perp m^*}, \mathbf{M}_{\perp m^*}$
		m'^*	fa, nme, ame	MP	AR	$\mathbf{P}_{\parallel m'^*}, \mathbf{M}_{\parallel m'^*}$	$\mathbf{P}_{\parallel m'^*}, \mathbf{M}_{\parallel m'^*}$
						$\mathbf{P}_{\perp m'^*}, \mathbf{M}_{\perp m'^*}$	$-\mathbf{P}_{\perp m'^*}, -\mathbf{M}_{\perp m'^*}$
2	$\mathbf{P} \parallel 2$ $\mathbf{M} \parallel 2$	$21'^*$	ame	P	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
		$2m^*m^*$	ame	P	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
		$2m'^*m'^*$	me	MP	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, \mathbf{M}$
		$2\underline{2}\underline{2}$	fa	0	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
$\underline{2}^*$	$\mathbf{P} \parallel \underline{2}^*$ $\mathbf{M} \parallel \underline{2}^*$	$\underline{2}^*$	fa, ame, me	MP	SI	$\mathbf{P}_1, \mathbf{M}_1$	$\mathbf{P}_2, \mathbf{M}_2$
		$\underline{2}'1'^*$	nme, ame	MP	SR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
		$\underline{2}^*/m'^*$	fa, ame, me, nme	0	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
		$\underline{2}^*/m^*$	nme	MP	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, \mathbf{M}$
		$2^*\underline{2}^*\underline{2}$	fa, ame, me	0	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$

Table 4.9 (continued)

T_{12}	M	$\overline{J}_{12}$	fa, ame me, nme	type of $\overline{J}_{12}$	sym. of wall	$\mathbf{P}, \mathbf{M}(W_{12})$	$\mathbf{P}, \mathbf{M}(W_{21})$
$\underline{2}'^*$	$\mathbf{P} \parallel \underline{2}'^*$ $\mathbf{M} \perp \underline{2}'^*$	$\underline{2}'^*$	fa, ame	MP	SI	$\mathbf{P}_1, \mathbf{M}_1$	$\mathbf{P}_2, \mathbf{M}_2$
		$\underline{2}1'^*$	nme, ame	P	SR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
		$\underline{2}'^*/m^*$	fa, ame, me, nme	0	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
		$\underline{2}'^*/m'^*$	nme	M	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, \mathbf{M}$
		$2'^*\underline{2}'^*\underline{2}$	nme, ame	M	SR	$\mathbf{P}, \mathbf{M}_{\parallel \underline{2}}$ $\mathbf{P}, \mathbf{M}_{\parallel 2'^*}$	$-\mathbf{P}, \mathbf{M}_{\parallel \underline{2}}$ $-\mathbf{P}, -\mathbf{M}_{\parallel 2'^*}$
m	$\mathbf{P} \parallel m$ $\mathbf{M} \perp m$	$m1'^*$	nme, ame	P	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
		$\underline{2}/m$	fa, amm	M	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, \mathbf{M}$
		$\underline{2}'/m$	fa, ame, me, nme	0	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
m'	$\mathbf{P} \parallel m'$ $\mathbf{M} \parallel m'$	m'	ame	MP	AI	$\mathbf{P}_1, \mathbf{M}_1$	$\mathbf{P}_2, \mathbf{M}_2$
		$m'1'^*$	ame	MP	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
		$\underline{2}/m'$	fa, ame, me	0	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
		$\underline{2}'/m'$	fa, nme	M	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, \mathbf{M}$
		$2^*m'^*m'$	ame	MP	AR	$\mathbf{P}_{\parallel 2^*}, \mathbf{M}_{\parallel 2^*}$ $\mathbf{P}_{\perp m'^*}, \mathbf{M}_{\perp m'^*}$	$\mathbf{P}_{\parallel 2^*}, \mathbf{M}_{\parallel 2^*}$ $-\mathbf{P}_{\perp m'^*}, -\mathbf{M}_{\perp m'^*}$
		$2'^*m^*m'$	ame	MP	AR	$\mathbf{P}_{\parallel 2'^*}, \mathbf{M}_{\parallel 2'^*}$ $\mathbf{P}_{\perp m'}, \mathbf{M}_{\perp m'}$	$\mathbf{P}_{\parallel 2'^*}, -\mathbf{M}_{\parallel 2'^*}$ $-\mathbf{P}_{\perp m'}, \mathbf{M}_{\perp m'}$
$\underline{m}^*$	$\mathbf{P} \parallel \underline{m}^*$ $\mathbf{M} \perp \underline{m}^*$	$2'^*/\underline{m}^*$	fa, ame	0	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
		$\underline{m}^*m^*\underline{2}$	fa, ame	P	SR	$\mathbf{P}_{\parallel \underline{2}}, \mathbf{M}$ $\mathbf{P}_{\perp m^*}, \mathbf{M}$	$\mathbf{P}_{\parallel \underline{2}}, -\mathbf{M}$ $-\mathbf{P}_{\perp m^*}, -\mathbf{M}$

Table 4.9 (continued)

T_{12}	M	$\overline{J}_{12}$	fa, ame me, nme	type of $\overline{J}_{12}$	sym. of wall	$\mathbf{P}, \mathbf{M}(W_{12})$	$\mathbf{P}, \mathbf{M}(W_{21})$
$\underline{m}'^{*}$	$\mathbf{P} \parallel \underline{m}'^{*}$	$2^{*}/\underline{m}'^{*}$	fa, ame	0	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
	$\mathbf{M} \parallel \underline{m}'^{*}$	$\underline{m}'^{*}m'^{*}\underline{2}$	fa, ame, me	MP	SR	$\mathbf{P}_{\parallel\underline{2}}, \mathbf{M}_{\parallel\underline{2}}$	$\mathbf{P}_{\parallel\underline{2}}, \mathbf{M}_{\parallel\underline{2}}$
						$\mathbf{P}_{\perp\mathbf{m}'^{*}}, \mathbf{M}_{\perp\mathbf{m}'^{*}}$	$-\mathbf{P}_{\perp\mathbf{m}'^{*}}, -\mathbf{M}_{\perp\mathbf{m}'^{*}}$
$m'm'2$	$\mathbf{P} \parallel 2$	$m'm'21'^{*}$	ame	P	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
	$\mathbf{M} \parallel 2$	$\underline{m}'m'm'$	fa	0	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
$m'm2'$	$\mathbf{P} \parallel 2'$	$m'm2'1'^{*}$	ame	MP	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, \mathbf{M}$
	$\mathbf{M} \perp m$	$\underline{m}mm'$	fa	0	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
		$\underline{m}'m'm$	fa	M	AR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, \mathbf{M}$
$m'\underline{m}^{*}\underline{2}'^{*}$	$\mathbf{P} \parallel \underline{2}'^{*}$	$m'\underline{m}'\underline{2}1'^{*}$	ame	P	SR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
	$\mathbf{M} \perp \underline{m}^{*}$	$m'\underline{m}^{*}\underline{2}'^{*}$	ame	MP	SI	$\mathbf{P}_1, \mathbf{M}_1$	$\mathbf{P}_2, \mathbf{M}_2$
		$m'\underline{m}^{*}m^{*}$	fa, ame	0	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
		$m'\underline{m}^{*}m'^{*}$	nme	M	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, \mathbf{M}$
$m\underline{m}'^{*}\underline{2}'^{*}$	$\mathbf{P} \parallel \underline{2}'^{*}$	$m\underline{m}\underline{2}1'^{*}$	nme, ame	P	SR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
	$\mathbf{M} \perp m$	$m\underline{m}'^{*}m^{*}$	fa, nme, ame	0	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
		$m\underline{m}'^{*}m'^{*}$	fa, nme	M	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, \mathbf{M}$
$m'\underline{m}'^{*}\underline{2}^{*}$	$\mathbf{P} \parallel \underline{2}^{*}$	$m'\underline{m}\underline{2}'1'^{*}$	ame	P	SR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
	$\mathbf{M} \parallel \underline{2}^{*}$	$m'\underline{m}'^{*}\underline{2}^{*}$	ame	MP	SI	$\mathbf{P}_1, \mathbf{M}_1$	$\mathbf{P}_2, \mathbf{M}_2$
		$m'\underline{m}'^{*}m'^{*}$	fa, ame, me, nme	0	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, -\mathbf{M}$
		$m'\underline{m}'^{*}m^{*}$	nme	M	SR	$\mathbf{P}, \mathbf{M}$	$-\mathbf{P}, \mathbf{M}$
3	$\mathbf{P} \parallel 3$	$31'^{*}$	ame	P	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
	$\mathbf{M} \parallel 3$	$3m^{*}$	ame	P	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$
		$3m'^{*}$	me	MP	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, \mathbf{M}$
$3m'$	$\mathbf{P}, \mathbf{M} \parallel 3$	$3m'1'^{*}$	ame	P	AR	$\mathbf{P}, \mathbf{M}$	$\mathbf{P}, -\mathbf{M}$

Chapter 5

Concluding summary

In this report the symmetry of domain walls associated with 380 magnetic completely transposable domain pairs was discussed. A special attention was given to 141 magnetoelectric and to 48 non-magnetic non-ferroelastic domain pairs. Symmetry group J_{12} of the unordered domain pair $\{S_1, S_2\}$ was expressed, in explicite notation, as

$$J_{12}[F_1] = F_1 + g^{\star}_{12} F_1, \tag{5.1}$$

where F_1 is the symmetry groups of the domain states S_1 and S_2, and $g^{\star}_{12}$ is an operation exchanging these domain states. For all these domain pairs the sectional magnetic point groups $\overline{J}_{12}$ were found for all crystallographic plane orbits $\{hkl\}$ represented by a sectional plane (hkl).

To find a symmetry layer group T_{12} describing a domain wall the standard process of left coset decomposition of sectional layer group $\overline{J}_{12}$ was used. Complete tables for non-ferroelastic domain pairs are presented in Appendix C.

As further analysis was restricted to possible existence of non-zero magnetization $\mathbf{M} \neq \mathbf{0}$ or polarization $\mathbf{P} \neq \mathbf{0}$ in a domain wall, 155 magnetic point layer groups were determined. It is possible to distinguish four parts of these magnetic layer groups:

a) 22 of them are pyromagnetic $\mathbf{M} \neq \mathbf{0}$ non-pyroelectric $\mathbf{P} = \mathbf{0}$,

b) 22 of them are non-pyromagnetic $\mathbf{M} = \mathbf{0}$ pyroelectric $\mathbf{P} \neq \mathbf{0}$,

c) 20 of them are pyromagnetic $\mathbf{M} \neq \mathbf{0}$ pyroelectric $\mathbf{P} \neq \mathbf{0}$,

d) 91 of them are non-pyromagnetic $\mathbf{M} = \mathbf{0}$ non-pyroelectric $\mathbf{P} = \mathbf{0}$.

A detailed description of the direction of magnetization and polarization with respect to the outer normal to the layer or their mutual direction is presented for all 64 magnetic layer groups from a), b), and c) .

Using these results it was possible to divide magnetic completely transposable domain pairs also into four types:

Type $\mathcal{MP}$ - groups $J_{12}[F_1]$ describing such domain pairs that some dmain walls can be simultaneously pyromagnetic and pyroelectric.
They cover 184 groups, 83 of them are magnetoelectric.

Type $\mathcal{M}$ - groups $J_{12}[F_1]$ describing such domain pairs that some domain walls can be pyromagnetic but *all* domain walls are non-pyroelectric.
They cover 69 groups, 58 of them are magnetoelectric.

Type $\mathcal{P}$ - groups $J_{12}[F_1]$ describing such domain pairs that domain walls can bea pyroelectric but *all* domain walls are non-pyromagnetic.
They cover 37 groups, none of them is magnetoelectric.

Type $\mathcal{O}$ - groups $J_{12}[F_1]$ describing such domain pairs that *all* domain walls are both non-pyromagnetic and non-pyroelectric.
They cover 90 groups, none of them is magnetoelectric.

A special attention was devoted to domain pairs with antiferromagnetic and non-ferroelectric domain states, i.e. with zero average magnetization and polarization. This situation appears in 169 cases:

- the type $\mathcal{MP}$ appears in 70 cases,
- the type $\mathcal{M}$ appears in 40 cases,
- the type $\mathcal{P}$ appears in 18 cases,
- the type $\mathcal{O}$ appears in 41 cases.

For domain pairs which belong to the types $\mathcal{MP}, \mathcal{M}$ and $\mathcal{P}$, i.e. for 128 cases, layer groups $T_{12} \in \overline{J}_{12}$ were determined. Complete tables are presented in Appendices G, H and I. Analysis of these tables enables one to determine all combinations of the symmetry group T_{12} of a domain wall and the sectional layer group $\overline{J}_{12}$ and to study some properties of a domain wall separately from the concrete plane (hkl) parallel to the domain wall.

Appendix A

Coordinate system and symmetry elements

Next four figures present coordinate systems, set of the symmetry elements and their notation used for the following crystallographic systems:

- orthorhombic
- tetragonal
- trigonal and hexagonal
- cubic

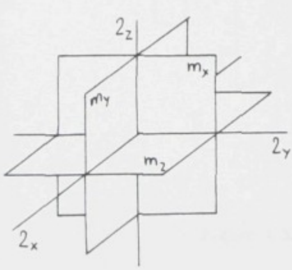


Figure A.1: Orthorhombic system

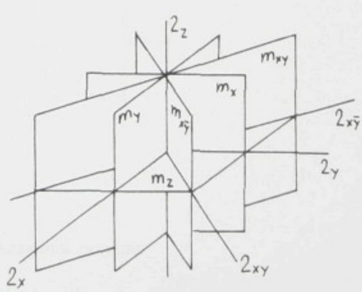


Figure A.2: Tetragonal system

Table A.1: **Symbols of symmetry operations of 6/mmm point group**

This report	1	2_z	3	6	3^2	6^5	2_{10}	2_{01}	$2_{\bar{1}\bar{1}}$	2_{12}	$2_{\bar{2}\bar{1}}$	$2_{1\bar{1}}$
Rectangular coor.sys.	1	2_z	3	6	3^2	6^5	2_x	$2_{x'}$	$2_{x''}$	2_y	$2_{y'}$	$2_{y''}$
This report	$\bar{1}$	m_z	$\bar{3}$	$\bar{6}$	$\bar{3}^2$	$\bar{6}^5$	$m_{2\bar{1}}$	$m_{\bar{1}2}$	$m_{\bar{1}\bar{1}}$	m_{01}	$m_{\bar{1}0}$	$m_{1\bar{1}}$
Rectangular coor.sys.	$\bar{1}$	m_z	$\bar{3}$	$\bar{6}$	$\bar{3}^2$	$\bar{6}^5$	m_x	$m_{x'}$	$m_{x''}$	m_y	$m_{y'}$	$m_{y''}$

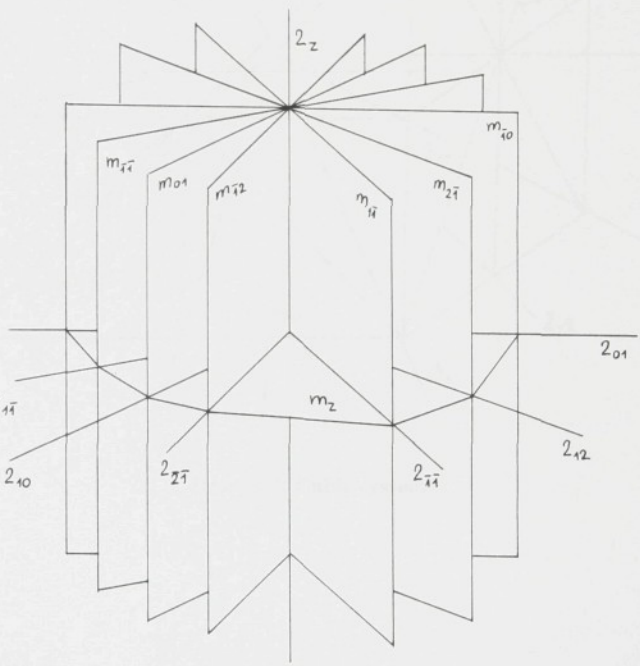


Figure A.3: Trigonal and hexagonal systems

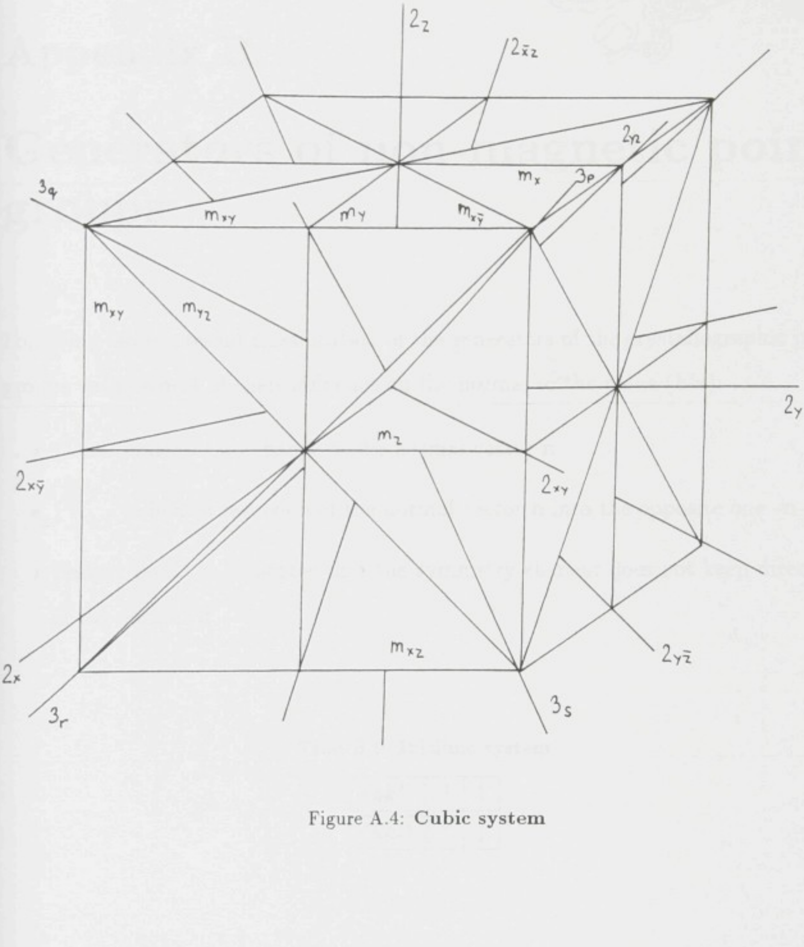


Figure A.4: **Cubic system**

Appendix B

Generators of non-magnetic point groups

Following tables present classification of the generators of the crystallographic point groups with respect of their influence on the normal to the plane (hkl):

- $\uparrow$ ..... indicates no change of the normal vector **n**
- $\downarrow$ ..... indicates reversion of the normal vector **n** into the opposite one -**n**
- "empty box" ... indicates that the symmetry element does not keep direction of the normal **n**

Table B.1: **Triclinic system**

(hkl)	1	$\bar{1}$
(hkl)	$\uparrow$	$\downarrow$

Table B.2: **Monoclinic system**

(hkl)	1	$\bar{1}$	2_z	m_z
(001)	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
$(hk0)$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
(hkl)	$\uparrow$	$\downarrow$		

Table B.3: **Orthorhombic system**

(hkl)	1	$\bar{1}$	2_x	2_y	2_z	m_x	m_y	m_z
(001)	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
(010)	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
(100)	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$
$2(hk0)$	$\uparrow$	$\downarrow$			$\downarrow$			$\uparrow$
$(h0l)$	$\uparrow$	$\downarrow$		$\downarrow$			$\uparrow$	
$(0kl)$	$\uparrow$	$\downarrow$	$\downarrow$			$\uparrow$		
(hkl)	$\uparrow$	$\downarrow$						

Table B.4: **Tetragonal system**

(hkl)	1	$\bar{1}$	2_x	2_y	2_z	2_{xy}	$2_{x\bar{y}}$	4_z	4_z^{-1}	$\bar{4}_z$	$\bar{4}_z^{-1}$	m_x	m_y	m_z	m_{xy}	$m_{x\bar{y}}$
(001)	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
(100)	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$							$\downarrow$	$\uparrow$	$\uparrow$		
(110)	$\uparrow$	$\downarrow$			$\downarrow$	$\uparrow$	$\downarrow$							$\uparrow$	$\downarrow$	$\uparrow$
$(hk0)$	$\uparrow$	$\downarrow$			$\downarrow$									$\uparrow$		
$(h0l)$	$\uparrow$	$\downarrow$		$\downarrow$									$\uparrow$			
(hhl)	$\uparrow$	$\downarrow$					$\downarrow$									$\uparrow$
(hkl)	$\uparrow$	$\downarrow$														

Table B.5: **Trigonal system**

(hkl)	1	$\bar{1}$	2_{10}	$2_{\bar{1}\bar{1}}$	2_{01}	3_z	3_z^{-1}	$\bar{3}_z$	$\bar{3}_z^{-1}$	$m_{2\bar{1}}$	$m_{\bar{1}2}$	$m_{\bar{1}\bar{1}}$
(0001)	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$(2\bar{1}\bar{1}0)$	$\uparrow$	$\downarrow$	$\uparrow$							$\downarrow$		
$(01\bar{1}0)$	$\uparrow$	$\downarrow$	$\downarrow$							$\uparrow$		
$(2h\bar{h}\bar{h}l)$	$\uparrow$	$\downarrow$										
$(0h\bar{h}l)$	$\uparrow$	$\downarrow$	$\downarrow$							$\uparrow$		
$(hki0)$	$\uparrow$	$\downarrow$										
$(hkil)$	$\uparrow$	$\downarrow$										

Table B.6: **Hexagonal system**

(hkl)	1	$\bar{1}$	2_{10}	$2_{\bar{1}\bar{1}}$	2_{01}	2_{12}	$2_{\bar{2}\bar{1}}$	$2_{1\bar{1}}$	2_z	3_z	3_z^{-1}	$\bar{3}_z$	$\bar{3}_z^{-1}$
(0001)	↑	↓	↓	↓	↓	↓	↓	↓	↑	↑	↑	↓	↓
$(2\bar{1}\bar{1}0)$	↑	↓	↑			↓			↓				
$(01\bar{1}0)$	↑	↓	↓			↑			↓				
$(2h\bar{h}\bar{h}l)$	↑	↓				↓							
$(0h\bar{h}l)$	↑	↓	↓										
$(hki0)$	↑	↓							↓				
$(hkil)$	↑	↓											

$(hkl)\|g$	6_z	6_z^{-1}	$\bar{6}_z$	$\bar{6}_z^{-1}$	$m_{2\bar{1}}$	$m_{\bar{1}2}$	$m_{\bar{1}\bar{1}}$	$m_{\bar{1}0}$	m_{01}	$m_{1\bar{1}}$	m_z
(0001)	↑	↑	↓	↓	↑	↑	↑	↑	↑	↑	↓
$(2\bar{1}\bar{1}0)$					↓				↑		↑
$(01\bar{1}0)$					↑				↓		↑
$(2h\bar{h}\bar{h}l)$									↑		
$(0h\bar{h}l)$					↑						
$(hki0)$											↑
$(hkil)$											

Table B.7: **Cubic system**

(hkl)	1	$\bar{1}$	2_x	2_y	2_z	2_{xy}	$2_{x\bar{y}}$	2_{xz}	2_{yz}	$2_{\bar{x}z}$	$2_{y\bar{z}}$
(001)	↑	↓	↓	↓	↑	↓	↓				
(110)	↑	↓			↓	↑	↓				
$(hk0)$	↑	↓			↓						
(hhl)	↑	↓					↓				
(hkl)	↑	↓									
(111)	↑	↓					↓			↓	↓

(hkl)	4_x	4_y	4_z	4_x^{-1}	4_y^{-1}	4_z^{-1}	$\bar{4}_x$	$\bar{4}_y$	$\bar{4}_z$	$\bar{4}_x^{-1}$	$\bar{4}_y^{-1}$	$\bar{4}_z^{-1}$
(001)			↑			↑			↓			↓
(110)												
$(hk0)$												
(hhl)												
(hkl)												
(111)												

(hkl)	3_p	3_q	3_r	3_s	3_p^{-1}	3_q^{-1}	3_r^{-1}	3_s^{-1}	$\bar{3}_p$	$\bar{3}_q$	$\bar{3}_r$	$\bar{3}_s$	$\bar{3}_p^{-1}$	$\bar{3}_q^{-1}$	$\bar{3}_r^{-1}$	$\bar{3}_s^{-1}$
(001)																
(110)																
$(hk0)$																
(hhl)																
(hkl)																
(111)	↑				↑				↓				↓			

(hkl)	m_x	m_y	m_z	m_{xy}	$m_{x\bar{y}}$	m_{xz}	m_{yz}	$m_{\bar{x}z}$	$m_{y\bar{z}}$
(001)	↑	↑	↓	↑	↑				
(110)			↑	↓	↑				
$(hk0)$			↑						
(hhl)					↑				
(hkl)									
(111)					↑			↑	↑

Appendix C

Symmetry groups of domain walls in non-magnetic non-ferroelastic domain pairs

The following table presents the sectional layer groups $\overline{J}_{12}$ and the symmetry groups T_{12} of the domain walls for all crystallographic non-equivalent planes (hkl) when the domain pairs $\{S_1, S_2\}$ are non-ferroelastic.

- F_1 .... symmetry group of the domain state S_1
- J_{12} .... symmetry group of the unordered domain pair $\{S_1, S_2\}$
- $\underline{s}_{12}, r^*_{12}$ and $\underline{t}^*_{12}$ .... side, state and side&state reversing operations
- $\widehat{F}_1$ .... one-sided sectional layer group containing all trivial symmetry operations of the domain wall
- $\overline{F}_{12} = \widehat{F}_1 + \underline{s}_{12}\widehat{F}_1$ .... sectional layer group of the group F_1
- $\widehat{J}_{12} = \widehat{F}_1 + r^*_{12}\widehat{F}_1$ .... one-sided layer group of the group J_{12}
- $T_{12} = \widehat{F}_1 + \underline{t}^*_{12}\widehat{F}_1$ .... symmetry group of the wall
- $\overline{J}_{12} = \widehat{F}_1 + \underline{t}^*_{12}\widehat{F}_1 + r^*_{12}\widehat{F}_1 + \underline{s}_{12}\widehat{F}_1$ ... the complete sectional layer group

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
1	$\bar{1}^*$	(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
2	$2/m^*$	(001)	2_z		2_z		2_z	$\underline{\bar{1}}^*$	$2_z/\underline{m}^*_z$	$2_z/\underline{m}^*_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
m	$2^*/m$	(001)	1	$\underline{m}_z$	$\underline{m}_z$	2^*_z	2^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$2^*_z/\underline{m}_z$
		$(hk0)$	m_z		m_z		m_z	$\underline{\bar{1}}^*$	$\underline{2}^*_z/m_z$	$\underline{2}^*_z/m_z$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
222	$m^*m^*m^*$	(001)	2_z	$\underline{2}_y$	$\underline{2}_x\underline{2}_y2_z$	m^*_x	$m^*_xm^*_y2_z$	$\underline{\bar{1}}^*$	$2_z/\underline{m}^*_z$	$\underline{2}_x/m^*_x\underline{2}_y/m^*_y2_z/\underline{m}^*_z$
		(010)	2_y	$\underline{2}_x$	$\underline{2}_x2_y\underline{2}_z$	m^*_z	$m^*_x2_ym^*_z$	$\underline{\bar{1}}^*$	$2_y/\underline{m}^*_y$	$\underline{2}_x/m^*_x2_y/\underline{m}^*_y\underline{2}_z/m^*_z$
		(100)	2_x	$\underline{2}_y$	$2_x\underline{2}_y\underline{2}_z$	m^*_y	$2_xm^*_ym^*_z$	$\underline{\bar{1}}^*$	$2_x/\underline{m}^*_x$	$2_x/\underline{m}^*_x\underline{2}_y/m^*_y\underline{2}_z/m^*_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		$(h0l)$	1	$\underline{2}_y$	$\underline{2}_y$	m^*_y	m^*_y	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_y/m^*_y$
		$(0kl)$	1	$\underline{2}_x$	$\underline{2}_x$	m^*_x	m^*_x	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_x/m^*_x$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
$mm2$	mmm^*	(001)	$m_xm_y2_z$		$m_xm_y2_z$		$m_xm_y2_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_x/m_x\underline{2}^*_y/m_y2_z/\underline{m}^*_z$	$\underline{2}^*_x/m_x\underline{2}^*_y/m_y2_z/\underline{m}^*_z$
		(010)	m_x	$\underline{2}_z$	$m_x\underline{m}_y\underline{2}_z$	2^*_y	$m_x2^*_ym^*_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_x/m_x$	$\underline{2}^*_x/m_x2^*_y/\underline{m}_y\underline{2}_z/m^*_z$
		(100)	m_y	$\underline{2}_z$	$\underline{m}_xm_y\underline{2}_z$	2^*_x	$2^*_xm_ym^*_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_y/m_y$	$2^*_x/\underline{m}_x\underline{2}^*_y/m_y\underline{2}_z/m^*_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		$(h0l)$	m_y		m_y		m_y	$\underline{\bar{1}}^*$	$\underline{2}^*_y/m_y$	$\underline{2}^*_y/m_y$
		$(0kl)$	m_x		m_x		m_x	$\underline{\bar{1}}^*$	$\underline{2}^*_x/m_x$	$\underline{2}^*_x/m_x$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
4	$4/m^*$	(001)	4_z		4_z		4_z	$\underline{\bar{1}}^*$	$4_z/\underline{m}^*_z$	$4_z/\underline{m}^*_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
4	42^*2^*	(001)	4_z		4_z		4_z	$\underline{2}^*_x$	$4_z\underline{2}^*_x\underline{2}^*_{xy}$	$4_z\underline{2}^*_x\underline{2}^*_{xy}$
		(100)	1	$\underline{2}_z$	$\underline{2}_z$	2^*_x	2^*_x	$\underline{2}^*_y$	$\underline{2}^*_y$	$2^*_x\underline{2}^*_y\underline{2}_z$
		(110)	1	$\underline{2}_z$	$\underline{2}_z$	2^*_{xy}	2^*_{xy}	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$2^*_{xy}\underline{2}^*_{x\bar{y}}\underline{2}_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$		1		1	$\underline{2}_z$
		(hhl)	1		1		1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$
		$(h0l)$	1		1		1	$\underline{2}^*_y$	$\underline{2}^*_y$	$\underline{2}^*_y$
		(hkl)	1		1		1		1	1
4	$4m^*m^*$	(001)	4_z		4_z	m^*_x	$4_zm^*_xm^*_{xy}$		4_z	$4_zm^*_xm^*_{xy}$
		(100)	1	$\underline{2}_z$	$\underline{2}_z$	m^*_y	m^*_y	$\underline{m}^*_x$	$\underline{m}^*_x$	$\underline{m}^*_xm^*_y\underline{2}_z$
		(110)	1	$\underline{2}_z$	$\underline{2}_z$	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$	$\underline{m}^*_{xy}$	$\underline{m}^*_{xy}$	$\underline{m}^*_{xy}m^*_{x\bar{y}}\underline{2}_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$		1		1	$\underline{2}_z$
		(hhl)	1		1	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$		1	$m^*_{x\bar{y}}$
		$(h0l)$	1		1	m^*_y	m^*_y		1	m^*_y
		(hkl)	1		1		1		1	1
$\bar{4}$	$4^*/m^*$	(001)	2_z	$\underline{\bar{4}}_z$	$\underline{\bar{4}}_z$	4^*_z	4^*_z	$\underline{\bar{1}}^*$	$4^*_z/\underline{m}^*_z$	$2_z/\underline{m}^*_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\hat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
$\bar{4}$	$\bar{4}2^*m^*$	(001)	2_z	$\underline{\bar{4}}_z$	$\underline{\bar{4}}_z$	m^*_{xy}	$m^*_{xy}m^*_{x\bar{y}}2_z$	$\underline{2}^*_x$	$\underline{2}^*_x\underline{2}^*_y2_z$	$\underline{\bar{4}}_z\underline{2}^*_xm^*_{xy}$
		(100)	1	$\underline{2}_z$	$\underline{2}_z$	2^*_x	2^*_x	$\underline{2}^*_y$	$\underline{2}^*_y$	$2^*_x\underline{2}^*_y\underline{2}_z$
		(110)	1	$\underline{2}_z$	$\underline{2}_z$	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$	$\underline{m}^*_{xy}$	$\underline{m}^*_{xy}$	$\underline{m}^*_{xy}m^*_{x\bar{y}}\underline{2}_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$		1		1	$\underline{2}_z$
		(hhl)	1		1	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$		1	$m^*_{x\bar{y}}$
		$(h0l)$	1		1		1	$\underline{2}^*_y$	$\underline{2}^*_y$	$\underline{2}^*_y$
		(hkl)	1		1		1		1	1
$\bar{4}$	$\bar{4}m^*2^*$	(001)	2_z	$\underline{\bar{4}}_z$	$\underline{\bar{4}}_z$	m^*_x	$m^*_xm^*_y2_z$	$\underline{2}^*_{xy}$	$\underline{2}^*_{xy}\underline{2}^*_{x\bar{y}}2_z$	$\underline{\bar{4}}_zm^*_x\underline{2}^*_{xy}$
		(100)	1	$\underline{2}_z$	$\underline{2}_z$	m^*_y	m^*_y	$\underline{m}^*_x$	$\underline{m}^*_x$	$\underline{m}^*_xm^*_y\underline{2}_z$
		(110)	1	$\underline{2}_z$	$\underline{2}_z$	2^*_{xy}	2^*_{xy}	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$2^*_{xy}\underline{2}^*_{x\bar{y}}\underline{2}_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$		1		1	$\underline{2}_z$
		(hhl)	1		1		1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$
		$(h0l)$	1		1	m^*_y	m^*_y		1	m^*_y
		(hkl)	1		1		1		1	1
$4/m$	$4/mm^*m^*$	(001)	4_z	$\underline{\bar{1}}$	$4_z/\underline{m}_z$	m^*_x	$4_zm^*_xm^*_{xy}$	$\underline{2}^*_x$	$4_z\underline{2}^*_x\underline{2}^*_{xy}$	$4_z/\underline{m}_z\underline{2}^*_x/m^*_x\underline{2}^*_{xy}/m^*_{xy}$
		(100)	m_z	$\underline{\bar{1}}$	$\underline{2}_z/m_z$	2^*_x	$2^*_xm^*_ym_z$	$\underline{2}^*_y$	$\underline{m}^*_x\underline{2}^*_ym_z$	$2^*_x/\underline{m}^*_x\underline{2}^*_y/m^*_y\underline{2}_z/m_z$
		(110)	m_z	$\underline{\bar{1}}$	$\underline{2}_z/m_z$	2^*_{xy}	$2^*_{xy}m^*_{x\bar{y}}m_z$	$\underline{2}^*_{x\bar{y}}$	$\underline{m}^*_{xy}\underline{2}^*_{x\bar{y}}m_z$	$2^*_{xy}/\underline{m}^*_{xy}\underline{2}^*_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m_z$
		$(hk0)$	m_z	$\underline{\bar{1}}$	$\underline{2}_z/m_z$		m_z		m_z	$\underline{2}_z/m_z$
		(hhl)	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m^*_{x\bar{y}}$
		$(h0l)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	m^*_y	m^*_y	$\underline{2}^*_y$	$\underline{2}^*_y$	$\underline{2}^*_y/m^*_y$
		(hkl)	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
422	$4/m^*m^*m^*$	(001)	4_z	$\underline{2}_x$	$4_z\underline{2}_x\underline{2}_{xy}$	m^*_x	$4_z m^*_x m^*_{xy}$	$\underline{\bar{1}}^*$	$4_z/\underline{m}^*_z$	$4_z/\underline{m}^*_z\underline{2}_x/m^*_x\underline{2}_{xy}/m^*_{xy}$
		(100)	2_x	$\underline{2}_z$	$2_x\underline{2}_y\underline{2}_z$	m^*_z	$2_x m^*_y m^*_z$	$\underline{\bar{1}}^*$	$2_x/\underline{m}^*_x$	$2_x/\underline{m}^*_x\underline{2}_y/m^*_y\underline{2}_z/m^*_z$
		(110)	2_{xy}	$\underline{2}_z$	$2_{xy}\underline{2}_{x\bar{y}}\underline{2}_z$	m^*_z	$2_{xy}m^*_{x\bar{y}}m^*_z$	$\underline{\bar{1}}^*$	$2_{xy}/\underline{m}^*_{xy}$	$2_{xy}/\underline{m}^*_{xy}\underline{2}_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m^*_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		(hhl)	1	$\underline{2}_{x\bar{y}}$	$\underline{2}_{x\bar{y}}$	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_{x\bar{y}}/m^*_{x\bar{y}}$
		$(h0l)$	1	$\underline{2}_y$	$\underline{2}_y$	m^*_y	m^*_y	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_y/m^*_y$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
$4mm$	$4/m^*mm$	(001)	$4_z m_x m_{xy}$		$4_z m_x m_{xy}$		$4_z m_x m_{xy}$	$\underline{\bar{1}}^*$	$4_z/\underline{m}^*_z\underline{2}^*_x/m_x\underline{2}^*_{xy}/m_{xy}$	$4_z/\underline{m}^*_z\underline{2}^*_x/m_x\underline{2}^*_{xy}/m_{xy}$
		(100)	m_y	$\underline{2}_z$	$\underline{m}_x m_y\underline{2}_z$	2^*_x	$2^*_x m_y m^*_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_y/m_y$	$2^*_x/\underline{m}_x\underline{2}^*_y/m_y\underline{2}_z/m^*_z$
		(110)	$m_{x\bar{y}}$	$\underline{2}_z$	$\underline{m}_{xy}m_{x\bar{y}}\underline{2}_z$	2^*_{xy}	$2^*_{xy}m_{x\bar{y}}m^*_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$2^*_{xy}/\underline{m}_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m^*_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		(hhl)	$m_{x\bar{y}}$		$m_{x\bar{y}}$		$m_{x\bar{y}}$	$\underline{\bar{1}}^*$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$
		$(h0l)$	m_y		m_y		m_y	$\underline{\bar{1}}^*$	$\underline{2}^*_y/m_y$	$\underline{2}^*_y/m_y$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
$\bar{4}2m$	$4^*/m^*m^*m$	(001)	$2_z m_{xy} m_{x\bar{y}}$	$\underline{2}_x$	$\underline{\bar{4}}_z\underline{2}_x m_{xy}$	m^*_x	$4^*_z m^*_x m_{xy}$	$\underline{\bar{1}}^*$	$\underline{2}^*_{xy}/m_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}2_z/\underline{m}^*_z$	$4^*_z/\underline{m}^*_z\underline{2}_x/m^*_x\underline{2}^*_{xy}/m_{xy}$
		(100)	2_x	$\underline{2}_z$	$2_x\underline{2}_y\underline{2}_z$	m^*_z	$2_x m^*_y m^*_z$	$\underline{\bar{1}}^*$	$2_x/\underline{m}^*_x$	$2_x/\underline{m}^*_x\underline{2}_y/m^*_y\underline{2}_z/m^*_z$
		(110)	$m_{x\bar{y}}$	$\underline{2}_z$	$\underline{m}_{xy}m_{x\bar{y}}\underline{2}_z$	m^*_z	$2^*_{xy}m_{x\bar{y}}m^*_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$2^*_{xy}/\underline{m}_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m^*_z$
		$(hk0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		(hhl)	$m_{x\bar{y}}$		$m_{x\bar{y}}$		$m_{x\bar{y}}$	$\underline{\bar{1}}^*$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$
		$(h0l)$	1	$\underline{2}_y$	$\underline{2}_y$	m^*_y	m^*_y	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_y/m^*_y$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
3	$\bar{3}^*$	(0001)	3_z		3_z		3_z	$\underline{\bar{1}}^*$	$\underline{\bar{3}}^*_z$	$\underline{\bar{3}}^*_z$
		$(hki0)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
		$(hkil)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
3	32^*1	(0001)	3_z		3_z		3_z	$\underline{2}^*_{10}$	$3_z\underline{2}^*_{10}1$	$3_z\underline{2}^*_{10}1$
		$(2\bar{1}\bar{1}0)$	1		1	2^*_{10}	2^*_{10}		1	2^*_{10}
		$(01\bar{1}0)$	1		1		1	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$
		$(\bar{2}hhhl)$	1		1		1		1	1
		$(0h\bar{h}\bar{l})$	1		1		1	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$
		$(hki0)$	1		1		1		1	1
		$(hkil)$	1		1		1		1	1
3	312^*	(0001)	3_z		3_z		3_z	$\underline{2}^*_{12}$	$3_z1\underline{2}^*_{12}$	$3_z1\underline{2}^*_{12}$
		$(2\bar{1}\bar{1}0)$	1		1		1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$
		$(01\bar{1}0)$	1		1	2^*_{12}	2^*_{12}		1	2^*_{12}
		$(2h\bar{h}\bar{h}\bar{l})$	1		1		1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$
		$(0\bar{h}hl)$	1		1		1		1	1
		$(hki0)$	1		1		1		1	1
		$(hkil)$	1		1		1		1	1

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
3	$3m^*1$	(0001)	3_z		3_z	$m^*_{2\bar{1}}$	$3_z m^*_{2\bar{1}} 1$		3_z	$3_z m^*_{2\bar{1}} 1$
		$(2\bar{1}\bar{1}0)$	1		1		1	$\underline{m}^*_{2\bar{1}}$	$\underline{m}^*_{2\bar{1}}$	$\underline{m}^*_{2\bar{1}}$
		$(01\bar{1}0)$	1		1	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$		1	$m^*_{2\bar{1}}$
		$(2h\bar{h}\bar{h}l)$	1		1		1		1	1
		$(0\bar{h}hl)$	1		1	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$		1	$m^*_{2\bar{1}}$
		$(hki0)$	1		1		1		1	1
		$(hkil)$	1		1		1		1	1
3	$31m^*$	(0001)	3_z		3_z	m^*_{01}	$3_z 1 m^*_{01}$		3_z	$3_z 1 m^*_{01}$
		$(2\bar{1}\bar{1}0)$	1		1	m^*_{01}	m^*_{01}		1	m^*_{01}
		$(01\bar{1}0)$	1		1		1	$\underline{m}^*_{01}$	$\underline{m}^*_{01}$	$\underline{m}^*_{01}$
		$(\bar{2}hhhl)$	1		1	m^*_{01}	m^*_{01}		1	m^*_{01}
		$(0h\bar{h}l)$	1		1		1		1	1
		$(hki0)$	1		1		1		1	1
		$(hkil)$	1		1		1		1	1
3	6^*	(0001)	3_z		3_z	2^*_z	6^*_z		3_z	6^*_z
		$(hki0)$	1		1		1	$\underline{2}^*_z$	$\underline{2}^*_z$	$\underline{2}^*_z$
		$(hkil)$	1		1		1		1	1
3	$\bar{6}^*$	(0001)	3_z		3_z		3_z	$\underline{m}^*_z$	$\underline{\bar{6}}^*_z$	$\underline{\bar{6}}^*_z$
		$(hki0)$	1		1	m^*_z	m^*_z		1	m^*_z
		$(hkil)$	1		1		1		1	1

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
$\bar{3}$	$\bar{3}m^*1$	(0001)	3_z	$\underline{\bar{1}}$	$\underline{\bar{3}}_z$	$m^*_{2\bar{1}}$	$3_z m^*_{2\bar{1}}1$	$\underline{2}^*_{10}$	$3_z\underline{2}^*_{10}1$	$\underline{\bar{3}}_z\underline{2}^*_{10}/m^*_{2\bar{1}}1$
		$(2\bar{1}\bar{1}0)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	2^*_{10}	2^*_{10}	$\underline{m}^*_{2\bar{1}}$	$\underline{m}^*_{2\bar{1}}$	$2^*_{10}/\underline{m}^*_{2\bar{1}}$
		$(01\bar{1}0)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}/m^*_{2\bar{1}}$
		$(2h\bar{h}\bar{h}l)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$
		$(0h\bar{h}\bar{l})$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}/m^*_{2\bar{1}}$
		$(hki0)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$
		$(hkil)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$
$\bar{3}$	$\bar{3}1m^*$	(0001)	3_z	$\underline{\bar{1}}$	$\underline{\bar{3}}_z$	m^*_{01}	$3_z1m^*_{01}$	$\underline{2}^*_{12}$	$3_z1\underline{2}^*_{12}$	$\underline{\bar{3}}_z1\underline{2}^*_{12}/m^*_{01}$
		$(2\bar{1}\bar{1}0)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	m^*_{01}	m^*_{01}	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m^*_{01}$
		$(01\bar{1}0)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	2^*_{12}	2^*_{12}	$\underline{m}^*_{01}$	$\underline{m}^*_{01}$	$2^*_{12}/\underline{m}^*_{01}$
		$(2h\bar{h}\bar{h}l)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	m^*_{01}	m^*_{01}	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m^*_{01}$
		$(0h\bar{h}l)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$
		$(hki0)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$
		$(hkil)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$
$\bar{3}$	$6^*/m^*$	(0001)	3_z	$\underline{\bar{1}}$	$\underline{\bar{3}}_z$	2^*_z	6^*_z	$\underline{m}^*_z$	$\underline{\bar{6}}^*_z$	$6^*_z/\underline{m}^*_z$
		$(hki0)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	m^*_z	m^*_z	$\underline{2}^*_z$	$\underline{2}^*_z$	$\underline{2}^*_z/m^*_z$
		$(hkil)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$
32	$\bar{3}^*m^*1$	(0001)	3_z	$\underline{2}_{10}$	$3_z\underline{2}_{10}1$	$m^*_{2\bar{1}}$	$3_z m^*_{2\bar{1}}1$	$\underline{\bar{1}}^*$	$\underline{\bar{3}}^*_z$	$\underline{\bar{3}}^*_z\underline{2}_{10}/m^*_{2\bar{1}}1$
		$(2\bar{1}\bar{1}0)$	2_{10}		2_{10}		2_{10}	$\underline{\bar{1}}^*$	$2_{10}/\underline{m}^*_{2\bar{1}}$	$2_{10}/\underline{m}^*_{2\bar{1}}$
		$(01\bar{1}0)$	1	$\underline{2}_{10}$	$\underline{2}_{10}$	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_{10}/m^*_{2\bar{1}}$
		$(2h\bar{h}\bar{h}l)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
		$(0h\bar{h}\bar{l})$	1	$\underline{2}_{10}$	$\underline{2}_{10}$	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_{10}/m^*_{2\bar{1}}$
		$(hki0)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
		$(hkil)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
32	6^*22^*	(0001)	3_z	$\underline{2}_{10}$	$3_z\underline{2}_{10}1$	2^*_z	6^*_z	$\underline{2}^*_{12}$	$3_z1\underline{2}^*_{12}$	$6^*_z\underline{2}_{10}\underline{2}^*_{12}$
		$(2\bar{1}\bar{1}0)$	2_{10}		2_{10}		2_{10}	$\underline{2}^*_z$	$2_{10}\underline{2}^*_{12}\underline{2}^*_z$	$2_{10}\underline{2}^*_{12}\underline{2}^*_z$
		$(01\bar{1}0)$	1	$\underline{2}_{10}$	$\underline{2}_{10}$	2^*_{12}	2^*_{12}	$\underline{2}^*_z$	$\underline{2}^*_z$	$\underline{2}_{10}2^*_{12}\underline{2}^*_z$
		$(2h\bar{h}\bar{h}l)$	1		1		1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$
		$(0h\bar{h}l)$	1	$\underline{2}_{10}$	$\underline{2}_{10}$		1		1	$\underline{2}_{10}$
		$(hki0)$	1		1		1	$\underline{2}^*_z$	$\underline{2}^*_z$	$\underline{2}^*_z$
		$(hkil)$	1		1		1		1	1
32	$\bar{6}^*2m^*$	(0001)	3_z	$\underline{2}_{10}$	$3_z\underline{2}_{10}1$	m^*_{01}	$3_z1m^*_{01}$	$\underline{m}^*_z$	$\underline{\bar{6}}^*_z$	$\underline{\bar{6}}^*_z\underline{2}_{10}m^*_{01}$
		$(2\bar{1}\bar{1}0)$	2_{10}		2_{10}	m^*_z	$2_{10}m^*_{01}m^*_z$		2_{10}	$2_{10}m^*_{01}m^*_z$
		$(01\bar{1}0)$	1	$\underline{2}_{10}$	$\underline{2}_{10}$	m^*_z	m^*_z	$\underline{m}^*_{01}$	$\underline{m}^*_{01}$	$\underline{2}_{10}\underline{m}^*_{01}m^*_z$
		$(\bar{2}hhl)$	1		1	m^*_{01}	m^*_{01}		1	m^*_{01}
		$(0h\bar{h}l)$	1	$\underline{2}_{10}$	$\underline{2}_{10}$		1		1	$\underline{2}_{10}$
		$(hki0)$	1		1	m^*_z	m^*_z		1	m^*_z
		$(hkil)$	1		1		1		1	1
$3m$	$\bar{3}^*m1$	(0001)	$3_zm_{2\bar{1}}1$		$3_zm_{2\bar{1}}1$		$3_zm_{2\bar{1}}1$	$\underline{\bar{1}}^*$	$\underline{\bar{3}}^*_z\underline{2}^*_{10}/m_{2\bar{1}}1$	$\underline{\bar{3}}^*_z\underline{2}^*_{10}/m_{2\bar{1}}1$
		$(2\bar{1}\bar{1}0)$	1	$\underline{m}_{2\bar{1}}$	$\underline{m}_{2\bar{1}}$	2^*_{10}	2^*_{10}	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$2^*_{10}/\underline{m}_{2\bar{1}}$
		$(01\bar{1}0)$	$m_{2\bar{1}}$		$m_{2\bar{1}}$		$m_{2\bar{1}}$	$\underline{\bar{1}}^*$	$\underline{2}^*_{10}/m_{2\bar{1}}$	$\underline{2}^*_{10}/m_{2\bar{1}}$
		$(2h\bar{h}\bar{h}l)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
		$(0h\bar{h}\bar{l})$	$m_{2\bar{1}}$		$m_{2\bar{1}}$		$m_{2\bar{1}}$	$\underline{\bar{1}}^*$	$\underline{2}^*_{10}/m_{2\bar{1}}$	$\underline{2}^*_{10}/m_{2\bar{1}}$
		$(hki0)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
		$(hkil)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
$3m$	6^*mm^*	(0001)	$3_z m_{2\bar{1}} 1$		$3_z m_{2\bar{1}} 1$	m^*_{01}	$6^*_z m_{2\bar{1}} m^*_{01}$		$3_z m_{2\bar{1}} 1$	$6^*_z m_{2\bar{1}} m^*_{01}$
		$(2\bar{1}\bar{1}0)$	1	$\underline{m}_{2\bar{1}}$	$\underline{m}_{2\bar{1}}$	m^*_{01}	m^*_{01}	$\underline{2}^*_z$	$\underline{2}^*_z$	$\underline{m}_{2\bar{1}} m^*_{01} \underline{2}^*_z$
		$(01\bar{1}0)$	$m_{2\bar{1}}$		$m_{2\bar{1}}$		$m_{2\bar{1}}$	$\underline{2}^*_z$	$m_{2\bar{1}} \underline{m}^*_{01} \underline{2}^*_z$	$m_{2\bar{1}} \underline{m}^*_{01} \underline{2}^*_z$
		$(2h\bar{h}\bar{h}l)$	1		1	m^*_{01}	m^*_{01}		1	m^*_{01}
		$(0h\bar{h}l)$	$m_{2\bar{1}}$		$m_{2\bar{1}}$		$m_{2\bar{1}}$		$m_{2\bar{1}}$	$m_{2\bar{1}}$
		$(hki0)$	1		1		1	$\underline{2}^*_z$	$\underline{2}^*_z$	$\underline{2}^*_z$
		$(hkil)$	1		1		1		1	1
$3m$	$\bar{6}^*m2^*$	(0001)	$3_z m_{2\bar{1}} 1$		$3_z m_{2\bar{1}} 1$		$3_z m_{2\bar{1}} 1$	$\underline{m}^*_z$	$\underline{\bar{6}}^*_z m_{2\bar{1}} \underline{2}^*_{12}$	$\underline{\bar{6}}^*_z m_{2\bar{1}} \underline{2}^*_{12}$
		$(2\bar{1}\bar{1}0)$	1	$\underline{m}_{2\bar{1}}$	$\underline{m}_{2\bar{1}}$	m^*_z	m^*_z	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12} \underline{m}_{2\bar{1}} m^*_z$
		$(01\bar{1}0)$	$m_{2\bar{1}}$		$m_{2\bar{1}}$	2^*_{12}	$2^*_{12} m_{2\bar{1}} m^*_z$		$m_{2\bar{1}}$	$2^*_{12} m_{2\bar{1}} m^*_z$
		$(2h\bar{h}\bar{h}l)$	1		1		1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$
		$(0\bar{h}hl)$	$m_{2\bar{1}}$		$m_{2\bar{1}}$		$m_{2\bar{1}}$		$m_{2\bar{1}}$	$m_{2\bar{1}}$
		$(hki0)$	1		1	m^*_z	m^*_z		1	m^*_z
		$(hkil)$	1		1		1		1	1
$\bar{3}m$	$6^*/m^*mm^*$	(0001)	$3_z m_{2\bar{1}}$	$\underline{\bar{1}}$	$\underline{\bar{3}}_z \underline{2}_{10}/m_{2\bar{1}} 1$	m^*_{01}	$6^*_z m_{2\bar{1}} m^*_{01}$	$\underline{m}^*_z$	$\underline{\bar{6}}^*_z m_{2\bar{1}} \underline{2}^*_{12}$	$6^*_z/\underline{m}^*_z \underline{2}_{10}/m_{2\bar{1}} \underline{2}^*_{12}/m^*_{01}$
		$(2\bar{1}\bar{1}0)$	2_{10}	$\underline{\bar{1}}$	$2_{10}/\underline{m}_{2\bar{1}}$	m^*_z	$2_{10} m^*_{01} m^*_z$	$\underline{2}^*_z$	$2_{10} \underline{2}^*_{12} \underline{2}^*_z$	$2_{10}/\underline{m}_{2\bar{1}} \underline{2}^*_{12}/m^*_{01} \underline{2}^*_z/m^*_z$
		$(01\bar{1}0)$	$m_{2\bar{1}}$	$\underline{\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}}$	m^*_z	$2^*_{12} m_{2\bar{1}} m^*_z$	$\underline{2}^*_z$	$\underline{m}^*_{01} m_{2\bar{1}} \underline{2}^*_z$	$\underline{2}_{10}/m_{2\bar{1}} 2^*_{12}/\underline{m}^*_{01} \underline{2}^*_z/m^*_z$
		$(2h\bar{h}\bar{h}l)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	m^*_{01}	m^*_{01}	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m^*_{01}$
		$(0h\bar{h}l)$	$m_{2\bar{1}}$	$\underline{\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}}$		$m_{2\bar{1}}$		$m_{2\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}}$
		$(hki0)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	m^*_z	m^*_z	$\underline{2}^*_z$	$\underline{2}^*_z$	$\underline{2}^*_z/m^*_z$
		$(hkil)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
6	$6/m^*$	(0001)	6_z		6_z		6_z	$\underline{\bar{1}}^*$	$6_z/\underline{m}^*_z$	$6_z/\underline{m}^*_z$
		$(hki0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		$(hkil)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
6	62^*2^*	(0001)	6_z		6_z		6_z	$\underline{2}^*_{10}$	$6_z\underline{2}^*_{10}\underline{2}^*_{12}$	$6_z\underline{2}^*_{10}\underline{2}^*_{12}$
		$(2\bar{1}\bar{1}0)$	1	$\underline{2}_z$	$\underline{2}_z$	2^*_{10}	2^*_{10}	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$2^*_{10}\underline{2}^*_{12}\underline{2}_z$
		$(01\bar{1}0)$	1	$\underline{2}_z$	$\underline{2}_z$	2^*_{12}	2^*_{12}	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}2^*_{12}\underline{2}_z$
		$(2h\bar{h}\bar{h}l)$	1		1		1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$
		$(0h\bar{h}l)$	1		1		1	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$
		$(hki0)$	1	$\underline{2}_z$	$\underline{2}_z$		1		1	$\underline{2}_z$
		$(hkil)$	1		1		1		1	1
6	$6m^*m^*$	(0001)	6_z		6_z	m^*_{01}	$6_zm^*_{2\bar{1}}m^*_{01}$		6_z	$6_zm^*_{2\bar{1}}m^*_{01}$
		$(2\bar{1}\bar{1}0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_{01}	m^*_{01}	$\underline{m}^*_{2\bar{1}}$	$\underline{m}^*_{2\bar{1}}$	$\underline{m}^*_{2\bar{1}}m^*_{01}\underline{2}_z$
		$(01\bar{1}0)$	1	$\underline{2}_z$	$\underline{2}_z$	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$	$\underline{m}^*_{01}$	$\underline{m}^*_{01}$	$m^*_{2\bar{1}}\underline{m}^*_{01}\underline{2}_z$
		$(2h\bar{h}\bar{h}l)$	1		1	m^*_{01}	m^*_{01}		1	m^*_{01}
		$(0h\bar{h}l)$	1		1	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$		1	$m^*_{2\bar{1}}$
		$(hki0)$	1	$\underline{2}_z$	$\underline{2}_z$		1		1	$\underline{2}_z$
		$(hkil)$	1		1		1		1	1
$\bar{6}$	$6^*/m$	(0001)	3_z	$\underline{m}_z$	$\underline{\bar{6}}_z$	2^*_z	6^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{3}}^*_z$	$6^*_z/\underline{m}_z$
		$(hki0)$	m_z		m_z		m_z	$\underline{\bar{1}}^*$	$\underline{2}^*_z/m_z$	$\underline{2}^*_z/m_z$
		$(hkil)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
$\bar{6}$	$\bar{6}2^*m^*$	(0001)	3_z	$\underline{m}_z$	$\underline{\bar{6}}_z$	m^*_{01}	$3_z1m^*_{01}$	$\underline{2}^*_{10}$	$3_z\underline{2}^*_{10}1$	$\underline{\bar{6}}_z\underline{2}^*_{10}m^*_{01}$
		$(2\bar{1}\bar{1}0)$	m_z		m_z	2^*_{10}	$2^*_{10}m^*_{01}m_z$		m_z	$2^*_{10}m^*_{01}m_z$
		$(01\bar{1}0)$	m_z		m_z		m_z	$\underline{2}^*_{10}$	$\underline{2}^*_{10}\underline{m}^*_{01}m_z$	$\underline{2}^*_{10}\underline{m}^*_{01}m_z$
		$(\bar{2}hhhl)$	1		1	m^*_{01}	m^*_{01}		1	m^*_{01}
		$(0h\bar{h}l)$	1		1		1	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$
		$(hki0)$	m_z		m_z		m_z		m_z	m_z
		$(hkil)$	1		1		1		1	1
$\bar{6}$	$\bar{6}m^*2^*$	(0001)	3_z	$\underline{m}_z$	$\underline{\bar{6}}_z$	$m^*_{2\bar{1}}$	$3_zm^*_{2\bar{1}}1$	$\underline{2}^*_{12}$	$3_z1\underline{2}^*_{12}$	$\underline{\bar{6}}_zm^*_{2\bar{1}}\underline{2}^*_{12}$
		$(2\bar{1}\bar{1}0)$	m_z		m_z		m_z	$\underline{2}^*_{12}$	$\underline{2}^*_{12}\underline{m}^*_{2\bar{1}}m_z$	$\underline{2}^*_{12}\underline{m}^*_{2\bar{1}}m_z$
		$(01\bar{1}0)$	m_z		m_z	2^*_{12}	$2^*_{12}m^*_{2\bar{1}}m_z$		m_z	$2^*_{12}m^*_{2\bar{1}}m_z$
		$(2h\bar{h}\bar{h}l)$	1		1		1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$
		$(0\bar{h}hl)$	1		1	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$		1	$m^*_{2\bar{1}}$
		$(hki0)$	m_z		m_z		m_z		m_z	m_z
		$(hkil)$	1		1		1		1	1
$6/m$	$6/mm^*m^*$	(0001)	6_z	$\underline{\bar{1}}$	$6_z/\underline{m}_z$	m^*_{01}	$6_zm^*_{2\bar{1}}m^*_{01}$	$\underline{2}^*_{10}$	$6_z\underline{2}^*_{10}\underline{2}^*_{12}$	$6_z/\underline{m}_z\underline{2}^*_{10}/m^*_{2\bar{1}}\underline{2}^*_{12}/m^*_{01}$
		$(2\bar{1}\bar{1}0)$	m_z	$\underline{\bar{1}}$	$\underline{2}_z/m_z$	2^*_{10}	$2^*_{10}m^*_{01}m_z$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}\underline{m}^*_{2\bar{1}}m_z$	$2^*_{10}/\underline{m}^*_{2\bar{1}}\underline{2}^*_{12}/m^*_{01}\underline{2}_z/m_z$
		$(01\bar{1}0)$	m_z	$\underline{\bar{1}}$	$\underline{2}_z/m_z$	$m^*_{2\bar{1}}$	$2^*_{12}m^*_{2\bar{1}}m_z$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}\underline{m}^*_{01}m_z$	$\underline{2}^*_{10}/m^*_{2\bar{1}}2^*_{12}/\underline{m}^*_{01}\underline{2}_z/m_z$
		$(2h\bar{h}\bar{h}l)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	m^*_{01}	m^*_{01}	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m^*_{01}$
		$(0h\bar{h}l)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}$	$\underline{2}^*_{10}/m^*_{2\bar{1}}$
		$(hki0)$	m_z	$\underline{\bar{1}}$	$\underline{2}_z/m_z$		m_z		m_z	$\underline{2}_z/m_z$
		$(hkil)$	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\hat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
622	$6/m^*m^*m^*$	(0001)	6_z	$\underline{2}_{10}$	$6_z\underline{2}_{10}\underline{2}_{12}$	m^*_{01}	$6_z m^*_{2\bar{1}} m^*_{01}$	$\underline{\bar{1}}^*$	$6_z/\underline{m}^*_z$	$6_z/\underline{m}^*_z\underline{2}_{10}/m^*_{2\bar{1}}\underline{2}_{12}/m^*_{01}$
		$(2\bar{1}\bar{1}0)$	2_{10}	$\underline{2}_z$	$2_{10}\underline{2}_{12}\underline{2}_z$	m^*_{01}	$2_{10}m^*_{01}m^*_z$	$\underline{\bar{1}}^*$	$2_{10}/\underline{m}^*_{2\bar{1}}$	$2_{10}/\underline{m}^*_{2\bar{1}}\underline{2}_{12}/m^*_{01}\underline{2}_z/m^*_z$
		$(01\bar{1}0)$	2_{12}	$\underline{2}_z$	$\underline{2}_{10}2_{12}\underline{2}_z$	m^*_z	$2_{12}m^*_{2\bar{1}}m^*_z$	$\underline{\bar{1}}^*$	$2_{12}/\underline{m}^*_{01}$	$\underline{2}_{10}/m^*_{2\bar{1}}2_{12}/\underline{m}^*_{01}\underline{2}_z/m^*_z$
		$(2\bar{h}\bar{h}hl)$	1	$\underline{2}_{12}$	$\underline{2}_{12}$	m^*_{01}	m^*_{01}	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_{12}/m^*_{01}$
		$(0h\bar{h}l)$	1	$\underline{2}_{10}$	$\underline{2}_{10}$	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_{10}/m^*_{2\bar{1}}$
		$(hki0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		$(hkil)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
$6mm$	$6/m^*mm$	(0001)	$6_z m_{2\bar{1}} m_{01}$		$6_z m_{2\bar{1}} m_{01}$		$6_z m_{2\bar{1}} m_{01}$	$\underline{\bar{1}}^*$	$6_z/\underline{m}^*_z\underline{2}^*_{10}/m_{2\bar{1}}\underline{2}^*_{12}/m_{01}$	$6_z/\underline{m}^*_z\underline{2}^*_{10}/m_{2\bar{1}}\underline{2}^*_{12}/m_{01}$
		$(2\bar{1}\bar{1}0)$	m_{01}	$\underline{2}_z$	$\underline{m}_{2\bar{1}}m_{01}\underline{2}_z$	2^*_{10}	$2^*_{10}m_{01}m^*_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_{12}/m_{01}$	$2^*_{10}/\underline{m}_{2\bar{1}}\underline{2}^*_{12}/m_{01}\underline{2}_z/m^*_z$
		$(01\bar{1}0)$	$m_{2\bar{1}}$	$\underline{2}_z$	$m_{2\bar{1}}\underline{m}_{01}\underline{2}_z$	2^*_{12}	$2^*_{12}m_{2\bar{1}}m^*_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_{10}/m_{2\bar{1}}$	$\underline{2}^*_{10}/m_{2\bar{1}}2^*_{12}/\underline{m}_{01}\underline{2}_z/m^*_z$
		$(2\bar{h}\bar{h}hl)$	m_{01}		m_{01}		m_{01}	$\underline{\bar{1}}^*$	$\underline{2}^*_{12}/m_{01}$	$\underline{2}^*_{12}/m_{01}$
		$(0h\bar{h}l)$	$m_{2\bar{1}}$		$m_{2\bar{1}}$		$m_{2\bar{1}}$	$\underline{\bar{1}}^*$	$\underline{2}^*_{10}/m_{2\bar{1}}$	$\underline{2}^*_{10}/m_{2\bar{1}}$
		$(hki0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		$(hkil)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
$\bar{6}2m$	$6^*/mm^*m$	(0001)	$3_z m_{01}$	$\underline{m}_z$	$\underline{\bar{6}}_z\underline{2}_{10}m_{01}$	$m^*_{2\bar{1}}$	$6^*_z m^*_{2\bar{1}} m_{01}$	$\underline{\bar{1}}^*$	$\underline{\bar{3}}^*_z 1\underline{2}^*_{12}/m_{01}$	$6^*_z/\underline{m}_z\underline{2}_{10}/m^*_{2\bar{1}}\underline{2}^*_{12}/m_{01}$
		$(2\bar{1}\bar{1}0)$	$m_z 2_{10} m_{01}$		$2_{10}m_{01}m_z$		$2_{10}m_{01}m_z$	$\underline{\bar{1}}^*$	$2_{10}/\underline{m}^*_{2\bar{1}}\underline{2}^*_{12}/m_{01}\underline{2}^*_z/m_z$	$2_{10}/\underline{m}^*_{2\bar{1}}\underline{2}^*_{12}/m_{01}\underline{2}^*_z/m_z$
		$(01\bar{1}0)$	m_z	$\underline{2}_{10}$	$\underline{2}_{10}\underline{m}_{01}m_z$	2^*_{12}	$2^*_{12}m^*_{2\bar{1}}m_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_z/m_z$	$\underline{2}_{10}/m^*_{2\bar{1}}2^*_{12}/\underline{m}_{01}\underline{2}^*_z/m_z$
		$(2\bar{h}\bar{h}hl)$	m_{01}		m_{01}		m_{01}	$\underline{\bar{1}}^*$	$\underline{2}^*_{12}/m_{01}$	$\underline{2}^*_{12}/m_{01}$
		$(0h\bar{h}l)$	1	$\underline{2}_{10}$	$\underline{2}_{10}$	$m^*_{2\bar{1}}$	$m^*_{2\bar{1}}$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_{10}/m^*_{2\bar{1}}$
		$(hki0)$	m_z		m_z		m_z	$\underline{\bar{1}}^*$	$\underline{2}^*_z/m_z$	$\underline{2}^*_z/m_z$
		$(hkil)$	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
23	$m^*\bar{3}^*$	(001)	2_z	$\underline{2}_x$	$\underline{2}_x\underline{2}_y 2_z$	m^*_x	$m^*_x m^*_y 2_z$	$\underline{\bar{1}}^*$	$2_z/\underline{m}^*_z$	$\underline{2}_x/m^*_x\underline{2}_y/m^*_y 2_z/\underline{m}^*_z$
		(110)	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		$(kl0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		(hhl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
		(111)	3_p		3_p		3_p	$\underline{\bar{1}}^*$	$\underline{\bar{3}}^*_p$	$\underline{\bar{3}}^*_p$
23	4^*32^*	(001)	2_z	$\underline{2}_x$	$\underline{2}_x\underline{2}_y 2_z$	4^*_z	4^*_z	$\underline{2}^*_{xy}$	$2^*_{xy}\underline{2}^*_{x\bar{y}}2_z$	$4^*_z\underline{2}_x\underline{2}^*_{xy}$
		(110)	1	$\underline{2}_z$	$\underline{2}_z$	2^*_{xy}	2^*_{xy}	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$2^*_{xy}\underline{2}^*_{x\bar{y}}\underline{2}_z$
		$(kl0)$	1	$\underline{2}_z$	$\underline{2}_z$		1		1	$\underline{2}_z$
		(hhl)	1		1		1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$
		(hkl)	1		1		1		1	1
		(111)	3_p		3_p		3_p	$\underline{2}^*_{x\bar{y}}$	$3_p\underline{2}^*_{x\bar{y}}$	$3_p\underline{2}^*_{x\bar{y}}$
23	$\bar{4}^*3m^*$	(001)	2_z	$\underline{2}_x$	$\underline{2}_x\underline{2}_y 2_z$	m^*_{xy}	$m^*_{xy}m^*_{x\bar{y}}2_z$	$\underline{\bar{4}}^*_z$	$\underline{\bar{4}}^*_z$	$\underline{\bar{4}}^*_z\underline{2}_x m^*_{xy}$
		(110)	1	$\underline{2}_z$	$\underline{2}_z$	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$	$\underline{m}^*_{xy}$	$\underline{m}^*_{xy}$	$\underline{m}^*_{xy}m^*_{x\bar{y}}\underline{2}_z$
		$(kl0)$	1	$\underline{2}_z$	$\underline{2}_z$		1		1	$\underline{2}_z$
		(hhl)	1		1	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$		1	$m^*_{x\bar{y}}$
		(hkl)	1		1		1		1	1
		(111)	3_p		3_p	$m^*_{x\bar{y}}$	$3_p m^*_{x\bar{y}}$		3_p	$3_p m^*_{x\bar{y}}$

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	$\underline{s}_{1j}$	$\overline{F}_1$	r^*_{1j}	$\widehat{J}_{1j}$	$\underline{t}^*_{1j}$	T_{1j}	$\overline{J}_{1j}$
$m\bar{3}$	$m\bar{3}m^*$	(001)	$2_z m_x m_y$	$\underline{\bar{1}}$	$\underline{2}_x/m_x\underline{2}_y/m_y 2_z/\underline{m}_z$	m^*_{xy}	$4^*_z m_x m^*_{xy}$	$\underline{2}^*_{xy}$	$\underline{\bar{4}}^*_z m_x \underline{2}^*_{xy}$	$4^*_z/\underline{m}_z\underline{2}_x/m_x\underline{2}^*_{xy}/m^*_{xy}$
		(110)	m_z	$\underline{\bar{1}}$	$\underline{2}_z/m_z$	2^*_{xy}	$2^*_{xy} m^*_{x\bar{y}} m_z$	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}\underline{m}^*_{xy} m_z$	$2^*_{xy}/\underline{m}^*_{xy}\underline{2}^*_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m_z$
		$(kl0)$	m_z	$\underline{\bar{1}}$	$\underline{2}_z/m_z$		m_z		m_z	$\underline{2}_z/m_z$
		(hhl)	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m^*_{x\bar{y}}$
		(hkl)	1	$\underline{\bar{1}}$	$\underline{\bar{1}}$		1		1	$\underline{\bar{1}}$
		(111)	3_p	$\underline{\bar{1}}$	$\underline{\bar{3}}_p$	$m^*_{x\bar{y}}$	$3_p m^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	$3_p\underline{2}^*_{x\bar{y}}$	$\underline{\bar{3}}_p\underline{2}^*_{x\bar{y}}/m^*_{x\bar{y}}$
432	$m^*\bar{3}^*m^*$	(001)	4_z	$\underline{2}_x$	$4_z\underline{2}_x\underline{2}_{xy}$	m^*_x	$4_z m^*_x m^*_y$	$\underline{\bar{1}}^*$	$4_z/\underline{m}^*_z$	$4_z/\underline{m}^*_z\underline{2}_x/m^*_x\underline{2}_{xy}/m^*_{xy}$
		(110)	2_{xy}	$\underline{2}_z$	$2_{xy}\underline{2}_{x\bar{y}}\underline{2}_z$	m^*_z	$2_{xy} m^*_{x\bar{y}} m^*_z$	$\underline{\bar{1}}^*$	$2_{xy}/\underline{m}^*_{xy}$	$2_{xy}/\underline{m}^*_{xy}\underline{2}_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m^*_z$
		$(kl0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		(hhl)	1	$\underline{2}_{x\bar{y}}$	$\underline{2}_{x\bar{y}}$	$m^*_{x\bar{y}}$	$m^*_{x\bar{y}}$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_{x\bar{y}}/m^*_{x\bar{y}}$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
		(111)	3_p	$\underline{2}_{x\bar{y}}$	$3_p\underline{2}_{x\bar{y}}$	$m^*_{x\bar{y}}$	$3_p m^*_{x\bar{y}}$	$\underline{\bar{1}}^*$	$\underline{\bar{3}}^*_p$	$\underline{\bar{3}}^*_p\underline{2}_{x\bar{y}}/m^*_{x\bar{y}}$
$\bar{4}3m$	$m^*\bar{3}^*m$	(001)	$2_z m_{xy} m_{x\bar{y}}$	$\underline{2}_x$	$\underline{\bar{4}}_z\underline{2}_x m_{xy}$	m^*_x	$4^*_z m^*_x m_{xy}$	$\underline{\bar{1}}^*$	$\underline{2}^*_{xy}/m_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}2_z/\underline{m}^*_z$	$4^*_z/\underline{m}^*_z\underline{2}_x/m^*_x\underline{2}^*_{xy}/m_{xy}$
		(110)	$m_{x\bar{y}}$	$\underline{2}_z$	$\underline{m}_{xy} m_{x\bar{y}}\underline{2}_z$	m^*_z	$2^*_{xy} m_{x\bar{y}} m^*_z$	$\underline{\bar{1}}^*$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$2^*_{xy}/\underline{m}_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m^*_z$
		$(kl0)$	1	$\underline{2}_z$	$\underline{2}_z$	m^*_z	m^*_z	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$
		(hhl)	$m_{x\bar{y}}$		$m_{x\bar{y}}$		$m_{x\bar{y}}$	$\underline{\bar{1}}^*$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$
		(hkl)	1		1		1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$
		(111)	$3_p m_{x\bar{y}}$		$3_p m_{x\bar{y}}$		$3_p m_{x\bar{y}}$	$\underline{\bar{1}}^*$	$\underline{\bar{3}}^*_p\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$\underline{\bar{3}}^*_p\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$

Appendix D

Symmetry classification of non-magnetic non-ferroelastic domain walls

Following tables present classification of walls for all crystallographic non-equivalent planes (hkl) for 48 non-ferroelastic domain pairs.

- F_1 .... symmetry of the single domain
- J_{12} ...... symmetry of the domain pair
- S .... symmetrical wall
- A .... asymmetrical wall
- R .... reversible wall
- I .... irreversible wall

F_1	J_{12}	(hkl)	$(hk0)$	(001)
1	$\bar{1}^*$	S	S	S
2	$2/m^*$	S	S	S
m	$2^*/m$	S	S	S

F_1	J_{12}	(hkl)	$(hk0)$	(001)
1	$\bar{1}^*$	I	I	I
2	$2/m^*$	I	R	I
m	$2^*/m$	I	I	R

F_1	J_{12}	(hkl)	$(0kl)$	$(h0l)$	$(hk0)$	(100)	(010)	(001)
222	$m^*m^*m^*$	S	S	S	S	S	S	S
$mm2$	mm^*m	S	S	S	S	S	S	S

F_1	J_{12}	(hkl)	$(0kl)$	$(h0l)$	$(hk0)$	(100)	(010)	(001)
222	$m^*m^*m^*$	I	R	R	R	R	R	R
$mm2$	mm^*m	I	I	I	R	R	R	I

F_1	J_{12}	(hkl)	(hhl)	$(h0l)$	$(hk0)$	(110)	(100)	(001)
4	$4/m^*$	S	S	S	S	S	S	S
$\bar{4}$	$4^*/m^*$	S	S	S	S	S	S	S
4	42^*2^*	A	S	S	A	S	S	S
4	$4m^*m^*$	A	A	A	A	S	S	A
$\bar{4}$	$\bar{4}2^*m^*$	A	A	S	A	S	S	S
$\bar{4}$	$\bar{4}m^*2^*$	A	S	A	A	S	S	S
$4/m$	$4/mm^*m^*$	A	S	S	A	S	S	S
422	$4/m^*m^*m^*$	S	S	S	S	S	S	S
$4mm$	$4/m^*mm$	S	S	S	S	S	S	S
$\bar{4}2m$	$4^*/m^*m^*m$	S	S	S	S	S	S	S

F_1	J_{12}	(hkl)	(hhl)	$(h0l)$	$(hk0)$	(110)	(100)	(001)
4	$4/m^*$	I	I	I	R	R	R	I
$\bar{4}$	$4^*/m^*$	I	I	I	R	R	R	R
4	42^*2^*	I	I	I	R	R	R	I
4	$4m^*m^*$	I	R	R	R	R	R	R
$\bar{4}$	$\bar{4}2^*m^*$	I	R	I	R	R	R	R
$\bar{4}$	$\bar{4}m^*2^*$	I	I	R	R	R	R	R
$4/m$	$4/mm^*m^*$	R	R	R	R	R	R	R
422	$4/m^*m^*m^*$	I	R	R	R	R	R	R
$4mm$	$4/m^*mm$	I	I	I	R	R	R	I
$\bar{4}2m$	$4^*/m^*m^*m$	I	I	R	R	R	R	R

F_1	J_{12}	(hkl)	(hhl)	(111)	$(hk0)$	(110)	(001)
23	$m^*\bar{3}^*$	S	S	S	S	S	S
23	4^*32^*	A	S	S	A	S	S
23	$\bar{4}^*3m^*$	A	A	A	A	S	S
$m\bar{3}$	$m\bar{3}m^*$	A	S	S	A	S	S
432	$m^*\bar{3}^*m^*$	S	S	S	S	S	S
$\bar{4}3m$	$m^*\bar{3}^*m$	S	S	S	S	S	S

F_1	J_{12}	(hkl)	(hhl)	(111)	$(hk0)$	(110)	(001)
23	$m^*\bar{3}^*$	I	I	I	R	R	R
23	4^*32^*	I	I	I	R	R	R
23	$\bar{4}^*3m^*$	I	R	R	R	R	R
$m\bar{3}$	$m\bar{3}m^*$	R	R	R	R	R	R
432	$m^*\bar{3}^*m^*$	I	R	R	R	R	R
$\bar{4}3m$	$m^*\bar{3}^*m$	I	I	I	R	R	R

F_1	J_{12}	$(hkil)$	$(h0\bar{h}l)$	$(hh\bar{2}\bar{h}l)$	$(hki0)$	$(10\bar{1}0)$	$(11\bar{2}0)$	(0001)
3	$\bar{3}^*$	S	S	S	S	S	S	S
3	32^*1	A	S	A	A	S	A	S
3	312^*	A	A	S	A	A	S	S
3	$3m^*1$	A	A	A	A	A	S	A
3	$31m^*$	A	A	A	A	S	A	A
$\bar{3}$	$\bar{3}m^*1$	A	S	A	A	S	S	S
$\bar{3}$	$\bar{3}1m^*$	A	A	S	A	S	S	S
32	$\bar{3}^*m^*1$	S	S	S	S	S	S	S
$3m$	$\bar{3}^*m1$	S	S	S	S	S	S	S
3	6^*	A	A	A	S	S	S	A
3	$\bar{6}^*$	A	A	A	A	A	A	S
$\bar{3}$	$6^*/m^*$	A	A	A	S	S	S	S
6	$6/m^*$	S	S	S	S	S	S	S
$\bar{6}$	$6^*/m$	S	S	S	S	S	S	S
32	6^*22^*	A	A	S	S	S	S	S
6	62^*2^*	A	S	S	A	S	S	S
$3m$	6^*mm^*	A	A	A	S	S	S	A
6	$6m^*m^*$	A	A	A	A	S	S	A
32	$\bar{6}2m^*$	A	A	A	A	S	A	S
$\bar{6}$	$\bar{6}2^*m^*$	A	S	A	A	S	A	S
$3m$	$\bar{6}^*m2^*$	A	A	S	A	A	S	S
$\bar{6}$	$\bar{6}m^*2^*$	A	A	S	A	A	S	S
$\bar{3}m$	$6^*/m^*mm^*$	A	A	S	S	S	S	S
$6/m$	$6/mm^*m^*$	A	S	S	A	S	S	S
622	$6/m^*m^*m^*$	S	S	S	S	S	S	S
$6mm$	$6/m^*mm$	S	S	S	S	S	S	S
$\bar{6}2m$	$6^*/mm^*m$	S	S	S	S	S	S	S

F_1	$J_{12}1$	$(hkil)$	$(h0\bar{h}l)$	$(hh\overline{2h}l)$	$(hki0)$	$(10\bar{1}0)$	$(11\bar{2}0)$	(0001)
3	$\bar{3}^*$	I	I	I	I	I	I	I
3	32^*1	I	I	I	I	I	R	I
3	312^*	I	I	I	I	R	I	I
3	$3m^*1$	I	R	I	I	R	I	I
3	$31m^*$	I	I	R	I	I	R	R
$\bar{3}$	$\bar{3}m^*1$	R	R	R	R	R	R	R
$\bar{3}$	$\bar{3}1m^*$	R	R	R	R	R	R	R
32	$\bar{3}^*m^*1$	I	R	I	I	R	I	R
$3m$	$\bar{3}^*m1$	I	I	I	I	I	R	I
3	6^*	I	I	I	I	I	I	R
3	$\bar{6}^*$	I	I	I	R	R	R	I
$\bar{3}$	$6^*/m^*$	R	R	R	R	R	R	R
6	$6/m^*$	I	I	I	R	R	R	I
$\bar{6}$	$6^*/m$	I	I	I	I	I	I	R
32	6^*22^*	I	R	I	I	R	I	R
6	62^*2^*	I	I	I	R	R	R	I
$3m$	6^*mm^*	I	I	R	I	I	R	R
6	$6m^*m^*$	I	R	R	R	R	R	R
32	$\bar{6}2m^*$	I	R	R	R	R	R	R
$\bar{6}$	$\bar{6}2^*m^*$	I	I	R	I	I	R	R
$3m$	$\bar{6}^*m2^*$	I	I	I	R	R	R	I
$\bar{6}$	$\bar{6}m^*2^*$	I	R	I	I	R	I	R
$\bar{3}m$	$6^*/m^*mm^*$	R	R	R	R	R	R	R
$6/m$	$6/mm^*m^*$	R	R	R	R	R	R	R
622	$6/m^*m^*m^*$	I	R	R	R	R	R	R
$6mm$	$6/m^*mm$	I	I	I	R	R	R	I
$\bar{6}2m$	$6^*/mm^*m$	I	R	I	I	R	I	R

Appendix E

Magnetic sectional layer groups

The following tables present sectional layer groups of 122 magnetic point groups, sectional planes (hkl) represent all plane orbits $\{hkl\}$.

Table E.1: **The family of the magnetic point group 1**

1	1	$11'$
(hkl)	1	$11'$

Table E.2: **The family of the magnetic point group $\bar{1}$**

$\bar{\mathbf{1}}$	$\bar{1}$	$\bar{1}'$	$\bar{1}1'$
(hkl)	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$

Table E.3: **The family of the magnetic point group 2**

2	2_z	$2'_z$	$2_z1'$
(001)	2_z	$2'_z$	$2_z1'$
$(hk0)$	$\underline{2}_z$	$\underline{2}'_z$	$\underline{2}_z1'$
(hkl)	1	1	$11'$

Table E.4: **The family of the magnetic point group m**

m	m_z	m'_z	m_z1'
(001)	$\underline{m}_z$	$\underline{m}'_z$	$\underline{m}_z1'$
$(hk0)$	m_z	m'_z	m_z1'
(hkl)	1	1	$11'$

Table E.5: **The family of the magnetic point group 2/m**

2/m	$2_z/m_z$	$2'_z/m'_z$	$2_z/m'_z$	$2'_z/m_z$	$2_z/m_z1'$
(001)	$2_z/\underline{m}_z$	$2'_z/\underline{m}'_z$	$2_z/\underline{m}'_z$	$2'_z/\underline{m}_z$	$2_z/\underline{m}_z1'$
$(hk0)$	$\underline{\underline{2}}_z/m_z$	$\underline{\underline{2}}'_z/m'_z$	$\underline{\underline{2}}_z/m'_z$	$\underline{\underline{2}}'_z/m_z$	$\underline{\underline{2}}_z/m_z1'$
(hkl)	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$

Table E.6: **The family of the magnetic group 222**

222	$2_x2_y2_z$	$2_x2'_y2'_z$	$2_x2_y2_z1'$
(001)	$\underline{\underline{2}}_x\underline{\underline{2}}_y2_z$	$\underline{\underline{2}}_x\underline{\underline{2}}'_y2'_z$	$\underline{\underline{2}}_x\underline{\underline{2}}_y2_z1'$
(010)	$\underline{\underline{2}}_x2_y\underline{\underline{2}}_z$	$\underline{\underline{2}}_x2'_y\underline{\underline{2}}'_z$	$\underline{\underline{2}}_x2_y\underline{\underline{2}}_z1'$
(100)	$2_x\underline{\underline{2}}_y\underline{\underline{2}}_z$	$2_x\underline{\underline{2}}'_y\underline{\underline{2}}'_z$	$2_x\underline{\underline{2}}_y\underline{\underline{2}}_z1'$
$(hk0)$	$\underline{\underline{2}}_z$	$\underline{\underline{2}}'_z$	$\underline{\underline{2}}_z1'$
$(h0l)$	$\underline{\underline{2}}_y$	$\underline{\underline{2}}'_y$	$\underline{\underline{2}}_y1'$
$(0kl)$	$\underline{\underline{2}}_x$	$\underline{\underline{2}}_x$	$\underline{\underline{2}}_x1'$
(hkl)	1	1	$11'$

Table E.7: **The family of the magnetic group mm2**

mm2	$m_xm_y2_z$	$m'_xm'_y2_z$	$m'_xm_y2'_z$	$m_xm_y2_z1'$
(001)	$m_xm_y2_z$	$m'_xm'_y2_z$	$m'_xm_y2'_z$	$m_xm_y2_z1'$
(010)	$m_x\underline{m}_y\underline{\underline{2}}_z$	$m'_x\underline{m}'_y\underline{\underline{2}}_z$	$m'_x\underline{m}_y\underline{\underline{2}}'_z$	$m_x\underline{m}_y\underline{\underline{2}}_z1'$
(100)	$\underline{m}_xm_y\underline{\underline{2}}_z$	$\underline{m}'_xm'_y\underline{\underline{2}}_z$	$\underline{m}'_xm_y\underline{\underline{2}}'_z$	$\underline{m}_xm_y\underline{\underline{2}}_z1'$
$(hk0)$	$\underline{\underline{2}}_z$	$\underline{\underline{2}}_z$	$\underline{\underline{2}}'_z$	$\underline{\underline{2}}_z1'$
$(h0l)$	m_y	m'_y	m_y	m_y1'
$(0kl)$	m_x	m'_x	m'_x	m_x1'
(hkl)	1	1	1	$11'$

Table E.8: **The family of the magnetic group mmm**

mmm	$m_xm_ym_z$	$m_xm'_ym'_z$
(001)	$\underline{2}_x/m_x\underline{2}_y/m_y2_z/\underline{m}_z$	$\underline{2}_x/m_x\underline{2}'_y/m'_y2'_z/\underline{m}'_z$
(010)	$\underline{2}_x/m_x2_y/\underline{m}_y\underline{2}_z/m_z$	$\underline{2}_x/m_x2'_y/\underline{m}'_y\underline{2}'_z/m'_z$
(100)	$2_x/\underline{m}_x\underline{2}_y/m_y\underline{2}_z/m_z$	$2_x/\underline{m}_x\underline{2}'_y/m'_y\underline{2}'_z/m'_z$
$(hk0)$	$\underline{2}_z/m_z$	$\underline{2}'_z/m'_z$
$(h0l)$	$\underline{2}_y/m_y$	$\underline{2}'_y/m'_y$
$(0kl)$	$\underline{2}_x/m_x$	$\underline{2}_x/m_x$
(hkl)	$\underline{\bar{1}}$	$\underline{\bar{1}}$

mmm	$m'_xm'_ym'_z$	$m'_xm_ym_z$	$m_xm_ym_z1'$
(001)	$\underline{2}_x/m'_x\underline{2}_y/m'_y2_z/\underline{m}'_z$	$\underline{2}_x/m'_x\underline{2}'_y/m_y2'_z/\underline{m}_z$	$\underline{2}_x/m_x\underline{2}_y/m_y2_z/\underline{m}_z1'$
(010)	$\underline{2}_x/m'_x2_y/\underline{m}'_y\underline{2}_z/m'_z$	$\underline{2}_x/m'_x2'_y/\underline{m}_y\underline{2}'_z/m_z$	$\underline{2}_x/m_x2_y/\underline{m}_y\underline{2}_z/m_z1'$
(100)	$2_x/\underline{m}'_x\underline{2}_y/m'_y\underline{2}_z/m'_z$	$2_x/\underline{m}'_x\underline{2}'_y/m_y\underline{2}'_z/m_z$	$2_x/\underline{m}_x\underline{2}_y/m_y\underline{2}_z/m_z1'$
$(hk0)$	$\underline{2}_z/m'_z$	$\underline{2}'_z/m_z$	$\underline{2}_z/m_z1'$
$(h0l)$	$\underline{2}_y/m'_y$	$\underline{2}'_y/m_y$	$\underline{2}_y/m_y1'$
$(0kl)$	$\underline{2}_x/m'_x$	$\underline{2}_x/m'_x$	$\underline{2}_x/m_x1'$
(hkl)	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$

Table E.9: **The family of the magnetic point group 4**

4	4_z	$4'_z$	$4_z1'$
(001)	4_z	$4'_z$	$4_z1'$
(hkl)	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z1'$
(hkl)	1	1	$11'$

Table E.10: **The family of the magnetic point group $\bar{4}$**

$\bar{\mathbf{4}}$	$\bar{4}_z$	$\bar{4}'_z$	$\bar{4}_z1'$
(001)	$\underline{\bar{4}}_z$	$\underline{\bar{4}}'_z$	$\underline{\bar{4}}_z1'$
(hkl)	1	1	$11'$

Table E.11: **The family of the magnetic point group 4/m**

4/m	$4_z/m_z$	$4'_z/m_z$	$4/m'$	$4'/m'$	$4/m1'$
(001)	$4_z/\underline{m}_z$	$4'_z/\underline{m}_z$	$4_z/\underline{m}'_z$	$4'_z/\underline{m}'_z$	$4_z/\underline{m}_z1'$
$(hk0)$	$\underline{2}_z/m_z$	$\underline{2}_z/m_z$	$\underline{2}_z/m'_z$	$\underline{2}_z/m'_z$	$\underline{2}_z/m_z1'$
(hkl)	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$

Table E.12: **The family of the magnetic point group 422**

422	$4_z 2_x 2_{xy}$	$4'_z 2'_x 2_{xy}$	$4_z 2'_x 2'_{xy}$	$4_z 2'_x 2'_{xy}$
(001)	$4_z \underline{2}_x \underline{2}_{xy}$	$4'_z \underline{2}'_x \underline{2}_{xy}$	$4_z \underline{2}'_x \underline{2}'_{xy}$	$4_z \underline{2}_x \underline{2}_{xy} 1'$
(100)	$2_x \underline{2}_y \underline{2}_z$	$2'_x \underline{2}'_y \underline{2}_z$	$2'_x \underline{2}'_y \underline{2}_z$	$2_x \underline{2}_y \underline{2}_z 1'$
(110)	$2_{xy} \underline{2}_{x\bar{y}} \underline{2}_z$	$2_{xy} \underline{2}_{x\bar{y}} \underline{2}_z$	$2'_{xy} \underline{2}'_{x\bar{y}} \underline{2}_z$	$2_{xy} \underline{2}_{x\bar{y}} \underline{2}_z 1'$
$(hk0)$	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z 1'$
(hhl)	$\underline{2}_{x\bar{y}}$	$\underline{2}_{x\bar{y}}$	$\underline{2}'_{x\bar{y}}$	$\underline{2}_{x\bar{y}} 1'$
$(h0l)$	$\underline{2}_y$	$\underline{2}'_y$	$\underline{2}'_y$	$\underline{2}_y 1'$
(hkl)	1	1	1	$11'$

Table E.13: **The family of the magnetic point group 4mm**

4mm	$4_z m_x m_{xy}$	$4'_z m_x m'_{xy}$	$4_z m'_x m'_{xy}$	$4_z m_x m_{xy} 1'$
(001)	$4_z m_x m_{xy}$	$4'_z m_x m'_{xy}$	$4_z m'_x m'_{xy}$	$4_z m_x m_{xy} 1'$
(100)	$\underline{m}_x m_y \underline{2}_z$	$\underline{m}_x m_y \underline{2}_z$	$\underline{m}'_x m'_y \underline{2}_z$	$\underline{m}_x m_y \underline{2}_z 1'$
(110)	$\underline{m}_{xy} m_{x\bar{y}} \underline{2}_z$	$\underline{m}'_{xy} m'_{x\bar{y}} \underline{2}_z$	$\underline{m}'_{xy} m'_{x\bar{y}} \underline{2}_z$	$\underline{m}_{xy} m_{x\bar{y}} \underline{2}_z 1'$
$(hk0)$	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z 1'$
(hhl)	$m_{x\bar{y}}$	$m'_{x\bar{y}}$	$m'_{x\bar{y}}$	$m_{x\bar{y}}$
$(h0l)$	m_y	m_y	m'_y	$m_y 1'$
(hkl)	1	1	1	$11'$

Table E.14: **The family of the magnetic point group $\bar{4}$2m**

$\bar{4}$2m	$\bar{4}_z 2_x m_{xy}$	$\bar{4}'_z 2_x m_{xy}$	$\bar{4}'_z 2'_x m_{xy}$	$\bar{4}_z 2'_x m'_{xy}$	$\bar{4}_z 2_x m_{xy} 1'$
(001)	$\underline{\bar{4}}_z \underline{2}_x m_{xy}$	$\underline{\bar{4}}'_z \underline{2}_x m_{xy}$	$\underline{\bar{4}}'_z \underline{2}'_x m_{xy}$	$\underline{\bar{4}}_z \underline{2}'_x m'_{xy}$	$\underline{\bar{4}}_z \underline{2}_x m_{xy} 1'$
(100)	$2_x \underline{2}_y \underline{2}_z$	$2_x \underline{2}_y \underline{2}_z$	$2'_x \underline{2}'_y \underline{2}_z$	$2'_x \underline{2}'_y \underline{2}_z$	$2_x \underline{2}_y \underline{2}_z 1'$
(110)	$\underline{m}_{xy} m_{x\bar{y}} \underline{2}_z$	$\underline{m}_{xy} m_{x\bar{y}} \underline{2}_z$	$\underline{m}_{xy} m_{x\bar{y}} \underline{2}_z$	$\underline{m}'_{xy} m'_{x\bar{y}} \underline{2}_z$	$\underline{m}_{xy} m_{x\bar{y}} \underline{2}_z 1'$
$(hk0)$	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z 1'$
(hhl)	$m_{x\bar{y}}$	$m_{x\bar{y}}$	$m_{x\bar{y}}$	$m'_{x\bar{y}}$	$m_{x\bar{y}} 1'$
$(h0l)$	$\underline{2}_y$	$\underline{2}_y$	$\underline{2}'_y$	$\underline{2}'_y$	$\underline{2}_y 1'$
(hkl)	1	1	1	1	$11'$

Table E.15: **The family of the magnetic point group 4/mmm**

4/mmm	$4_z/m_z m_x m_{xy}$	$4'_z/m_z m_x m'_{xy}$
(001)	$4_z/\underline{m}_z\underline{2}_x/m_x\underline{2}_{xy}/m_{xy}$	$4'_z/\underline{m}_z\underline{2}_x/m_x\underline{2}'_{xy}/m'_{xy}$
(100)	$2_x/\underline{m}_x\underline{2}_y/m_y\underline{2}_z/m_z$	$2_x/\underline{m}_x\underline{2}_y/m_y\underline{2}_z/m_z$
(110)	$2_{xy}/\underline{m}_{xy}\underline{2}_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m_z$	$2'_{xy}/\underline{m}'_{xy}\underline{2}'_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m_z$
$(hk0)$	$\underline{2}_z/m_z$	$\underline{2}_z/m_z$
(hhl)	$\underline{2}_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}_{x\bar{y}}/m'_{x\bar{y}}$
$(h0l)$	$\underline{2}_y/m_y$	$\underline{2}_y/m_y$
(hkl)	$\underline{\bar{1}}$	$\underline{\bar{1}}$

4/mmm	$4_z/m_z m'_x m'_{xy}$	$4_z/m'_z m'_x m'_{xy}$
(001)	$4_z/\underline{m}_z\underline{2}'_x/m'_x\underline{2}'_{xy}/m'_{xy}$	$4_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}_{xy}/m'_{xy}$
(100)	$2'_x/\underline{m}'_x\underline{2}'_y/m'_y\underline{2}_z/m_z$	$2_x/\underline{m}'_x\underline{2}_y/m'_y\underline{2}_z/m'_z$
(110)	$2'_{xy}/\underline{m}'_{xy}\underline{2}'_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m_z$	$2_{xy}/\underline{m}'_{xy}\underline{2}_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m_z$
$(hk0)$	$\underline{2}_z/m_z$	$\underline{2}_z/m'_z$
(hhl)	$\underline{2}'_{x\bar{y}}/m'_{x\bar{y}}$	$\underline{2}_{x\bar{y}}/m'_{x\bar{y}}$
$(h0l)$	$\underline{2}'_y/m'_y$	$\underline{2}_y/m'_y$
(hkl)	$\underline{\bar{1}}$	$\underline{\bar{1}}'$

4/mmm	$4_z/m'_z m_x m_{xy}$	$4'_z/m'_z m_x m'_{xy}$	$4_z/m_z m_x m_{xy}1'$
(001)	$4_z/\underline{m}'_z\underline{2}'_x/m_x\underline{2}'_{xy}/m_{xy}$	$4'_z/\underline{m}'_z\underline{2}'_x/m_x\underline{2}_{xy}/m'_{xy}$	$4_z/\underline{m}_z\underline{2}_x/m_x\underline{2}_{xy}/m_{xy}1'$
(100)	$2'_x/\underline{m}_x\underline{2}'_y/m_y\underline{2}_z/m'_z$	$2'_x/\underline{m}_x\underline{2}'_y/m_y\underline{2}_z/m'_z$	$2_x/\underline{m}_x\underline{2}_y/m_y\underline{2}_z/m_z1'$
(110)	$2'_{xy}/\underline{m}_{xy}\underline{2}'_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m'_z$	$2_{xy}/\underline{m}'_{xy}\underline{2}_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m'_z$	$2_{xy}/\underline{m}_{xy}\underline{2}_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m_z1'$
$(hk0)$	$\underline{2}_z/m'_z$	$\underline{2}_z/m'_z$	$\underline{2}_z/m_z1'$
(hhl)	$\underline{2}'_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}_{x\bar{y}}/m'_{x\bar{y}}$	$\underline{2}_{x\bar{y}}/m_{x\bar{y}}1'$
$(h0l)$	$\underline{2}'_y/m_y$	$\underline{2}'_y/m_y$	$\underline{2}_y/m_y1'$
(hkl)	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$

Table E.16: **The family of the magnetic point group 3**

3	3_z	$3_z1'$
(0001)	3_z	$3_z1'$
$(hkil)$	1	$11'$

Table E.17: **The family of the magnetic point group $\bar{3}$**

$\bar{\mathbf{3}}$	$\bar{3}_z$	$\bar{3}'_z$	$\bar{3}_z1'$
(0001)	$\underline{\bar{3}}_z$	$\underline{\bar{3}}'_z$	$\underline{\bar{3}}_z1'$
$(hkil)$	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$

Table E.18: **The family of the magnetic point group 32**

32	3_z2_{10}	$3_z2'_{10}$	$3_z2_{10}1'$
(0001)	$\underline{\bar{3}}_z\underline{2}_{10}$	$\underline{\bar{3}}_z\underline{2}_{10}$	$\underline{\bar{3}}_z\underline{2}_{10}1'$
$(2\bar{1}\bar{1}0)$	2_{10}	$2'_{10}$	$2_{10}1'$
$(01\bar{1}0)$	$\underline{2}_{10}$	$\underline{2}'_{10}$	$\underline{2}_{10}1'$
$(2h\bar{h}\bar{h})$	1	1	$11'$
$(0h\bar{h}l)$	$\underline{2}_{10}$	$\underline{2}'_{10}$	$\underline{2}_{10}1'$
$(hki0)$	1	1	$11'$
$(hkil)$	1	1	$11'$

Table E.19: **The family of the magnetic point group 3m**

3m	$3_zm_{2\bar{1}}$	$3_zm'_{2\bar{1}}$	$3_zm_{2\bar{1}}1'$
(0001)	$\underline{\bar{3}}_zm_{2\bar{1}}$	$\underline{\bar{3}}_zm'_{2\bar{1}}$	$\underline{\bar{3}}_zm_{2\bar{1}}1'$
$(2\bar{1}\bar{1}0)$	$\underline{m}_{2\bar{1}}$	$\underline{m}'_{2\bar{1}}$	$\underline{m}_{2\bar{1}}1'$
$(01\bar{1}0)$	$m_{2\bar{1}}$	$m'_{2\bar{1}}$	$m_{2\bar{1}}1'$
$(2h\bar{h}\bar{h})$	1	1	$11'$
$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m'_{2\bar{1}}$	$m_{2\bar{1}}1'$
$(hki0)$	1	1	$11'$
$(hkil)$	1	1	$11'$

Table E.20: **The family of the magnetic point group $\bar{3}$m**

$\bar{3}$m	$\bar{3}_zm_{2\bar{1}}$	$\bar{3}_zm'_{2\bar{1}}$	$\bar{3}'_zm'_{2\bar{1}}$	$\bar{3}'_zm_{2\bar{1}}$	$\bar{3}_zm_{2\bar{1}}1'$
(0001)	$\underline{\bar{3}}_z\underline{2}_{10}/m_{2\bar{1}}$	$\underline{\bar{3}}_z\underline{2}'_{10}/m'_{2\bar{1}}$	$\underline{\bar{3}}'_z\underline{2}_{10}/m'_{2\bar{1}}$	$\underline{\bar{3}}'_z\underline{2}'_{10}/m_{2\bar{1}}$	$\underline{\bar{3}}_z\underline{2}_{10}/m_{2\bar{1}}1'$
$(2\bar{1}\bar{1}0)$	$2_{10}/\underline{m}_{2\bar{1}}$	$2'_{10}/\underline{m}'_{2\bar{1}}$	$2_{10}/\underline{m}'_{2\bar{1}}$	$2'_{10}/\underline{m}_{2\bar{1}}$	$2_{10}/\underline{m}_{2\bar{1}}1'$
$(01\bar{1}0)$	$\underline{2}_{10}/m_{2\bar{1}}$	$\underline{2}'_{10}/m'_{2\bar{1}}$	$\underline{2}_{10}/m'_{2\bar{1}}$	$\underline{2}'_{10}/m_{2\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}}1'$
$(2h\bar{h}\bar{h})$	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$
$(0h\bar{h}l)$	$\underline{2}_{10}/m_{2\bar{1}}$	$\underline{2}'_{10}/m'_{2\bar{1}}$	$\underline{2}_{10}/m'_{2\bar{1}}$	$\underline{2}'_{10}/m_{2\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}}1'$
$(hki0)$	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$
$(hkil)$	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$

Table E.21: **The family of the magnetic point group 6**

6	6_z	$6'_z$	$6_z1'$
(0001)	6_z	$6'_z$	$6_z1'$
$(hkil)$	1	1	$11'$

Table E.22: **The family of the magnetic point group $\bar{6}$**

$\bar{6}$	$\bar{6}_z$	$\bar{6}'_z$	$\bar{6}_z1'$
(0001)	$\bar{6}_z$	$\bar{6}'_z$	$\bar{6}_z1'$
$(hki0)$	m_z	m'_z	m_z1'
$(hkil)$	1	1	$11'$

Table E.23: **The family of the magnetic point group 6/m**

6/m	$6_z/m_z$	$6'_z/m'_z$	$6_z/m'_z$	$6'_z/m_z$	$6_z/m_z1'$
(0001)	$6_z/\underline{m}_z$	$6'_z/\underline{m}'_z$	$6_z/\underline{m}'_z$	$6'_z/\underline{m}_z$	$6_z/\underline{m}_z1'$
$(hki0)$	$\underline{\underline{2}}_z/m_z$	$\underline{\underline{2}}'_z/m'_z$	$\underline{\underline{2}}_z/m'_z$	$\underline{\underline{2}}'_z/m_z$	$\underline{\underline{2}}_z/m_z1'$
$(hkil)$	$\underline{\bar{1}}$	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$

Table E.24: **The family of the magnetic point group 622**

622	$6_z2_{10}2_{12}$	$6'_z2_{10}2'_{12}$	$6_z2'_{10}2'_{12}$	$6_z2_{10}2_{12}1'$
(0001)	$6_z\underline{\underline{2}}_{10}\underline{\underline{2}}_{12}$	$6'_z\underline{\underline{2}}_{10}\underline{\underline{2}}'_{12}$	$6_z\underline{\underline{2}}'_{10}\underline{\underline{2}}'_{12}$	$6_z\underline{\underline{2}}_{10}\underline{\underline{2}}_{12}1'$
$(2\bar{1}\bar{1}0)$	$2_{10}\underline{\underline{2}}_{12}\underline{\underline{2}}_z$	$2_{10}\underline{\underline{2}}'_{12}\underline{\underline{2}}'_z$	$2'_{10}\underline{\underline{2}}'_{12}\underline{\underline{2}}_z$	$2_{10}\underline{\underline{2}}_{12}\underline{\underline{2}}_z1'$
$(01\bar{1}0)$	$\underline{\underline{2}}_{10}2_{12}\underline{\underline{2}}_z$	$\underline{\underline{2}}_{10}2'_{12}\underline{\underline{2}}'_z$	$\underline{\underline{2}}'_{10}2'_{12}\underline{\underline{2}}_z$	$\underline{\underline{2}}_{10}2_{12}\underline{\underline{2}}_z1'$
$(2h\bar{h}\bar{h})$	$\underline{\underline{2}}_{12}$	$\underline{\underline{2}}'_{12}$	$\underline{\underline{2}}'_{12}$	$\underline{\underline{2}}_{12}1'$
$(0h\bar{h}l)$	$\underline{\underline{2}}_{10}$	$\underline{\underline{2}}_{10}$	$\underline{\underline{2}}'_{10}$	$\underline{\underline{2}}_{10}1'$
$(hki0)$	$\underline{\underline{2}}_z$	$\underline{\underline{2}}'_z$	$\underline{\underline{2}}_z$	$\underline{\underline{2}}_z1'$
$(hkil)$	1	1	1	$11'$

Table E.25: **The family of the magnetic point group 6mm**

6mm	$6_zm_{2\bar{1}}m_{01}$	$6'_zm_{2\bar{1}}m'_{01}$	$6_zm'_{2\bar{1}}m'_{01}$	$6_zm_{2\bar{1}}m_{01}1'$
(0001)	$6_zm_{2\bar{1}}m_{01}$	$6'_zm_{2\bar{1}}m'_{01}$	$6_zm'_{2\bar{1}}m'_{01}$	$6_zm_{2\bar{1}}m_{01}1'$
$(2\bar{1}\bar{1}0)$	$\underline{m}_{2\bar{1}}m_{01}\underline{\underline{2}}_z$	$\underline{m}_{2\bar{1}}m'_{01}\underline{\underline{2}}'_z$	$\underline{m}'_{2\bar{1}}m'_{01}\underline{\underline{2}}_z$	$\underline{m}_{2\bar{1}}m_{01}\underline{\underline{2}}_z1'$
$(01\bar{1}0)$	$m_{2\bar{1}}\underline{m}_{01}\underline{\underline{2}}_z$	$m_{2\bar{1}}\underline{m}'_{01}\underline{\underline{2}}'_z$	$m'_{2\bar{1}}\underline{m}'_{01}\underline{\underline{2}}_z$	$m_{2\bar{1}}\underline{m}_{01}\underline{\underline{2}}_z1'$
$(2h\bar{h}\bar{h})$	m_{01}	m'_{01}	m'_{01}	$m_{01}1'$
$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m_{2\bar{1}}$	$m'_{2\bar{1}}$	$m_{2\bar{1}}1'$
$(hki0)$	$\underline{\underline{2}}_z$	$\underline{\underline{2}}'_z$	$\underline{\underline{2}}_z$	$\underline{\underline{2}}_z1'$
$(hkil)$	1	1	1	$11'$

Table E.26: **The family of the magnetic point group $\bar{6}$m2**

$\bar{6}$**m2**	$\bar{6}_z m_{2\bar{1}} 2_{12}$	$\bar{6}'_z m'_{2\bar{1}} 2_{12}$	$\bar{6}'_z m_{2\bar{1}} 2'_{12}$	$\bar{6}_z m'_{2\bar{1}} 2'_{12}$	$\bar{6}_z m_{2\bar{1}} 2_{12} 1'$
(0001)	$\underline{\bar{6}}_z m_{2\bar{1}} \underline{2}_{12}$	$\underline{\bar{6}}'_z m'_{2\bar{1}} \underline{2}_{12}$	$\underline{\bar{6}}'_z m_{2\bar{1}} \underline{2}'_{12}$	$\underline{\bar{6}}_z m'_{2\bar{1}} \underline{2}'_{12}$	$\underline{\bar{6}}_z m_{2\bar{1}} \underline{2}_{12} 1'$
$(2\bar{1}\bar{1}0)$	$\underline{m}_{2\bar{1}} \underline{2}_{12} m_z$	$\underline{m}'_{2\bar{1}} \underline{2}_{12} m'_z$	$\underline{m}_{2\bar{1}} \underline{2}'_{12} m'_z$	$\underline{m}'_{2\bar{1}} \underline{2}'_{12} m_z$	$\underline{m}_{2\bar{1}} \underline{2}_{12} m_z 1'$
$(01\bar{1}0)$	$m_{2\bar{1}} 2_{12} m_z$	$m'_{2\bar{1}} 2_{12} m'_z$	$m_{2\bar{1}} 2'_{12} m'_z$	$m'_{2\bar{1}} 2'_{12} m_z$	$m_{2\bar{1}} 2_{12} m_z 1'$
$(2\bar{h}\bar{h}h)$	$\underline{2}_{12}$	$\underline{2}_{12}$	$\underline{2}'_{12}$	$\underline{2}'_{12}$	$\underline{2}_{12} 1'$
$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m'_{2\bar{1}}$	$m_{2\bar{1}}$	$m'_{2\bar{1}}$	$m_{2\bar{1}} 1'$
$(hki0)$	m_z	m'_z	m'_z	m_z	$m_z 1'$
$(hkil)$	1	1	1	1	$11'$

Table E.27: **The family of the magnetic point group 6/mmm**

6/mmm	$6_z/m_z m_{2\bar{1}} m_{01}$	$6'_z/m'_z m_{2\bar{1}} m'_{01}$
(0001)	$6_z/\underline{m}_z \underline{2}_{10}/m_{2\bar{1}} \underline{2}_{12}/m_{01}$	$6'_z/\underline{m}'_z \underline{2}_{10}/m_{2\bar{1}} \underline{2}'_{12}/m'_{01}$
$(2\bar{1}\bar{1}0)$	$2_{10}/\underline{m}_{2\bar{1}} \underline{2}_{12}/m_{01} \underline{2}_z/m_z$	$2_{10}/\underline{m}_{2\bar{1}} \underline{2}'_{12}/m'_{01} \underline{2}'_z/m'_z$
$(01\bar{1}0)$	$\underline{2}_{10}/m_{2\bar{1}} 2_{12}/\underline{m}_{01} \underline{2}_z/m_z$	$\underline{2}_{10}/m_{2\bar{1}} 2'_{12}/\underline{m}'_{01} \underline{2}'_z/m'_z$
$(2\bar{h}\bar{h}h)$	$\underline{2}_{12}/m_{01}$	$\underline{2}'_{12}/m'_{01}$
$(0h\bar{h}l)$	$\underline{2}_{10}/m_{2\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}}$
$(hki0)$	$\underline{2}_z/m_z$	$\underline{2}'_z/m'_z$
$(hkil)$	$\underline{\bar{1}}$	$\underline{\bar{1}}$

6/mmm	$6_z/m_z m'_{2\bar{1}} m'_{01}$	$6_z/m'_z m'_{2\bar{1}} m'_{01}$
(0001)	$6_z/\underline{m}_z \underline{2}'_{10}/m'_{2\bar{1}} \underline{2}'_{12}/m'_{01}$	$6_z/\underline{m}'_z \underline{2}_{10}/m'_{2\bar{1}} \underline{2}_{12}/m'_{01}$
$(2\bar{1}\bar{1}0)$	$2'_{10}/\underline{m}'_{2\bar{1}} \underline{2}'_{12}/m'_{01} \underline{2}_z/m_z$	$2_{10}/\underline{m}'_{2\bar{1}} \underline{2}_{12}/m'_{01} \underline{2}_z/m'_z$
$(01\bar{1}0)$	$\underline{2}'_{10}/m'_{2\bar{1}} 2'_{12}/\underline{m}'_{01} \underline{2}_z/m_z$	$\underline{2}_{10}/m'_{2\bar{1}} 2_{12}/\underline{m}'_{01} \underline{2}_z/m'_z$
$(2\bar{h}\bar{h}h)$	$\underline{2}'_{12}/m'_{01}$	$\underline{2}_{12}/m'_{01}$
$(0h\bar{h}l)$	$\underline{2}'_{10}/m'_{2\bar{1}}$	$\underline{2}_{10}/m'_{2\bar{1}}$
$(hki0)$	$\underline{2}_z/m_z$	$\underline{2}_z/m'_z$
$(hkil)$	$\underline{\bar{1}}$	$\underline{\bar{1}}'$

6/mmm	$6_z/m'_z m_{2\bar{1}} m_{01}$	$6'_z/m_z m_{2\bar{1}} m'_{01}$	$6_z/m_z m_{2\bar{1}} m_{01} 1'$
(0001)	$6_z/\underline{m}'_z \underline{2}'_{10}/m_{2\bar{1}} \underline{2}'_{12}/m_{01}$	$6'_z/\underline{m}_z \underline{2}'_{10}/m_{2\bar{1}} \underline{2}_{12}/m'_{01}$	$6_z/\underline{m}_z \underline{2}_{10}/m_{2\bar{1}} \underline{2}_{12}/m_{01} 1'$
$(2\bar{1}\bar{1}0)$	$2'_{10}/\underline{m}_{2\bar{1}} \underline{2}'_{12}/m_{01} \underline{2}_z/m'_z$	$2'_{10}/\underline{m}_{2\bar{1}} \underline{2}_{12}/m'_{01} \underline{2}'_z/m_z$	$2_{10}/\underline{m}_{2\bar{1}} \underline{2}_{12}/m_{01} \underline{2}_z/m_z 1'$
$(01\bar{1}0)$	$\underline{2}'_{10}/m_{2\bar{1}} 2'_{12}/\underline{m}_{01} \underline{2}_z/m'_z$	$\underline{2}'_{10}/m_{2\bar{1}} 2_{12}/\underline{m}'_{01} \underline{2}'_z/m_z$	$\underline{2}_{10}/m_{2\bar{1}} 2_{12}/\underline{m}_{01} \underline{2}_z/m_z 1'$
$(2\bar{h}\bar{h}h)$	$\underline{2}'_{12}/m_{01}$	$\underline{2}_{12}/m'_{01}$	$\underline{2}_{12}/m_{01} 1'$
$(0h\bar{h}l)$	$\underline{2}'_{10}/m_{2\bar{1}}$	$\underline{2}'_{10}/m_{2\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}} 1'$
$(hki0)$	$\underline{2}_z/m'_z$	$\underline{2}'_z/m_z$	$\underline{2}_z/m_z 1'$
$3(hkil)$	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$

Table E.28: **The family of the magnetic point group 23**

23	2_z3_p	$2_z3_p1'$
(001)	2_z	$2_z1'$
$(hk0)$	$\underline{2}_z$	$\underline{2}_z1'$
(hkl)	1	$11'$
(111)	$\underline{\bar{3}}_p$	$\underline{\bar{3}}_p1'$

Table E.29: **The family of the magnetic point group m$\bar{3}$**

m$\bar{3}$	$m_x\bar{3}_p$	$m'_x\bar{3}_p$	$m_x\bar{3}_p1'$
(001)	$\underline{2}_x/m_x\underline{2}_y/m_y2_z/\underline{m}_z$	$\underline{2}_x/m'_x\underline{2}_y/m'_y2_z/\underline{m}'_z$	$\underline{2}_x/m_x\underline{2}_y/m_y2_z/\underline{m}_z1'$
(110)	$\underline{2}_z/m_z$	$\underline{2}_z/m'_z$	$\underline{2}_z/m_z1'$
$(hk0)$	$\underline{2}_z/m_z$	$\underline{2}_z/m'_z$	$\underline{2}_z/m_z1$
(hhl)	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$
(hkl)	$\underline{\bar{1}}$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$
(111)	$\underline{\bar{3}}_p$	$\underline{\bar{3}}'_p$	$\underline{\bar{3}}_p1'$

Table E.30: **The family of the magnetic point group 432**

432	$4_z3_p2_{xy}$	$4'_z3_p2'_{xy}$	$4_z3_p2_{xy}1'$
(001)	$4_z\underline{2}_x\underline{2}_{xy}$	$4'_z\underline{2}_x\underline{2}'_{xy}$	$4_z\underline{2}_x\underline{2}_{xy}1'$
(110)	$2_{xy}\underline{2}_{x\bar{y}}\underline{2}_z$	$2'_{xy}\underline{2}'_{x\bar{y}}\underline{2}_z$	$2_{xy}\underline{2}_{x\bar{y}}\underline{2}_z1'$
$(hk0)$	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z1'$
(hhl)	$\underline{2}_{x\bar{y}}$	$\underline{2}'_{x\bar{y}}$	$\underline{2}_{x\bar{y}}1'$
(hkl)	1	1	$11'$
(111)	$3_p\underline{2}_{x\bar{y}}$	$3_p\underline{2}'_{x\bar{y}}$	$3_p\underline{2}_{x\bar{y}}1'$

Table E.31: **The family of the magnetic point group $\bar{4}$3m**

$\bar{4}$3m	$\bar{4}_z3_pm_{xy}$	$\bar{4}'_z3_pm'_{xy}$	$\bar{4}_z3_pm_{xy}1'$
(001)	$\underline{\bar{4}}_z\underline{2}_xm_{xy}$	$\underline{\bar{4}}'_z\underline{2}_xm'_{xy}$	$\underline{\bar{4}}_z\underline{2}_xm_{xy}1'$
(110)	$\underline{m}_{xy}m_{x\bar{y}}\underline{2}_z$	$\underline{m}'_{xy}m'_{x\bar{y}}\underline{2}_z$	$\underline{m}_{xy}m_{x\bar{y}}\underline{2}_z1'$
$(hk0)$	$\underline{2}_z$	$\underline{2}_z$	$\underline{2}_z1'$
(hhl)	$m_{x\bar{y}}$	$m'_{x\bar{y}}$	$m_{x\bar{y}}1'$
(hkl)	1	1	$11'$
(111)	$3_pm_{x\bar{y}}$	$3_pm'_{x\bar{y}}$	$3_pm_{x\bar{y}}1'$

Table E.32: **The family of the magnetic point group m$\bar{3}$m**

m$\bar{3}$m	$m_z\bar{3}_pm_{xy}$	$m_z\bar{3}_pm'_{xy}$
(001)	$4_z/\underline{m}_z\underline{2}_x/m_x\underline{2}_{xy}/m_{xy}$	$4_z/\underline{m}_z\underline{2}_x/m_x\underline{2}'_{xy}/m'_{xy}$
(110)	$2_{xy}/\underline{m}_{xy}\underline{2}_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m_z$	$2'_{xy}/\underline{m}'_{xy}\underline{2}'_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m_z$
$(hk0)$	$\underline{2}_z/m_z$	$\underline{2}_z/m_z$
(hhl)	$\underline{2}_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}'_{x\bar{y}}/m'_{x\bar{y}}$
(hkl)	$\underline{\bar{1}}$	$\underline{\bar{1}}$
(111)	$\underline{\bar{3}}_p\underline{2}_{x\bar{y}}/m_{x\bar{y}}$	$\underline{\bar{3}}_p\underline{2}'_{x\bar{y}}/m'_{x\bar{y}}$

m$\bar{3}$m	$m'_z\bar{3}'_pm'_{xy}$	$m'_z\bar{3}'_pm_{xy}$	$m_z\bar{3}_pm_{xy}1'$
(001)	$4_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}_{xy}/m'_{xy}$	$4_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}'_{xy}/m_{xy}$	$4_z/\underline{m}_z\underline{2}_x/m_x\underline{2}_{xy}/m_{xy}1'$
(110)	$2_{xy}/\underline{m}'_{xy}\underline{2}_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m'_z$	$2'_{xy}/\underline{m}_{xy}\underline{2}'_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m'_z$	$2_{xy}/\underline{m}_{xy}\underline{2}_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m_z1'$
$(hk0)$	$\underline{2}_z/m'_z$	$\underline{2}_z/m'_z$	$\underline{2}_z/m_z1'$
(hhl)	$\underline{2}_{x\bar{y}}/m'_{x\bar{y}}$	$\underline{2}'_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}_{x\bar{y}}/m_{x\bar{y}}1'$
(hkl)	$\underline{\bar{1}}'$	$\underline{\bar{1}}'$	$\underline{\bar{1}}1'$
(111)	$\underline{\bar{3}}'_p\underline{2}_{x\bar{y}}/m'_{x\bar{y}}$	$\underline{\bar{3}}'_p\underline{2}'_{x\bar{y}}/m_{x\bar{y}}$	$\underline{\bar{3}}_p\underline{2}_{x\bar{y}}/m_{x\bar{y}}1'$

Appendix F

Pyroelectric and pyromagnetic domain walls in 380 domain pairs described by magnetic completely transposable twin laws

There are 380 classes of the *magnetic completely* transposable twin laws describing symmetry of the domain pairs. They are divided into 4 parts with respect to the type of the domain walls.

The symbols are given in the explicite notation $J[F]$ equally with notation of the corresponding type of these groups. The symbols **M**,**P**,**MP** and **0** are used in the following situations:

	magnetization $\mathbf{M} \neq 0$	magnetization $\mathbf{M} = 0$
polarization $\mathbf{P} \neq 0$	MP	P
polarization $\mathbf{P} = 0$	M	0

Inner structure of the $J[F]$ group is also presented in the explicite notation. Symbols $q, i, i', 1'$ represent:

- q . . . . . a group does not contain symmetry elements $\bar{1}, \bar{1}'$ and $1'$,
- i . . . . . a group contains space inversion $\bar{1}$ but does not $\bar{1}'$ and $1'$,
- i' . . . . . a group contains space-time inversion $\bar{1}'$ but does not $\bar{1}$ and $1'$,
- $1'$ . . . . . time inversion.

The symbol (a) is used for the ferroelastic domain pairs,
the symbol (b) for the non-ferroelastic magnetoelectric domain pairs.

1. **A domain wall can be simultaneously pyromagnetic and pyroelectric.**

(a) A group $J_{12}[F_1]$ is of the type **MP[MP]** ; 16 cases ; $q_2[q_1]$

- $2[1](a)$, $2'[1](a)$, $m[1](a)$, $m'[1](a)$,
- $m'm2'[m](a)$, $m'm2'[2'](a)$, $m'm2'[m'](a)$, $m'm'2[2](a)$, $m'm'2[m'](a)$,
- $4[2](a)$, $4m'm'[4](b)$, $4m'm'[m'm'2](a)$,
- $3m'[3](b)$,
- $6[3]$, $6m'm'[6](b)$, $6m'm'[3m']$.

(b) A group $J_{12}[F_1]$ is of the type **M[MP]** ; 9 cases ; $q_2[q_1]$

- $2'2'2[2](a)$, $2'2'2[2'](a)$,
- $\bar{4}[2](a)$, $42'2'[4](b)$, $\bar{4}2'm'[m'm'2](a)$,
- $32'[3](b)$,
- $\bar{6}[3](b)$, $62'2'[6](b)$, $\bar{6}m'2'[3m'](b)$.

(c) A group $J_{12}[F_1]$ is of the type **P[MP]** ; 22 cases ; $q_2[q_1], q_21'[q_1]$

- $11'[1](b)$,
- $21'[2](b)$, $21'[2'](b)$, $m1'[m](b)$, $m1'[m'](b)$,
- $mm2[m](a)$, $mm2[2](a)$, $mm21'[m'm2'](b)$, $mm21'[m'm'2](b)$,
- $41'[4'](b)$, $4'[2](a)$, $4mm[4](b)$, $4mm1'[4m'm'](b)$, $4'm'm[m'm'2](a)$,
- $31'[3](b)$, $3m[3](b)$, $3m1'[3m'](b)$,
- $61'[6](b)$, $6'[3](b)$, $6mm[6](b)$, $6mm1'[6m'm'](b)$, $6'mm'[3m'](b)$.

(d) A group $J_{12}[F_1]$ is of the type **0[MP]** ; 8 cases ; $q_2[q_1]$

- $222[2](a)$,
- $\bar{4}'[2](a)$, $422[4](b)$, $\bar{4}'2m'[m'm'2](a)$,
- $32[3](b)$,
- $\bar{6}'[3]$, $622[6](b)$, $\bar{6}'m'2[3m']$.

(e) A group $J_{12}[F_1]$ is of the type **M[M]** ; 17 cases ;$q_2[q_1], i_2[i_1]$

- $2/m[\bar{1}](a)$, $2'/m'[\bar{1}](a)$,
- $m'm'm[2/m](a)$, $m'm'm[2'/m'](a)$,
- $4/m[2/m](a)$, $42'2'[2'2'2](a)$, $\bar{4}2'm'[\bar{4}](b)$, $\bar{4}2'm'[2'2'2](a)$, $4/mm'm'[4/m]$, $4/mm'm'[m'm'm](a)$,
- $\bar{3}m'[\bar{3}]$
- $6/m[\bar{3}]$, $62'2'[32']$, $\bar{6}m'2'[\bar{6}]$, $\bar{6}m'2'[32'](b)$, $6/mm'm'[6/m]$, $6/mm'm'[\bar{3}m']$.

(f) A group $J_{12}[F_1]$ is of the type **0[M]** ; 22 cases ;$q_2[q_1], q_21'[q_1], i_2[i_1]$

- $2221'[2'2'2](b)$, $mmm[2/m](a)$,
- $\bar{4}1'[\bar{4}](b)$, $4'/m[2/m](a)$, $4221'[42'2'](b)$, $4'22'[2'2'2](a)$, $\bar{4}2m[\bar{4}](b)$, $\bar{4}2m1'[\bar{4}2'm'](b)$, $\bar{4}'2'm[2'2'2](a)$, $4/mmm[4/m]$, $4'/mmm'[m'm'm](a)$,
- $321'[32'](b)$, $\bar{3}m[\bar{3}]$,
- $\bar{6}1'[\bar{6}]$, $6'/m'[\bar{3}]$, $6221'[62'2'](b)$, $6'22'[32'](b)$, $\bar{6}m2[\bar{6}]$, $\bar{6}m21'[\bar{6}m'2']$, $\bar{6}'m2'[32']$, $6/mmm[6/m]$, $6'/m'mm'[\bar{3}m']$.

(g) A group $J_{12}[F_1]$ is of the type **P[P]** ; 14 cases ;$q_2[q_1], q_21'[q_1]$

- $mm21'[mm2](b)$,
- $41'[4'](b)$, $4mm[mm2](a)$, $4mm1'[4mm](b)$, $4mm1'[4'm'm](b)$, $4'm'm[mm2](a)$, $4'm'm[4'](b)$,
- $3m1'[3m](b)$,
- $61'[6']$, $6mm[3m]$, $6mm1'[6mm](b)$, $6mm1'[6'mm']$, $6'mm'[3m](b)$, $6'mm'[6']$.

(h) A group $J_{12}[F_1]$ is of the type $\mathbf{0[P]}$; 6 cases ;$q_2[q_1]$

- $4'22'[4'](b)$, $\bar{4}2m[mm2](a)$, $\bar{4}'2'm[mm2](a)$,
- $6'22'[6']$, $\bar{6}m2[3m](b)$, $\bar{6}'m2'[3m]$.

(i) A group $J_{12}[F_1]$ is of the type $\mathbf{0[0]}$; 70 cases ; $q_2[q_1], q_21'[q_1], i_2[i_1], i'_2[i'_1]$

- $2'/m[\bar{1}'](a)$, $2/m'[\bar{1}'](a)$,
- $2221'[222](b)$, $m'mm[2'/m](a)$, $m'mm[2/m'](a)$, $m'm'm'[2/m'](a)$,
- $\bar{4}1'[\bar{4}'](b)$, $4/m'[2/m'](a)$, $4'/m'[2/m'](a)$, $422[222](a)$,
  $4221'[422](b)$, $4221'[4'22'](b)$, $4'22'[222](a)$, $\bar{4}2m[222](a)$,
  $\bar{4}2m1'[\bar{4}2m](b)$, $\bar{4}2m1'[\bar{4}'2m'](b)$, $\bar{4}2m1'[\bar{4}'2'm](b)$, $\bar{4}'2m'[222](a)$,
  $\bar{4}'2m'[\bar{4}'](b)$, $\bar{4}'2'm[\bar{4}'](b)$, $4/mmm[mmm](a)$, $4'/mmm'[mmm](a)$,
  $4'/mmm'[4'/m]$, $4/m'm'm'[4/m'](b)$, $4/m'm'm'[m'm'm'](a)$,
  $4/m'mm[4/m'](b)$, $4/m'mm[mmm'](a)$, $4'/m'm'm[4'/m'](b)$,
  $4'/m'm'm[m'm'm'](a)$, $4'/m'm'm[mmm'](a)$,
- $321'[32](b)$, $\bar{3}'m[\bar{3}'](b)$, $\bar{3}'m'[\bar{3}'](b)$,
- $\bar{6}1'[\bar{6}'](b)$, $6/m'[\bar{3}']$, $6'/m[\bar{3}'](b)$, $622[32]$, $6221'[622](b)$,
  $6221'[6'22']$, $6'22'[32](b)$, $\bar{6}m2[32](b)$, $\bar{6}m21'[\bar{6}m2]$,
  $\bar{6}m21'[\bar{6}'m'2](b)$, $\bar{6}m21'[\bar{6}'m2'](b)$, $\bar{6}'m'2[32]$,
  $\bar{6}'m2'[\bar{6}'](b)$, $\bar{6}'m'2[\bar{6}'](b)$, $6/mmm[\bar{3}m]$,
  $6'/m'mm'[\bar{3}m]$, $6'/m'mm'[6'/m']$, $6/m'm'm'[6/m'](b)$,
  $6/m'm'm'[\bar{3}'m']$, $6/m'mm[6/m'](b)$, $6/m'mm[\bar{3}'m]$,
  $6'/mmm'[6'/m]$, $6'/mmm'[\bar{3}'m](b)$, $6'/mmm'[\bar{3}'m'](b)$,
- $231'[23](b)$, $432[23]$, $4321'[432](b)$, $4321'[4'32']$, $4'32'[23](b)$,
  $\bar{4}3m[23](b)$, $\bar{4}3m1'[\bar{4}3m]$, $\bar{4}'3m1'[\bar{4}'3m'](b)$, $\bar{4}3m'[23]$,
  $m\bar{3}m[m\bar{3}]$, $m\bar{3}m'[m\bar{3}]$, $m'\bar{3}'m'[m'\bar{3}']$, $m'\bar{3}'m[m'\bar{3}'](b)$.

2. **A domain wall is non-pyroelectric but can be pyromagnetic**

(a) A group $J_{12}[F_1]$ is of the type **M[MP]** ; 13 cases ;$i[q]$

- $\bar{1}[1](b)$,
- $2/m[2](b)$, $2/m[m](b)$, $2'/m'[2'](b)$, $2'/m'[m'](b)$,
- $m'm'm[m'm'2](b)$, $m'm'm[m'm2'](b)$,
- $4/m[4](b)$, $4/mm'm'[4m'm'](b)$,
- $\bar{3}[3](b)$, $\bar{3}m'[3m'](b)$,
- $6/m[6](b)$, $6/mm'm'[6m'm'](b)$.

(b) A group $J_{12}[F_1]$ is of the type **M[M]** ; 8 cases ; $i[q]$

- $m'm'm[2'2'2](b)$,
- $4/m[\bar{4}](b)$, $4/mm'm'[42'2'](b)$, $4/mm'm'[\bar{4}2'm'](b)$,
- $\bar{3}m'[32'](b)$,
- $6/m[\bar{6}]$, $6/mm'm'[62'2'](b)$, $6/mm'm'[\bar{6}m'2']$.

(c) A group $J_{12}[F_1]$ is of the type **0[P]** ; 8 cases ; $i[q]$

- $mmm[mm2](b)$,
- $4'/m[4'](b)$, $4/mmm[4mm](b)$, $4'mmm'[4'mm'](b)$,
- $\bar{3}m[3m](b)$,
- $6'/m'[6']$, $6/mmm[6mm](b)$, $6'/m'mm'[6'mm']$.

(d) A group $J_{12}[F_1]$ is of the type **0[0]** ; 40 cases ; $i[q], i_21'[i_1']$

- $\bar{1}1'[\bar{1}'](b)$,
- $2/m1'[2/m'](b)$, $2/m1'[2'/m](b)$,
- $mmm[222](b)$, $mmm1'[m'mm](b)$, $mmm1'[m'm'm'](b)$,
- $4/m1'[4/m'](b)$, $4/m1'[4'/m'](b)$, $4'/m[\bar{4}'](b)$, $4/mmm[422](b)$, $4/mmm1'[4/m'm'm'](b)$, $4/mmm1'[4/m'mm](b)$, $4/mmm[\bar{4}2m](b)$,

$4/mmm1'[4'/m'm'm](b)$, $4'/mmm'[4'22'](b)$,

$4'/mmm'[\bar{4}'2m'](b)$, $4'/mmm'[\bar{4}'m2'](b)$,

- $\bar{3}1'[\bar{3}'](b)$, $\bar{3}m[32](b)$, $\bar{3}m1'[\bar{3}'m](b)$, $\bar{3}m1'[\bar{3}'m'](b)$,
- $6/m1'[6/m'](b)$, $6/m1'[6'/m]$, $6'/m'[\bar{6}'](b)$, $6/mmm[622](b)$,

  $6/mmm[\bar{6}m2]$, $6/mmm1'[6/m'm'm'](b)$, $6/mmm1'[6/m'mm](b)$,

  $6/mmm1'[6'/mmm']$, $6'/m'mm'[6'22']$,

  $6'/m'mm'[\bar{6}'m2'](b)$, $6'/m'mm'[\bar{6}'2m'](b)$,
- $m\bar{3}[23](b)$, $m\bar{3}1'[m'\bar{3}'](b)$, $m\bar{3}m[\bar{4}3m]$, $m\bar{3}m[432](b)$, $m\bar{3}m1'[m'\bar{3}'m]$,

  $m\bar{3}m1'[m'\bar{3}'m'](b)$, $m\bar{3}m'[4'32']$, $m\bar{3}m'[\bar{4}'3m'](b)$.

3. **A domain wall is non-pyromagnetic but can be pyroelectric**

(a) A group $J_{12}[F_1]$ is of the type $\mathbf{P}[\mathbf{P}]$; 11 cases ; $q_2 1'[q_1 1']$

- $21'[11'](a)$, $m1'[11'](a)$,
- $mm21'[m1'](a)$, $mm21'[21'](a)$,
- $41'[21'](a)$, $4mm1'[mm21'](a)$, $4mm1'[41']$,
- $3m1'[31']$,
- $61'[31']$, $6mm1'[3m1']$, $6mm1'[61']$.

(b) A group $J_{12}[F_1]$ is of the type $\mathbf{0}[\mathbf{P}]$; 8 cases ; $q_2 1'[q_1 1']$

- $2221'[21'](a)$,
- $\bar{4}1'[21'](a)$, $4221'[41']$, $\bar{4}2m1'[mm21'](a)$,
- $321'[31']$,
- $\bar{6}1'[31']$, $6221'[61']$, $\bar{6}m21'[3m1']$.

(c) A group $J_{12}[F_1]$ is of the type $\mathbf{0}[\mathbf{0}]$; 18 cases ; $q_2 1'[q_1 1'], i_2 1'[i_1 1']$

- $2/m1'[\bar{1}1'](a)$,
- $mmm1'[2/m1'](a)$,

- $4/m1'[2/m1'](a)$, $4221'[2221'](a)$, $\bar{4}2m1'[2221'](a)$, $\bar{4}2m1'[\bar{4}1']$, $4/mmm1'[mmm1'](a)$, $4/mmm1'[4/m1']$,
- $\bar{3}m1'[\bar{3}1']$,
- $6/m1'[\bar{3}1']$, $6221'[321']$, $\bar{6}m21'[321']$, $\bar{6}m21'[\bar{6}1']$, $6/mmm1'[\bar{3}m1']$, $6/mmm1'[6/m1']$,
- $4321'[231']$, $\bar{4}3m1'[231']$, $m\bar{3}m1'[m\bar{3}1']$.

4. **A domain wall is non-pyroelectric and non-pyromagnetic**

(a) $J_{12}[F_1]$ is of the type **0[MP]** ; 13 cases ; $i'[q]$

- $\bar{1}'[1]$,
- $2'/m[m]$, $2'/m[2']$, $2/m'[2]$, $2/m'[m']$,
- $m'mm[m'm2']$, $m'm'm'[m'm'2]$,
- $4/m'[4]$, $4/m'm'm'[4m'm']$,
- $\bar{3}'m'[3m']$,
- $6/m'[6]$, $6/m'm'm'[6m'm']$.

(b) $J_{12}[F_1]$ is of the type **0[M]** ; 18 cases ; $i_2 1'[i_1], i'[q]$

- $\bar{1}1'[\bar{1}]$,
- $2/m1'[2/m]$, $2/m1'[2'/m']$,
- $mmm1'[m'm'm]$, $m'mm[22'2']$,
- $4'/m'[\bar{4}]$, $4/m1'[4/m]$, $4/mmm1'[4/mm'm']$, $4/m'mm[42'2']$, $4'/m'm'm[\bar{4}m'2']$,
- $\bar{3}1'[\bar{3}]$, $\bar{3}'[3]$, $\bar{3}m1'[\bar{3}m']$, $\bar{3}'m[32']$,
- $6'/m[\bar{6}], 6/m1'[6/m]$, $6/mmm1'[6/mm'm']$, $6/m'mm[62'2']$, $6'/mmm'[\bar{6}2'm']$,
- $m'\bar{3}'m[4'32']$.

(c) $J_{12}[F_1]$ is of the type **0[P]** ; 18 cases ; $i1'[q1'], i'[q]$

- $\bar{1}1'[11']$,
- $2/m1'[21'],\ \ 2/m1'[m1']$,
- $mmm1'[mm21'],\ \ m'mm[mm2]$,
- $4/m1'[41'],\ \ 4'/m'[4'],\ \ 4/mmm1'[4mm1']$,
  $4/m'mm[4mm],\ \ 4'/m'm'm[4'm'm]$,
- $\bar{3}1'[31'],\ \ \bar{3}m1'[3m1'],\ \ \bar{3}'m[3m]$,
- $6/m1'[61'],\ \ 6'/m[6'],\ \ 6/mmm1'[6mm1']$,
  $6/m'mm[6mm],\ \ 6'/mmm'[6'mm']$,

(d) $J_{12}[F_1]$ is of the type **0[0]** ; 41 cases ; $i_2 1'[i_1], i1'[q1'], i'[q]$

- $mmm1'[mmm],\ \ mmm1'[2221'],\ \ m'm'm'[222]$,
- $4/m1'[\bar{4}1'],\ \ 4/m1'[4'/m],\ \ 4/m'[\bar{4}'],\ \ 4/mmm1'[4/mmm]$,
  $4/mmm1'[4221'],\ \ 4/mmm1'[\bar{4}2m1'],\ \ 4/mmm1'[4'/mmm']$,
  $4/m'm'm'[422],\ \ 4/m'm'm'[\bar{4}'2m']$,
  $4/m'mm[\bar{4}'2'm],\ \ 4'/m'm'm[\bar{4}'2m],\ \ 4'/m'm'm[4'22']$,
- $\bar{3}m1'[\bar{3}m],\ \ \bar{3}m1'[321'],\ \ \bar{3}'m'[32]$,
- $6/m1'[\bar{6}1'],\ \ 6m1'[6'/m'],\ \ 6/m'[\bar{6}'],\ \ 6/mmm1'[6/mmm]$,
  $6/mmm1'[6221'],\ \ 6/mmm1'[\bar{6}m21'],\ \ 6/mmm1'[6'/m'mm']$,
  $6/m'm'm'[622],\ \ 6/m'm'm'[\bar{6}'m'2],\ \ 26/m'mm[\bar{6}'m2']$,
  $6'/mmm'[\bar{6}m2],\ \ 6'/mmm'[6'2'2]$,
- $m\bar{3}1'[m\bar{3}],\ \ m\bar{3}1'[231'],\ \ m'\bar{3}'[23],\ \ m\bar{3}m1'[m\bar{3}m],\ \ m\bar{3}m1'[\bar{4}3m1']$,
  $m\bar{3}m1'[4321'],\ \ m\bar{3}m1'[m\bar{3}m'],\ \ m'\bar{3}m'[432],\ \ m'\bar{3}'m'[\bar{4}'3m']$,
  $m'\bar{3}'m[\bar{4}3m], m'\bar{3}'m[4'32']$.

Appendix G

Symmetry of pyroelectric domain walls in non-pyromagnetic and non-pyroelectric domain pairs

The following table presents sectional layer groups $\overline{J}_{12}$ and the symmetry group of the domain walls for all crystallographic non-equivalent planes (hkl) for 18 cases non-pyromagnetic and non-pyroelectric domain pairs with pyroelectric walls (type $\mathcal{P}$).

- F_1 .... symmetry group of the domain state S_1
- J_{12} .... symmetry group of the unordered domain pair $\{S_1, S_2\}$
- $\widehat{F}_1$ .... one-sided sectional layer group containing all trivial symmetry operations of the domain wall
- T_{12} .... symmetry group of the wall
- $\overline{J}_{12}$ .... the complete sectional layer group of J_{12}
- P .... polarization $\mathbf{P} \neq \mathbf{0}$
- T .... type of the group T_{12}
- $\overline{J}$ .... type of the group $\overline{J}_{12}$

The digraph $*)$ is used for the ferroelastic domain pairs.

Numbers in J box are taken from the numbering of the 380 transposable magnetic twin laws [21].

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{1}$	$2^*/m^*$	(001)	1	$\underline{m}^*_z$	$2^*_z/\underline{m}^*_z$	P	
	$*)$	$(hk0)$	1	$\underline{2}^*_z$	$\underline{2}^*_z/m^*_z$	P	
	5.7	(hkl)	1	1	1	P	P
$2/m$	m^*m^*m	(001)	2_z	$\underline{2}^*_x\underline{2}^*_y 2_z$	$\underline{2}^*_x/m^*_x\underline{2}^*_y/m^*_y 2_z/\underline{m}_z$		
	$*)$	(010)	m_z	$\underline{2}^*_x\underline{m}^*_y m_z$	$\underline{2}^*_x/m^*_x 2^*_y/\underline{m}^*_y\underline{2}_z/m_z$	P	
		(100)	m_z	$\underline{m}^*_x\underline{2}^*_y m_z$	$2^*_x/\underline{m}^*_x\underline{2}^*_y/m^*_y\underline{2}_z/m_z$	P	
		$(hk0)$	m_z	m_z	m_z	P	P
		$(h0l)$	1	$\underline{2}^*_y$	$\underline{2}^*_y/m^*_y$	P	
		$(0kl)$	1	$\underline{2}^*_x$	$\underline{2}^*_x/m^*_x$	P	
	8.6	(hkl)	1	1	1	P	P
$2/m$	$4^*/m$	(001)	2_z	$\underline{\bar{4}}^*_z$	$4^*_z/\underline{m}_z$		
	$*)$	$(hk0)$	m_z	m_z	$\underline{2}_z/m_z$	P	
	11.5	(hkl)	1	1	1	P	P
222	4^*22^*	(001)	2_z	$\underline{2}^*_{xy}\underline{2}^*_{x\bar{y}}2_z$	$4^*_z\underline{2}_x\underline{2}^*_{xy}$		
	$*)$	(100)	2_x	2_x	$2_x\underline{2}_y\underline{2}_z$	P	
		(110)	1	$\underline{2}^*_{x\bar{y}}$	$2^*_{xy}\underline{2}^*_{x\bar{y}}\underline{2}_z$	P	
		$(hk0)$	1	1	$\underline{2}_z$	P	P
		(hhl)	1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	P	P
		$(h0l)$	1	1	$\underline{2}_y$	P	P
	12.4	(hkl)	1	1	1	P	P
222	$\bar{4}^*2m^*$	(001)	2_z	$\underline{\bar{4}}^*_z$	$\underline{\bar{4}}^*_z 2_z m^*_{x\bar{y}}$		
	$*)$	(100)	2_x	2_x	$2_x\underline{2}_y\underline{2}_z$	P	
		(110)	1	$\underline{m}^*_{xy}$	$\underline{m}^*_{xy}m^*_{x\bar{y}}\underline{2}_z$	P	P
		$(hk0)$	1	1	$\underline{2}_z$	P	P
		(hhl)	1	1	$m^*_{x\bar{y}}$	P	P
		$(h0l)$	1	1	$\underline{2}_y$	P	P
	14.5	(hkl)	1	1	1	P	P
mmm	$4^*/mmm^*$	(001)	$m_x m_y 2_z$	$\underline{\bar{4}}^*_z m_x\underline{2}^*_{xy}$	$4^*_z/\underline{m}_z\underline{2}_x/m_x\underline{2}^*_{xy}/m^*_{xy}$		
	$*)$	(100)	$2_x m_y m_z$	$2_x m_y m_z$	$2_x/\underline{m}_x\underline{2}_z/m_y\underline{2}_z/m_z$	P	
		(110)	m_z	$\underline{2}^*_{x\bar{y}}\underline{m}^*_{xy}m_z$	$2^*_{xy}/\underline{m}^*_{xy}\underline{2}^*_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m_z$	P	
		$(hk0)$	m_z	m_z	m_z	P	P
		(hhl)	1	1	1	P	P
		$(h0l)$	m_y	m_y	m_y	P	P
	15.7	(hkl)	1	1	1	P	P

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{4}$	$\bar{4}2^*m^*$	(001)	2_z	$\underline{2}_x^*\underline{2}_y^*2_z$	$\underline{\bar{4}}_z\underline{2}_x^*m_{xy}^*$		
		(100)	1	$\underline{2}_y^*$	$2_x^*\underline{2}_y^*\underline{2}_z$	P	
		(110)	1	$\underline{m}_{xy}^*$	$\underline{m}_{xy}^*m_{x\bar{y}}^*\underline{2}_z$	P	P
		$(hk0)$	1	1	$\underline{2}_z$	P	P
		(hhl)	1	1	$m_{x\bar{y}}^*$	P	P
		$(h0l)$	1	$\underline{2}_y^*$	$\underline{2}_y^*$	P	P
	14.7	(hkl)	1	1	1	P	P
$4/m$	$4/mm^*m^*$	(001)	4_z	$4_z\underline{2}_x^*\underline{2}_{xy}^*$	$4_z/\underline{m}_z\underline{2}_x^*/m_x^*\underline{2}_{xy}^*/m_{xy}^*$		
		(100)	m_z	$\underline{m}_x^*\underline{2}_y^*m_z$	$2_x^*/\underline{m}_x^*\underline{2}_y^*/m_y^*\underline{2}_z/m_z$	P	
		(110)	m_z	$\underline{m}_{xy}^*\underline{2}_{x\bar{y}}^*m_z$	$2_{xy}^*/\underline{m}_{xy}^*\underline{2}_{x\bar{y}}^*/m_{x\bar{y}}^*\underline{2}_z/m_z$	P	
		$(hk0)$	m_z	m_z	$\underline{2}_z/m_z$	P	
		(hhl)	1	$\underline{2}_{x\bar{y}}^*$	$\underline{2}_{x\bar{y}}^*/m_{x\bar{y}}^*$	P	
		$(h0l)$	1	$\underline{2}_y^*$	$\underline{2}_y^*/m_y^*$	P	
	15.8	(hkl)	1	1	$\underline{\bar{1}}$	P	
$\bar{3}$	$\bar{3}m^*1$	(0001)	3_z	$3_z\underline{2}_{10}^*1$	$\underline{\bar{3}}_z\underline{2}_{10}^*/m_{2\bar{1}}^*1$		
		$(2\bar{1}\bar{1}0)$	1	$\underline{m}_{2\bar{1}}^*$	$2_{10}^*/\underline{m}_{2\bar{1}}^*$	P	
		$(01\bar{1}0)$	1	$\underline{2}_{10}^*$	$\underline{2}_{10}^*/m_{2\bar{1}}^*$	P	
		$(2h\bar{h}\bar{h}l)$	1	1	$\underline{\bar{1}}$	P	
		$(0h\bar{h}\bar{l})$	1	$\underline{2}_{10}^*$	$\underline{2}_{10}^*/m_{2\bar{1}}^*$	P	
		$(hki0)$	1	1	$\underline{\bar{1}}$	P	
	20.5	$(hkil)$	1	1	$\underline{\bar{1}}$	P	
$\bar{3}$	$6^*/m^*$	(0001)	3_z	$\underline{\bar{6}}_z^*$	$6_z^*/\underline{m}_z^*$		
		$(hki0)$	1	$\underline{2}_z^*$	$\underline{2}_z^*/m_z^*$	P	
	23.7	$(hkil)$	1	1	$\underline{\bar{1}}$	P	
32	6^*22^*	(0001)	3_z	$3_z1\underline{2}_{12}^*$	$6_z^*\underline{2}_{10}\underline{2}_{12}^*$		
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}\underline{2}_{12}^*\underline{2}_z^*$	$2_{10}\underline{2}_{12}^*\underline{2}_z^*$		
		$(01\bar{1}0)$	1	$\underline{2}_z^*$	$\underline{2}_{10}2_{12}^*\underline{2}_z^*$	P	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}_{12}^*$	$\underline{2}_{12}^*$	P	P
		$(0h\bar{h}l)$	1	1	$\underline{2}_{10}$	P	P
		$(hki0)$	1	$\underline{2}_z^*$	$\underline{2}_z^*$	P	P
	24.4	$(hkil)$	1	1	1	P	P
32	$\bar{6}^*m^*2$	(0001)	3_z	$\underline{\bar{6}}_z^*$	$\underline{\bar{6}}_z^*m_{2\bar{1}}^*\underline{2}_{12}$		
		$(2\bar{1}\bar{1}0)$	1	$\underline{m}_{2\bar{1}}^*$	$\underline{2}_{12}\underline{m}_{2\bar{1}}^*m_z^*$	P	P
		$(01\bar{1}0)$	2_{12}	2_{12}	$2_{12}m_{2\bar{1}}^*m_z^*$	P	P
		$(\bar{2}hhhl)$	1	1	$\underline{2}_{12}$	P	P
		$(0h\bar{h}l)$	1	1	$m_{2\bar{1}}^*$	P	P
		$(hki0)$	1	1	m_z^*	P	P
	26.5	$(hkil)$	1	1	1	P	P

F_1	J_{1j}	(hkl)	$\hat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{3}m$	$6^*/m^*mm^*$	(0001)	$3_z m_{2\bar{1}}$	$\underline{\bar{6}}^*_z m_{2\bar{1}}\underline{2}^*_{12}$	$6^*_z/\underline{m}^*_z\underline{2}_{10}/m_{2\bar{1}}\underline{2}^*_{12}/m^*_{01}$		
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}\underline{2}^*_{12}\underline{2}^*_z$	$2_{10}/\underline{m}_{2\bar{1}}\underline{2}^*_{12}/m^*_{01}\underline{2}^*_z/m^*_z$		
		$(01\bar{1}0)$	$m_{2\bar{1}}$	$\underline{m}^*_{01}m_{2\bar{1}}\underline{2}^*_z$	$\underline{2}_{10}/m_{2\bar{1}}2^*_{12}/\underline{m}^*_{01}\underline{2}^*_z/m^*_z$	P	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m^*_{01}$	P	
		$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m_{2\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}}$	P	
		$(hki0)$	1	$\underline{2}^*_z$	$\underline{2}^*_z/m^*_z$	P	
	27.7	$(hkil)$	1	1	$\underline{\bar{1}}$	P	
$\bar{6}$	$\bar{6}m^*2^*$	(0001)	3_z	$3_z1\underline{2}^*_{12}$	$\underline{\bar{6}}_z m^*_{2\bar{1}}\underline{2}^*_{12}$		
		$(2\bar{1}\bar{1}0)$	m_z	$\underline{2}^*_{12}\underline{m}^*_{2\bar{1}}m_z$	$\underline{2}^*_{12}\underline{m}^*_{2\bar{1}}m_z$	P	P
		$(01\bar{1}0)$	m_z	m_z	$2^*_{12}m^*_{2\bar{1}}m_z$	P	P
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	P	P
		$(0\bar{h}hl)$	1	1	$m^*_{2\bar{1}}$	P	P
		$(hki0)$	m_z	m_z	m_z	P	P
	26.7	$(hkil)$	1	1	1	P	P
$6/m$	$6/mm^*m^*$	(0001)	6_z	$6_z\underline{2}^*_{10}\underline{2}^*_{12}$	$6_z/\underline{m}_z\underline{2}^*_{10}/m^*_{2\bar{1}}\underline{2}^*_{12}/m^*_{01}$		
		$(2\bar{1}\bar{1}0)$	m_z	$\underline{2}^*_{12}\underline{m}^*_{2\bar{1}}m_z$	$2^*_{10}/\underline{m}^*_{2\bar{1}}2^*_{12}/m^*_{01}\underline{2}_z/m_z$	P	
		$(01\bar{1}0)$	m_z	$\underline{2}^*_{10}\underline{m}^*_{01}m_z$	$\underline{2}^*_{10}/m^*_{2\bar{1}}2^*_{12}/\underline{m}^*_{01}\underline{2}_z/m_z$	P	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m^*_{01}$	P	
		$(0h\bar{h}l)$	1	$\underline{2}^*_{10}$	$\underline{2}^*_{10}/m^*_{2\bar{1}}$	P	
		$(hki0)$	m_z	m_z	$\underline{2}_z/m_z$	P	
	27.8	$(hkil)$	1	1	$\underline{\bar{1}}$	P	
23	4^*32^*	(001)	2_z	$\underline{2}^*_{xy}\underline{2}^*_{x\bar{y}}2_z$	$4^*_z\underline{2}_x\underline{2}^*_{xy}$		
		(110)	1	$\underline{2}^*_{x\bar{y}}$	$2^*_{xy}\underline{2}^*_{x\bar{y}}\underline{2}_z$	P	
		$(kl0)$	1	1	$\underline{2}_z$	P	P
		(hhl)	1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	P	P
		(hkl)	1	1	1	P	P
	30.3	(111)	3_p	$3_p\underline{2}^*_{x\bar{y}}$	$3_p\underline{2}^*_{x\bar{y}}$		
23	$\bar{4}^*3m^*$	(001)	2_z	$\underline{\bar{4}}^*_z$	$\underline{\bar{4}}^*_z\underline{2}_x m^*_{xy}$		
		(110)	1	$\underline{m}^*_{xy}$	$\underline{m}^*_{xy}m^*_{x\bar{y}}\underline{2}_z$	P	P
		$(kl0)$	1	1	$\underline{2}_z$	P	P
		(hhl)	1	1	$m^*_{x\bar{y}}$	P	P
		(hkl)	1	1	1	P	P
	31.3	(111)	3_p	3_p	$3_p m^*_{x\bar{y}}$	P	P
$m\bar{3}$	$m\bar{3}m^*$	(001)	$2_z m_x m_y$	$\underline{\bar{4}}^*_z m_x\underline{2}^*_{xy}$	$4^*_z/\underline{m}_z\underline{2}_x/m_x\underline{2}^*_{xy}/m^*_{xy}$		
		(110)	m_z	$\underline{2}^*_{x\bar{y}}\underline{m}^*_{xy}m_z$	$2^*_{xy}/\underline{m}^*_{xy}\underline{2}^*_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m_z$	P	
		$(kl0)$	m_z	m_z	$\underline{2}_z/m_z$	P	
		(hhl)	1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m^*_{x\bar{y}}$	P	
		(hkl)	1	1	$\underline{\bar{1}}$	P	
	32.5	(111)	3_p	$3_p\underline{2}^*_{x\bar{y}}$	$\underline{\bar{3}}_p\underline{2}^*_{x\bar{y}}/m^*_{x\bar{y}}$		

Appendix H

Symmetry of pyromagnetic domain walls in non-pyromagnetic and non-pyroelectric domain pairs

The following table presents *sectional layer groups* $\overline{J}_{12}$ and the symmetry group of the domain walls T_{12} for all crystallographic non-equivalent planes (hkl) for 40 cases of the non-pyromagnetic and non-pyroelectric domain pairs with pyromagnetic walls (type $\mathcal{M}$).

- F_1 .... symmetry group of the domain state S_1
- J_{12} .... symmetry group of the unordered domain pair $\{S_1, S_2\}$
- $\widehat{F}_1$ .... one-sided sectional layer group containing all trivial symmetry operations of the domain wall
- T_{12} .... symmetry group of the wall
- $\overline{J}_{12}$ .... the complete sectional layer group of J_{12}
- M .... magnetization $\mathbf{M} \neq \mathbf{0}$
- T .... type of the group T_{12}
- $\overline{J}$ .... type of the group $\overline{J}_{12}$

The digraph +) is used for the non-magnetoelectric domain pairs, the others are non-ferroelastic magnetoelectric domain pairs.

Numbers in J box are taken from the numbering of the 380 transposable magnetic twin laws [21].

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{1}'$	$\bar{1}'1'^{*}$ 2.4	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}'1'^{*}$	M	
$2/m'$	$2/m'1'^{*}$	(001)	2_z	$2_z/\underline{m}^{*}_z$	$2_z/\underline{m}'_z1'^{*}$	M	
		$(hk0)$	m'_z	$\underline{2}'^{*}_z/m'_z$	$\underline{2}_z/m'_z1'^{*}$	M	
	5.8	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}'1'^{*}$	M	
$2'/m$	$2/m1'^{*}$	(001)	$2'_z$	$2'_z/\underline{m}'^{*}_z$	$2'_z/\underline{m}_z1'^{*}$	M	
		$(hk0)$	m_z	$\underline{2}^{*}_z/m_z$	$\underline{2}'_z/m_z1'^{*}$	M	
	5.9	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}'1'^{*}$	M	
222	$m^{*}m^{*}m^{*}$	(001)	2_z	$2_z/\underline{m}^{*}_z$	$\underline{2}_x/m^{*}_x\underline{2}_y/m^{*}_y2_z/\underline{m}^{*}_z$	M	
		(010)	2_y	$2_y/\underline{m}^{*}_y$	$\underline{2}_x/m^{*}_x2_y/\underline{m}^{*}_y\underline{2}_z/m^{*}_z$	M	
		(100)	2_x	$2_x/\underline{m}^{*}_x$	$2_x/\underline{m}^{*}_x\underline{2}_y/m^{*}_y\underline{2}_z/m^{*}_z$	M	
		$(hk0)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_z/m^{*}_z$	M	M
		$(h0l)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_y/m^{*}_y$	M	M
		$(0kl)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_x/m^{*}_x$	M	M
	8.3	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}^{*}$	M	M

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$m'mm$	$m'mm1'^{*}$	(001)	$m'_x m_y 2'_z$	$\underline{2}'^{*}_x/m'_x\underline{2}^{*}_y/m_y 2'_z/\underline{m}'^{*}_z$	$\underline{2}_x/m'_x\underline{2}'_y/m_y 2'_z/\underline{m}_z 1'^{*}$	M	
		(010)	$m'_x 2'_y m_z$	$\underline{2}'^{*}_x/m'_x 2'_y/\underline{m}'^{*}_y\underline{2}^{*}_z/m_z$	$\underline{2}_x/m'_x 2'_y/x\underline{m}_y\underline{2}'_z/m_z 1'^{*}$	M	
		(100)	$2_x m_y m_z$	$2_x/\underline{m}^{*}_x\underline{2}^{*}_y/m_y\underline{2}^{*}_z/m_z$	$2_x/\underline{m}'_x\underline{2}'_y/m_y\underline{2}'_z/m_z 1'^{*}$		
		$(hk0)$	m_z	$\underline{2}^{*}_z/m_z$	$\underline{2}'_z/m_z 1'^{*}$	M	
		$(h0l)$	m_y	$\underline{2}^{*}_y/m_y$	$\underline{2}'_y/m_y 1'^{*}$	M	
		$(0kl)$	m'_x	$\underline{2}'^{*}_x/m'_x$	$\underline{2}'_x/m_x 1'^{*}$	M	
	8.8	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}'1'^{*}$	M	
$m'm'm'$	$m'm'm'1'^{*}$	(001)	$m'_x m'_y 2_z$	$\underline{2}'^{*}_x/m'_x\underline{2}'^{*}_y/m'_y 2_z/\underline{m}^{*}_z$	$\underline{2}_x m'_x\underline{2}'_y/m'_y 2_z/\underline{m}'_z 1'^{*}$	M	
		(010)	$m'_x 2_y m'_z$	$\underline{2}'^{*}_x/m'_x 2_y/\underline{m}^{*}_y\underline{2}'^{*}_z/m'_z$	$\underline{2}_x/m'_x 2_y/\underline{m}'_y\underline{2}_z/m'_z 1'^{*}$	M	
		(100)	$2_x m'_y m'_z$	$2_x/\underline{m}^{*}_x\underline{2}'^{*}_y/m'_y\underline{2}'^{*}_z/m'_z$	$2_x/\underline{m}'_x\underline{2}_y/m'_y\underline{2}_z/m'_z 1'^{*}$	M	
		$(hk0)$	m'_z	$\underline{2}'^{*}_z/m'_z$	$\underline{2}_z/m'_z 1'^{*}$	M	
		$(h0l)$	m'_y	$\underline{2}'^{*}_y/m'_y$	$\underline{2}_y/m'_y 1'^{*}$	M	
		$(0kl)$	m'_x	$\underline{2}'^{*}_x/m'_x$	$\underline{2}_x/m'_x 1'^{*}$	M	
	8.10	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}'1'^{*}$	M	

F_1	J_{1j}	(hkl)	$\hat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$4/m'$	$4/m'1'^{*}$	(001)	4_z	$4_z/\underline{m}_z^{*}$	$4_z/\underline{m}_z'1'^{*}$	M	
		$(hk0)$	m_z'	$\underline{2}_z'^{*}/m_z'$	$\underline{2}_z/m_z'1'^{*}$	M	
	11.9	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}'1'^{*}$	M	
$4'/m'$	$4'/m'1'^{*}$	(001)	$4_z'$	$4_z'/\underline{m}_z^{*}$	$4_z'/\underline{m}_z'1'^{*}$		
		$(hk0)$	m_z'	$\underline{2}_z'^{*}/m_z'$	$\underline{2}_z/m_z'1'^{*}$	M	
	11.10	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}'1'^{*}$	M	
$\bar{4}'$	$4'^{*}/m^{*}$	(001)	2_z	$2_z/\underline{m}_z^{*}$	$4_z'^{*}/\underline{m}_z^{*}$	M	
		$(hk0)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_z/m_z^{*}$	M	M
	11.13	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}^{*}$	M	M
422	$4/m^{*}m^{*}m^{*}$	(001)	4_z	$4_z/\underline{m}_z^{*}$	$4_z/\underline{m}_z^{*}\underline{2}_x/m_x^{*}\underline{2}_{xy}/m_{xy}^{*}$	M	
		(100)	2_x	$2_x/\underline{m}_x^{*}$	$2_x/\underline{m}_x^{*}\underline{2}_y/m_y^{*}\underline{2}_z/m_z^{*}$	M	
		(110)	2_{xy}	$2_{xy}/\underline{m}_{xy}^{*}$	$2_{xy}/\underline{m}_{xy}^{*}\underline{2}_{x\bar{y}}/m_{x\bar{y}}^{*}\underline{2}_z/m_z^{*}$	M	
		$(hk0)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_z/m_z^{*}$	M	M
		(hhl)	1	$\underline{\bar{1}}^{*}$	$\underline{2}_{x\bar{y}}/m_{x\bar{y}}^{*}$	M	M
		$(h0l)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_y/m_y^{*}$	M	M
	15.3	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}^{*}$	M	M

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{4}2m$	$4^*/m^*m^*m$	(001)	$m_{xy}m_{x\bar{y}}2_z$	$\underline{2}^*_{xy}/m_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}2_z/\underline{m}^*_z$	$4^*_z/\underline{m}^*_z\underline{2}_x/m^*_x\underline{2}^*_{xy}/m_{xy}$		
		(100)	2_x	$2_x/\underline{m}^*_x$	$2_x/\underline{m}^*_x\underline{2}_y/m^*_y\underline{2}_z/m^*_z$	M	
		(110)	$m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$2^*_{xy}/\underline{m}_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m^*_z$	M	
		$(hk0)$	1	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$	M	M
		(hhl)	$m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	M	M
		$(h0l)$	1	$\underline{\bar{1}}^*$	$\underline{2}_y/m^*_y$	M	M
	15.5	(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
$4/m'm'm'$	$4/m'm'm'1'^*$	(001)	$4_zm'_xm'_{xy}$	$4_z/\underline{m}^*_z\underline{2}'^*_x/m'_x\underline{2}'^*_{xy}/m'_{xy}$	$4_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}_{xy}/m'_{xy}1'^*$	M	
		(100)	$2_xm'_ym'_z$	$2_x/\underline{m}^*_x\underline{2}'^*_y/m'_y\underline{2}'^*_z/m'_z$	$2_x/\underline{m}'_x\underline{2}_y/m'_y\underline{2}_z/m'_z1'^*$	M	
		(110)	$2_{xy}m'_{x\bar{y}}m'_z$	$2_{xy}/\underline{m}^*_{xy}\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}\underline{2}'^*_z/m'_z$	$2_{xy}/\underline{m}'_{xy}\underline{2}_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m'_z1'^*$	M	
		$(hk0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z1'^*$	M	
		(hhl)	m'_{xy}	$\underline{2}'^*_{xy}/m'_{xy}$	$\underline{2}_{xy}/m'_{xy}1'^*$	M	
		$(h0l)$	m'_y	$\underline{2}'^*_y/m'_y$	$\underline{2}_y/m'_y1'^*$	M	
	15.14	(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}'1'^*$	M	
$4/m'mm$	$4/m'mm1'^*$	(001)	$4_zm_xm_{xy}$	$4_z/\underline{m}^*_z\underline{2}^*_x/m_x\underline{2}^*_{xy}/m_{xy}$	$4_z/\underline{m}'_z\underline{2}'_x/m_x\underline{2}'_{xy}/m_{xy}1'^*$		
		(100)	$2'_xm_ym'_z$	$2'_x/\underline{m}'^*_x\underline{2}^*_y/m_y\underline{2}'^*_z/m'_z$	$2'_x/\underline{m}_x\underline{2}'_y/m_y\underline{2}_z/m'_z1'^*$	M	
		(110)	$2'_{xy}m_{x\bar{y}}m'_z$	$2'_{xy}/\underline{m}'^*_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}\underline{2}'^*_z/m'_z$	$2'_{xy}/\underline{m}_{xy}\underline{2}'_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m'_z1'^*$	M	
		$(hk0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z1'^*$	M	
		(hhl)	m_{xy}	$\underline{2}^*_{xy}/m_{xy}$	$\underline{2}'_{xy}/m_{xy}1'^*$	M	
		$(h0l)$	m_y	$\underline{2}^*_y/m_y$	$\underline{2}'_y/m_y1'^*$	M	
	15.15	(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}'1'^*$	M	

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$4'/m'm'm$	$4'/m'm'm1'^{*}$	(001)	$4'_z m'_x m_{xy}$	$4'_z/\underline{m}^{*}_z\underline{2}'^{*}_x/m'_x\underline{2}^{*}_{xy}/m_{xy}$	$4'_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}'^{*}_{xy}/m_{xy}1'^{*}$		
		(100)	$2_x m'_y m'_z$	$2_x/\underline{m}^{*}_x\underline{2}'^{*}_y/m'_y\underline{2}'^{*}_z/m'_z$	$2_x/\underline{m}'^{*}_x\underline{2}_y/m'_y\underline{2}_z/m'_z1'^{*}$	M	
		(110)	$2'_{xy}m_{x\bar{y}}m'_z$	$2'_{xy}/\underline{m}'^{*}_{xy}\underline{2}^{*}_{x\bar{y}}/m_{x\bar{y}}\underline{2}'^{*}_z/m'_z$	$2'_{xy}/\underline{m}_{xy}\underline{2}'_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m'_z1'^{*}$	M	
		$(hk0)$	m'_z	$\underline{2}'_z/m'_z$	$\underline{2}_z/m'_z1'^{*}$	M	
		(hhl)	m_{xy}	$\underline{2}^{*}_{xy}/m_{xy}$	$\underline{2}'_{xy}/m_{xy}1'^{*}$	M	
		$(h0l)$	m'_y	$\underline{2}'^{*}_z/m'_y$	$\underline{2}_y/m'_y1'^{*}$	M	
	15.16	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}'1'^{*}$	M	
$4'22'$	$4'/m^{*}m^{*}m'^{*}$	(001)	$4'_z$	$4'_z/\underline{m}^{*}_z$	$4'_z/\underline{m}^{*}_z\underline{2}_x/m^{*}_x\underline{2}'_{xy}/m'^{*}_{xy}$		
		(100)	2_x	$2_x/\underline{m}^{*}_x$	$2_x/\underline{m}^{*}_x\underline{2}_y/m^{*}_y\underline{2}_z/m^{*}_z$	M	
		(110)	$2'_{xy}$	$2'_{xy}/\underline{m}'^{*}_{xy}$	$2'_{xy}/\underline{m}'^{*}_{xy}\underline{2}'_{x\bar{y}}/m'^{*}_{x\bar{y}}\underline{2}_z/m^{*}_z$	M	M
		$(hk0)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_z/m^{*}_z$	M	M
		(hhl)	1	$\underline{\bar{1}}^{*}$	$\underline{2}'_{x\bar{y}}/m'^{*}_{x\bar{y}}$	M	M
		$(h0l)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_y/m^{*}_y$	M	M
	15.25	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}^{*}$	M	M
$\bar{4}'2m'$	$4'^{*}/m^{*}m^{*}m'$	(001)	$2_z m'_{xy}m'_{x\bar{y}}$	$\underline{2}'^{*}_{xy}/m'_{xy}\underline{2}'^{*}_{x\bar{y}}/m'_{x\bar{y}}2_z/\underline{m}^{*}_z$	$4'_z/\underline{m}^{*}_z\underline{2}_x/m^{*}_x\underline{2}'^{*}_{xy}/m'_{xy}$	M	
		(100)	2_x	$2_x/\underline{m}^{*}_x$	$2_x/\underline{m}^{*}_x\underline{2}_y/m^{*}_y\underline{2}_z/m^{*}_z$	M	
		(110)	$m'_{x\bar{y}}$	$\underline{2}'^{*}_{x\bar{y}}/m'_{x\bar{y}}$	$2'_{xy}/\underline{m}'_{xy}\underline{2}'^{*}_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m^{*}_z$	M	M
		$(hk0)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_z/m^{*}_z$	M	M
		(hhl)	$m'_{x\bar{y}}$	$\underline{2}'^{*}_{x\bar{y}}/m'_{x\bar{y}}$	$\underline{2}'^{*}_{x\bar{y}}/m'_{x\bar{y}}$	M	M
		$(h0l)$	1	$\underline{\bar{1}}^{*}$	$\underline{2}_y/m^{*}_y$	M	M
	15.27	(hkl)	1	$\underline{\bar{1}}^{*}$	$\underline{\bar{1}}^{*}$	M	M

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{4}'m2'$	$4'^{*}/m^{*}mm'^{*}$	(001)	$2_z m_x m_y$	$\underline{2}^*_x/m_x\underline{2}^*_y/m_y 2_z/\underline{m}^*_z$	$4'^*_z/\underline{m}^*_z\underline{2}_x/m^*_x\underline{2}'^*_{xy}/m'_{xy}$		
		(100)	m_y	$\underline{2}^*_y/m_y$	$2^*_x/\underline{m}_x\underline{2}^*_y/m_y\underline{2}_z/m^*_z$	M	
		(110)	$2'_{xy}$	$2'_{xy}/\underline{m}'^*_{xy}$	$2'_{xy}/\underline{m}'^*_{xy}\underline{2}'_{x\bar{y}}/m'^*_{x\bar{y}}\underline{2}_z/m^*_z$	M	M
		$(hk0)$	1	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$	M	M
		(hhl)	1	$\underline{\bar{1}}^*$	$\underline{2}'_{xy}/m'^*_{xy}$	M	M
		$(h0l)$	m_y	$\underline{2}^*_y/m_y$	$\underline{2}^*_y/m_y$	M	M
	15.28	(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
$\bar{3}'$	$\bar{3}'1'^*$	(0001)	3_z	$\underline{\bar{3}}^*_z$	$\underline{\bar{3}}'_z1'^*$	M	
		$(hki0)$	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}'1'^*$	M	
	17.4	$(hkil)$	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}'1'^*$	M	
32	$\bar{3}^*m^*1$	(0001)	3_z	$\underline{\bar{3}}^*_z$	$\underline{\bar{3}}^*_z\underline{2}_{10}/m^*_{2\bar{1}}1$	M	
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}/\underline{m}^*_{2\bar{1}}$	$2_{10}/\underline{m}^*_{2\bar{1}}$	M	M
		$(01\bar{1}0)$	1	$\underline{\bar{1}}^*$	$\underline{2}_{10}/m^*_{2\bar{1}}$	M	M
		$(2h\bar{h}\bar{h}l)$	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
		$(0h\bar{h}\bar{l})$	1	$\underline{\bar{1}}^*$	$\underline{2}_{10}/m^*_{2\bar{1}}$	M	M
		$(hki0)$	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
	20.3	$(hkil)$	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar3'm$	$\bar3'm1'^*$	(0001)	$3_zm_{2\bar1}$	$\underline{\bar3}^*_z\underline{2}^*_{10}/m_{2\bar1}$	$\underline{\bar3}'_z\underline{2}'_{10}/m_{2\bar1}1'^*$		
		$(2\bar1\bar10)$	$2'_{10}$	$2'_{10}/\underline{m}'^*_{2\bar1}$	$2'_{10}/\underline{m}_{2\bar1}1'^*$	M	
		$(01\bar10)$	$m_{2\bar1}$	$\underline{2}_{10}/m_{2\bar1}$	$\underline{2}'_{10}/m_{2\bar1}1'^*$	M	
		$(2h\bar h\bar hl)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
		$(0h\bar h\bar l)$	$m_{2\bar1}$	$\underline{2}^*_{10}/m_{2\bar1}$	$\underline{2}'_{10}/m_{2\bar1}1'^*$	M	
		$(hki0)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
	20.9	$(hkil)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
$\bar3'm'$	$\bar3'm'1'^*$	(0001)	$3_zm'_{2\bar1}$	$\underline{\bar3}^*_z\underline{2}'^*_{10}/m'_{2\bar1}$	$\underline{\bar3}'_z\underline{2}_{10}/m'_{2\bar1}1'^*$	M	
		$(2\bar1\bar10)$	2_{10}	$2_{10}/\underline{m}^*_{2\bar1}$	$2_{10}/\underline{m}'_{2\bar1}1'^*$	M	
		$(01\bar10)$	$m'_{2\bar1}$	$\underline{2}'^*_{10}/m'_{2\bar1}$	$\underline{2}_{10}/m'_{2\bar1}1'^*$	M	
		$(2h\bar h\bar hl)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
		$(0h\bar h\bar l)$	$m'_{2\bar1}$	$\underline{2}'^*_{10}/m'_{2\bar1}$	$\underline{2}_{10}/m'_{2\bar1}1'^*$	M	
		$(hki0)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
	20.10	$(hkil)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
$6/m'$	$6/m1'^*$	(0001)	6_z	$6_z/\underline{m}^*_z$	$6_z/\underline{m}'_z1'^*$	M	
		$(2\bar1\bar10)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z1'^*$	M	
		$(01\bar10)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z1'^*$	M	
		$(2h\bar h\bar hl)$	1	$\underline{\bar1}^*$	$\underline{\bar1}1'^*$	M	
		$(0h\bar h\bar l)$	1	$\underline{\bar1}^*$	$\underline{\bar1}1'^*$	M	
		$(hki0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z1'^*$	M	
	23.8	$(hkil)$	1	$\underline{\bar1}^*$	$\underline{\bar1}1'^*$	M	

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$6'/m$	$6'/m1'^*$	(0001)	$6'_z$	$6'_z/\underline{m}'^*_z$	$6'_z/\underline{m}_z1'^*$		
		$(2\bar1\bar10)$	m_z	$\underline{2}^*_z/m_z$	$\underline{2}'_z/m_z1'^*$	M	
	$+)$	$(01\bar10)$	m_z	$\underline{2}^*_z/m_z$	$\underline{2}'_z/m_z1'^*$	M	
		$(2h\bar h\bar hl)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
		$(0h\bar h\bar l)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
		$(hki0)$	m_z	$\underline{2}^*_z/m_z$	$\underline{2}'_z/m_z1'^*$	M	
	23.9	$(hkil)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
$\bar6'$	$6'^*/m'$	(0001)	3_z	$\underline{\bar3}^*_z$	$6'^*_z/\underline{m}'_z$	M	
		$(hki0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}'^*_z/m'_z$	M	M
	23.19	$(hkil)$	1	$\underline{\bar1}^*$	$\underline{\bar1}^*$	M	M
622	$6/m^*m^*m^*$	(0001)	6_z	$6_z/\underline{m}^*_z$	$6_z/\underline{m}^*_z\underline{2}_{10}/m^*_{2\bar1}\underline{2}_{12}/m^*_{01}$	M	
		$(2\bar1\bar10)$	2_{10}	$2_{10}/\underline{m}^*_{2\bar1}$	$2_{10}/\underline{m}^*_{2\bar1}\underline{2}_{12}/m^*_{01}\underline{2}_z/m^*_z$	M	
		$(01\bar10)$	2_{12}	$2_{12}/\underline{m}^*_{01}$	$\underline{2}_{10}/m^*_{2\bar1}2_{12}/\underline{m}^*_{01}\underline{2}_z/m^*_z$	M	
		$(2h\bar h\bar hl)$	1	$\underline{\bar1}^*$	$\underline{2}_{12}/m^*_{01}$	M	M
		$(0h\bar hl)$	1	$\underline{\bar1}^*$	$\underline{2}_{10}/m^*_{2\bar1}$	M	M
		$(hki0)$	1	$\underline{\bar1}^*$	$\underline{2}_z/m^*_z$	M	M
	27.3	$(hkil)$	1	$\underline{\bar1}^*$	$\underline{\bar1}^*$	M	M

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{6}m2$	$6^*/mmm^*$	(0001)	$3_z m_{2\bar{1}}$	$\underline{\bar{3}}^*\underline{2}^*_{10}/m_{2\bar{1}}$	$6^*_z/\underline{m}_z\underline{2}^*_{10}/m_{2\bar{1}}\underline{2}_{12}/m^*_{01}$		
		$(2\bar{1}\bar{1}0)$	m_z	$\underline{2}^*_z/m_z$	$2^*_{10}/\underline{m}_{2\bar{1}}\underline{2}_{12}/m^*_{01}\underline{2}^*_z/m_z$	M	
	+)	$(01\bar{1}0)$	$m_{2\bar{1}}2_{12}m_z$	$\underline{2}^*_{10}/m_{2\bar{1}}2_{12}/\underline{m}^*_{01}\underline{2}^*_z/m_z$	$\underline{2}^*_{10}/m_{2\bar{1}}2_{12}/\underline{m}^*_{01}\underline{2}^*_z/m_z$		
		$(2h\bar{h}\bar{h}l)$	1	$\underline{\bar{1}}^*$	$\underline{2}_{12}/m^*_{01}$	M	M
		$(0h\bar{h}l)$	$m_{2\bar{1}}$	$\underline{2}^*_{10}/m_{2\bar{1}}$	$\underline{2}^*_{10}/m_{2\bar{1}}$	M	M
		$(hki0)$	m_z	$\underline{2}^*_z/m_z$	$\underline{2}^*_z/m_z$	M	M
	27.5	$(hkil)$	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
$6/m'm'm'$	$6/m'm'm'1'^*$	(0001)	$6_z m'_{2\bar{1}}m'_{01}$	$6_z/\underline{m}^*_z\underline{2}'^*_{10}/m'_{2\bar{1}}\underline{2}'^*_{12}/m'_{01}$	$6_z/\underline{m}'_z\underline{2}_{10}/m'_{2\bar{1}}\underline{2}_{12}/m'_{01}1'^*$	M	
		$(2\bar{1}\bar{1}0)$	$m'_{01}2_{10}m'_z$	$2_{10}/\underline{m}^*_{2\bar{1}}\underline{2}'^*_{12}/m'_{01}\underline{2}'^*_z/m'_z$	$2_{10}/\underline{m}'_{2\bar{1}}\underline{2}_{12}/m'_{01}\underline{2}_z/m'_z1'^*$	M	
		$(01\bar{1}0)$	$2_{12}m'_{2\bar{1}}m'_z$	$\underline{2}'^*_{10}/m'_{2\bar{1}}2_{12}/\underline{m}^*_{01}\underline{2}'^*_z/m'_z$	$\underline{2}_{10}/m'_{2\bar{1}}2_{12}/\underline{m}'_{01}\underline{2}_z/m'_z1'^*$	M	
		$(2h\bar{h}\bar{h}l)$	m'_{01}	$\underline{2}'^*/m'_{01}$	$\underline{2}_{12}/m'_{01}1'^*$	M	
		$(0h\bar{h}\bar{l})$	$m'_{2\bar{1}}$	$\underline{2}'^*_{10}/m'_{2\bar{1}}$	$\underline{2}_{10}/m'_{2\bar{1}}1'^*$	M	
		$(hki0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z1'^*$	M	
	27.14	$(hkil)$	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}'1'^*$	M	
$6/m'mm$	$6/m'mm1'^*$	(0001)	$6_z m_{2\bar{1}}m_{01}$	$6_z/\underline{m}^*_z\underline{2}^*_{10}/m_{2\bar{1}}\underline{2}^*_{12}/m_{01}$	$6_z/\underline{m}'_z\underline{2}'_{10}/m_{2\bar{1}}\underline{2}'_{12}/m_{01}1'^*$		
		$(2\bar{1}\bar{1}0)$	$m_{01}2'_{10}m'_z$	$2'_{10}/\underline{m}'^*_{2\bar{1}}\underline{2}^*_{12}/m_{01}\underline{2}'^*_z/m'_z$	$2'_{10}/\underline{m}_{2\bar{1}}\underline{2}'_{12}/m_{01}\underline{2}_z/m'_z1'^*$	M	
		$(01\bar{1}0)$	$2'_{12}m_{2\bar{1}}m'_z$	$\underline{2}^*_{10}/m_{2\bar{1}}2'_{12}/\underline{m}'^*_{01}\underline{2}'^*_z/m'_z$	$\underline{2}'_{10}/m_{2\bar{1}}2'_{12}/\underline{m}_{01}\underline{2}_z/m'_z1'^*$	M	
		$(2h\bar{h}\bar{h}l)$	m_{01}	$\underline{2}^*/m_{01}$	$\underline{2}'_{12}/m_{01}1'^*$	M	
		$(0h\bar{h}\bar{l})$	$m_{2\bar{1}}$	$\underline{2}^*_{10}/m_{2\bar{1}}$	$\underline{2}'_{10}/m_{2\bar{1}}1'^*$	M	
		$(hki0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z1'^*$	M	
	27.15	$(hkil)$	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}'1'^*$	M	

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$6'/mmm'$	$6'/mmm'1'^*$	(0001)	$6'_z m_{2\bar1} m'_{01}$	$6'_z/\underline{m}'^*_z\underline{2}^*_{10}/m_{2\bar1}\underline{2}'^*_{12}/m'_{01}$	$6'_z/\underline{m}_z\underline{2}'_{10}/m_{2\bar1}\underline{2}_{12}/m'_{01}1'^*$		
		$(2\bar1\bar10)$	$m'_{01}2'_{10}m_z$	$2'_{10}/\underline{m}'^*_{2\bar1}\underline{2}'^*_{12}/m'_{01}\underline{2}^*_z/m_z$	$2'_{10}/\underline{m}_{2\bar1}\underline{2}_{12}/m'_{01}\underline{2}'_z/m_z1'^*$	M	
	+)	$(01\bar10)$	$2_{12}m_{2\bar1}m_z$	$\underline{2}^*_{10}/m_{2\bar1}2_{12}/\underline{m}^*_{01}\underline{2}^*_z/m_z$	$\underline{2}'_{10}/m_{2\bar1}2_{12}/\underline{m}'_{01}\underline{2}'_z/m_z1'^*$		
		$(2h\bar h\bar hl)$	m'_{01}	$\underline{2}'^*_{12}/m'_{01}$	$\underline{2}_{12}/m'_{01}1'^*$	M	
		$(0h\bar h\bar l)$	$m_{2\bar1}$	$\underline{2}^*_{10}/m_{2\bar1}$	$\underline{2}'_{10}/m_{2\bar1}1'^*$	M	
		$(hki0)$	m_z	$\underline{2}^*_z/m_z$	$\underline{2}'_z/m_z1'^*$	M	
	27.16	$(hkil)$	1	$\underline{\bar1}^*$	$\underline{\bar1}'1'^*$	M	
$\bar6'm2'$	$6'^*/m'mm'^*$	(0001)	$3_z m_{2\bar1}$	$\underline{\bar3}^*\underline{2}^*_{10}/m_{2\bar1}$	$6'^*_z/\underline{m}'_z\underline{2}^*_{10}/m_{2\bar1}\underline{2}'_{12}/m'^*_{01}$		
		$(2\bar1\bar10)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}^*_{10}/\underline{m}_{2\bar1}\underline{2}'_{12}/m'^*_{01}\underline{2}'^*_z/m'_z$	M	M
		$(01\bar10)$	$m_{2\bar1}2'_{12}m_z$	$\underline{2}^*_{10}/m_{2\bar1}2'_{12}/\underline{m}'^*_{01}\underline{2}'^*_z/m'_z$	$\underline{2}^*_{10}/m_{2\bar1}\underline{2}'_{12}/\underline{m}'^*_{01}\underline{2}'^*_z/m'_z$	M	M
		$(2h\bar h\bar hl)$	1	$\underline{\bar1}^*$	$\underline{2}'_{12}/m'^*_{01}$	M	M
		$(0h\bar hl)$	$m_{2\bar1}$	$\underline{2}^*_{10}/m_{2\bar1}$	$\underline{2}^*_{10}/m_{2\bar1}$	M	M
		$(hki0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}'^*_z/m'_z$	M	M
	27.22	$(hkil)$	1	$\underline{\bar1}^*$	$\underline{\bar1}^*$	M	M
$6'22'$	$6'/m'^*m^*m'^*$	(0001)	$6'_z$	$6'_z/\underline{m}'^*_z$	$6'_z/\underline{m}'^*_z2_{10}/m^*_{2\bar1}\underline{2}'_{12}/m'^*_{01}$		
		$(2\bar1\bar10)$	2_{10}	$2_{10}/\underline{m}^*_{2\bar1}$	$2_{10}/\underline{m}^*_{2\bar1}\underline{2}'_{12}/m'^*_{01}\underline{2}'_z/m'^*_z$	M	M
	+)	$(01\bar10)$	$2'_{12}$	$2'_{12}/\underline{m}'^*_{01}$	$\underline{2}_{10}/m^*_{2\bar1}2'_{12}/\underline{m}'^*_{01}\underline{2}'_z/m'^*_z$	M	M
		$(2h\bar h\bar hl)$	1	$\underline{\bar1}^*$	$\underline{2}'_{12}/m'^*_{01}$	M	M
		$(0h\bar hl)$	1	$\underline{\bar1}^*$	$\underline{2}_{10}/m^*_{2\bar1}$	M	M
		$(hki0)$	1	$\underline{\bar1}^*$	$\underline{2}'_z/m'^*_z$	M	M
	27.20	$(hkil)$	1	$\underline{\bar1}^*$	$\underline{\bar1}^*$	M	M

F_1	J_{1j}	(hkl)	$\hat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{6}'2m'$	$6'^*/m'm^*m'$	(0001)	$3_z m'_{01}$	$\underline{\bar{3}}^*_z 1 \underline{2}'^*_{12}/m'_{01}$	$6'^*_z/\underline{m}'_z \underline{2}_{10}/m^*_{2\bar{1}} \underline{2}'^*_{12}/m'_{01}$	M	
		$(2\bar{1}\bar{1}0)$	$m'_z 2_{10} m'_{01}$	$2_{10}/\underline{m}^*_{2\bar{1}} \underline{2}'^*_{12}/m'_{01} \underline{2}'^*_z/m'_z$	$2_{10}/\underline{m}^*_{2\bar{1}} \underline{2}'^*_{12}/m'_{01} \underline{2}'^*_z/m'_z$	M	M
		$(01\bar{1}0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_{10}/m^*_{2\bar{1}} 2'^*_{12}/\underline{m}'_{01} \underline{2}'^*_z/m'_z$	M	M
		$(2h\bar{h}\bar{h}l)$	m'_{01}	$\underline{2}'^*_{12}/m'_{01}$	$\underline{2}'^*_{12}/m'_{01}$	M	M
		$(0h\bar{h}l)$	1	$\underline{\bar{1}}^*$	$\underline{2}_{10}/m^*_{2\bar{1}}$	M	M
		$(hki0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}'^*_z/m'_z$	M	M
	27.23	$(hkil)$	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
23	$m^*\bar{3}^*$	(001)	2_z	$2_z/\underline{m}^*_z$	$\underline{2}_x/m^*_x \underline{2}_y/m^*_y 2_z/\underline{m}^*_z$	M	
		(110)	1	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$	M	M
		$(kl0)$	1	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$	M	M
		(hhl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
		(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
	29.1	(111)	3_p	$\underline{\bar{3}}^*_p$	$\underline{\bar{3}}^*_p$	M	M
$m'\bar{3}'$	$m'\bar{3}'1'^*$	(001)	$m_x m_y 2_z$	$\underline{2}^*_x/m_x \underline{2}^*_y/m_y 2_z/\underline{m}^*_z$	$\underline{2}'_x/m_x \underline{2}'_y/m_y 2_z/\underline{m}'_z 1'^*$		
		(110)	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z 1'^*$	M	
		$(kl0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z 1'^*$	M	
		(hhl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}' 1'^*$	M	
		(hkl)	1	$\underline{\bar{1}}'^*$	$\underline{\bar{1}}' 1^*$	M	
	29.4	(111)	3_p	$\underline{\bar{3}}^*_p$	$\underline{\bar{3}}'_p 1'^*$	M	

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{4}3m$	$m^*\bar{3}^*m$	(001)	$2_z m_{xy} m_{x\bar{y}}$	$\underline{2}^*_{xy}/m_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}2_z/\underline{m}^*_z$	$4^*_z/\underline{m}^*_z\underline{2}_x/m^*_x\underline{2}^*_{xy}/m_{xy}$		
		(110)	$m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$2^*_{xy}/\underline{m}_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m^*_z$	M	
	$+)$	$(kl0)$	1	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$	M	M
		(hhl)	$m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	M	M
		(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
	32.2	(111)	$3_p m_{x\bar{y}}$	$\underline{\bar{3}}^*_p\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$\underline{\bar{3}}^*_p\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$		
432	$m^*\bar{3}^*m^*$	(001)	4_z	$4_z/\underline{m}^*_z$	$4_z/\underline{m}^*_z\underline{2}_x/m^*_x\underline{2}_{xy}/m^*_{xy}$	M	
		(110)	2_{xy}	$2_{xy}/\underline{m}^*_{xy}$	$2_{xy}/\underline{m}^*_{xy}\underline{2}_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m^*_z$	M	
		$(kl0)$	1	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$	M	M
		(hhl)	1	$\underline{\bar{1}}^*$	$\underline{2}_{x\bar{y}}/m^*_{x\bar{y}}$	M	M
		(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
	32.3	(111)	3_p	$\underline{\bar{3}}^*_p$	$\underline{\bar{3}}^*_p\underline{2}_{x\bar{y}}/m^*_{x\bar{y}}$	M	
$m'\bar{3}'m$	$m'\bar{3}'m1'^*$	(001)	$4_z m'_x m_{xy}$	$4_z/\underline{m}^*_z\underline{2}^*_x/m'_x\underline{2}'^*_{xy}/m_{xy}$	$4_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}_{xy}/m_{xy}1'^*$		
		(110)	$2'_{xy} m_{x\bar{y}} m'_z$	$2'_{xy}/\underline{m}'^*_{xy}\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}\underline{2}'^*_z/m'_z$	$2'_{xy}/\underline{m}_{xy}\underline{2}'_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m'_z1'^*$	M	
	$+)$	$(kl0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m'_z1'^*$	M	
		(hhl)	$m_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$\underline{2}'_{x\bar{y}}/m_{x\bar{y}}1'^*$	M	
		(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}'1'^*$	M	
	32.9	(111)	$3_p m_{x\bar{y}}$	$\underline{\bar{3}}'_p\underline{2}^*_{x\bar{y}}/m_{x\bar{y}}$	$\underline{\bar{3}}'_p\underline{2}'_{x\bar{y}}/m_{x\bar{y}}1'^*$		

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$m'\bar{3}'m'$	$m'\bar{3}'m'1'^*$	(001)	$4_z m'_x m'_{xy}$	$4_z/\underline{m}^*_z\underline{2}'^*_x/m'_x\underline{2}'^*_{xy}/m'_{xy}$	$4_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}_{xy}/m'_{xy}1'^*$	M	
		(110)	$2_{xy}m'_{x\bar{y}}m'_z$	$2_{xy}/\underline{m}^*_{xy}\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}\underline{2}'^*_z/m'_z$	$2_{xy}/\underline{m}'_{xy}\underline{2}_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m'_z1'^*$	M	
		$(kl0)$	m'_z	$\underline{2}'^*_z/m'_z$	$\underline{2}_z/m_z1'^*$	M	
		(hhl)	$m'_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}$	$\underline{2}_{x\bar{y}}/m_{x\bar{y}}1'^*$	M	
		(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}'1'^*$	M	
	32.10	(111)	$3_p m'_{x\bar{y}}$	$\underline{\bar{3}}^*_p\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}$	$\underline{\bar{3}}_p\underline{2}_{x\bar{y}}/m'_{x\bar{y}}1'^*$	M	
$4'32'$	$m^*\bar{3}^*m'^*$	(001)	$4'_z$	$4'_z/\underline{m}^*_z$	$4'_z/\underline{m}^*_z\underline{2}'_x/m'^*_x\underline{2}'_{xy}/m'^*_{xy}$		
		(110)	$2'_{xy}$	$2'_{xy}/\underline{m}'^*_{xy}$	$2'_{xy}/\underline{m}'^*_{xy}\underline{2}'_{x\bar{y}}/m'^*_{x\bar{y}}\underline{2}_z/m^*_z$	M	M
	+)	$(kl0)$	1	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$	M	M
		(hhl)	1	$\underline{\bar{1}}^*$	$\underline{2}'_{x\bar{y}}/m'^*_{x\bar{y}}$	M	M
		(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
	32.12	(111)	3_p	$\underline{\bar{3}}^*_p$	$\underline{\bar{3}}^*_p\underline{2}'_{x\bar{y}}/m'^*_{x\bar{y}}$	M	M
$\bar{4}'3m'$	$m^*\bar{3}^*m'$	(001)	$2_z m'_{xy} m'_{x\bar{y}}$	$\underline{2}'^*_{xy}/m'_{xy}\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}2_z/\underline{m}^*_z$	$4'^*_z/\underline{m}^*_z\underline{2}_x/m^*_x\underline{2}'^*_{xy}/m'_{xy}$	M	
		(110)	$m'_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}$	$2'^*_{xy}/\underline{m}'_{xy}\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}\underline{2}_z/m^*_z$	M	M
		$(kl0)$	1	$\underline{\bar{1}}^*$	$\underline{2}_z/m^*_z$	M	M
		(hhl)	$m'_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}$	M	M
		(hkl)	1	$\underline{\bar{1}}^*$	$\underline{\bar{1}}^*$	M	M
	32.13	(111)	$3_p m'_{x\bar{y}}$	$\underline{\bar{3}}^*_p\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}$	$\underline{\bar{3}}^*_p\underline{2}'^*_{x\bar{y}}/m'_{x\bar{y}}$	M	M

Appendix I

Symmetry of pyroelectric and pyromagnetic domain walls in non-pyromagnetic and non-pyroelectric domain pairs

The following table presents sectional layer groups $\overline{J}_{12}$ and the symmetry group of the domain walls T_{12} for all crystallographic non-equivalent planes (hkl) for 70 cases of the non-pyromagnetic and non-pyroelectric domain pairs with pyromagnetic and pyroelectric walls (type $\mathcal{MP}$).

- F_1 .... symmetry group of the domain state S_1
- J_{12} .... symmetry group of the unordered domain pair $\{S_1, S_2\}$
- $\widehat{F}_1$ .... one-sided sectional layer group containing all trivial symmetry operations of the domain wall
- T_{12} .... symmetry group of the wall
- $\overline{J}_{12}$ .... the *complete* sectional layer group of J_{12}
- M .... magnetization $\mathbf{M} \neq \mathbf{0}$
- P .... polarization $\mathbf{P} \neq \mathbf{0}$

- T .... type of the group T_{12}
- $\overline{J}$ .... type of the group $\overline{J}_{12}$

The digraph

$*$) is used for the ferroelastic domain pairs,

$+$) is used for the non-magnetoelectric domain pairs,

$\star$) is used for the magnetoelectric domain pairs,

the others are non-ferroelastic magnetoelectric domain pairs.

Numbers in J box are taken from the numbering of the 380 transposable magnetic twin laws [21].

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{1}'$	$2'^*/m^*$	(001)	1	$\underline{m}_z^*$	$2_z'^*/\underline{m}_z^*$	PM	
	$*)$	$(hk0)$	1	$\underline{2}_z'^*$	$\underline{2}_z'^*/m_z^*$	PM	
	5.13	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
$\bar{1}'$	$2^*/m'^*$	(001)	1	$\underline{m}_z'^*$	$2_z^*/\underline{m}_z'^*$	PM	
	$*)$	$(hk0)$	1	$\underline{2}_z^*$	$\underline{2}_z^*/m_z'^*$	PM	
	5.15	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
222	$2221'^*$	(001)	2_z	$\underline{2}_x'^*\underline{2}_y'^*2_z$	$\underline{2}_x\underline{2}_y2_z1'^*$	M	
		(010)	2_y	$\underline{2}_x'^*2_y\underline{2}_z'^*$	$\underline{2}_x2_y\underline{2}_z1'^*$	M	
		(100)	2_x	$2_x\underline{2}_y'^*\underline{2}_z'^*$	$2_x\underline{2}_y\underline{2}_z1'^*$	M	
		$(hk0)$	1	$\underline{2}_z'^*$	$\underline{2}_z1'^*$	PM	P
		$(h0l)$	1	$\underline{2}_y'^*$	$\underline{2}_y1'^*$	PM	P
		$(0kl)$	1	$\underline{2}_x'^*$	$\underline{2}_x1'^*$	PM	P
	6.2	(hkl)	1	1	$1'^*$	PM	P

F_1	J_{1j}	(hkl)	$\hat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$2'/m$	$m'^{*}m^{*}m$	(001)	$2'_z$	$\underline{2}^*_x\underline{2}'^*_y2'_z$	$\underline{2}^*_x/m'^*_x\underline{2}'^*_y/m^*_y2'_z/\underline{m}_z$	M	
		(010)	m_z	$\underline{2}^*_x\underline{m}^*_ym_z$	$\underline{2}^*_x/m'^*_x2'^*_y/\underline{m}^*_y\underline{2}'_z/m_z$	P	
	*)	(100)	m_z	$\underline{m}'^*_x\underline{2}'^*_ym_z$	$2^*_x/\underline{m}'^*_x\underline{2}'^*_y/m^*_y\underline{2}'_z/m_z$	PM	
		$(hk0)$	m_z	m_z	$\underline{2}'_z/m_z$	PM	
		$(h0l)$	1	$\underline{2}'^*_y$	$\underline{2}'^*_y/m^*_y$	PM	
		$(0kl)$	1	$\underline{2}^*_x$	$\underline{2}^*_x/m'^*_x$	PM	
	8.13	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
$2/m'$	$m'm^{*}m^{*}$	(001)	m'_x	$m'_x\underline{2}'^*_y\underline{m}^*_z$	$\underline{2}_x/m'_x\underline{2}'^*_y/m^*_y2'^*_z/\underline{m}^*_z$	PM	
		(010)	m'_x	$m'_x\underline{m}^*_y\underline{2}'^*_z$	$\underline{2}_x/m'_x2'^*_y/\underline{m}^*_y\underline{2}'^*_z/m^*_z$	PM	
	*)	(100)	2_x	$2_x\underline{2}'^*_y\underline{2}'^*_z$	$2_x/\underline{m}'_x\underline{2}'^*_y/m^*_y\underline{2}'^*_z/m^*_z$	M	
		$(hk0)$	1	$\underline{2}'^*_z$	$\underline{2}'^*_z/m^*_z$	PM	
		$(h0l)$	1	$\underline{2}'^*_y$	$\underline{2}'^*_y/m^*_y$	PM	
		$(0kl)$	m'_x	m'_x	$\underline{2}_x/m'_x$	PM	
	8.14	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
$2/m'$	$m'^{*}m'^{*}m$	(001)	2_z	$\underline{2}^*_x\underline{2}^*_y2_z$	$\underline{2}^*_x/m'^*_x\underline{2}^*_y/m'^*_y2_z/\underline{m}'_z$		
		(010)	m'_z	$\underline{2}^*_x\underline{m}'^*_ym'_z$	$\underline{2}^*_x/m'^*_x2^*_y/\underline{m}'^*_y\underline{2}_z/m'_z$	PM	
	*)	(100)	m'_z	$\underline{m}'^*_x\underline{2}^*_ym'_z$	$2^*_x/\underline{m}'^*_x\underline{2}^*_y/m'^*_y\underline{2}_z/m'_z$	PM	
		$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		$(h0l)$	1	$\underline{2}^*_y$	$\underline{2}^*_y/m'^*_y$	PM	
		$(0kl)$	1	$\underline{2}^*_x$	$\underline{2}^*_x/m'^*_x$	PM	
	8.23	(hkl)	1	1	$\underline{\bar{1}}'$	PM	

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{4}'$	$\bar{4}'1'^*$	(001)	2_z	$\underline{\bar{4}}^*_z$	$\underline{\bar{4}}'_z1'^*$	M	
		$(hk0)$	1	$\underline{2}'^*_z$	$\underline{2}_z1'^*$	PM	P
	10.4	(hkl)	1	1	$1'^*$	PM	P
$2/m'$	$4^*/m'$	(001)	2_z	$\underline{\bar{4}}'^*_z$	$4^*_z/\underline{m}'_z$		
	$*)$	$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
	11.16	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
$2/m'$	$4'^*/m'$	(001)	2_z	$\underline{\bar{4}}^*_z$	$4'^*_z/\underline{m}'_z$	M	
	$*)$	$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
	11.19	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
222	4^*22^*	(001)	2_z	$\underline{2}^*_{xy}\underline{2}^*_{x\bar{y}}2_z$	$4^*_z\underline{2}_x\underline{2}^*_{xy}$		
		(100)	2_x	2_x	$2_x\underline{2}_y\underline{2}_z$	PM	
	$*)$	(110)	1	$\underline{2}^*_{x\bar{y}}$	$2^*_{xy}\underline{2}^*_{x\bar{y}}\underline{2}_z$	PM	
		$(hk0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	PM	PM
		$(h0l)$	1	1	$\underline{2}_y$	PM	PM
	12.1	(hkl)	1	1	1	PM	PM
422	$4221'^*$	(001)	4_z	$4_z\underline{2}'^*_x\underline{2}'^*_{xy}$	$4_z\underline{2}_x\underline{2}_{xy}1'^*$	M	
		(100)	2_x	$2_x\underline{2}'^*_y\underline{2}'^*_z$	$2_x\underline{2}_y\underline{2}_z1'^*$	M	
		(110)	2_{xy}	$2_{xy}\underline{2}'^*_{x\bar{y}}\underline{2}'^*_z$	$2_{xy}\underline{2}_{x\bar{y}}\underline{2}_z1'^*$	M	
		$(hk0)$	1	$\underline{2}'^*_z$	$\underline{2}_z1'^*$	PM	P
		(hhl)	1	$\underline{2}'^*_{x\bar{y}}$	$\underline{2}_{x\bar{y}}1'^*$	PM	P
		$(h0l)$	1	$\underline{2}'^*_y$	$\underline{2}_y1'^*$	PM	P
	12.3	(hkl)	1	1	$1'^*$	PM	P

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$4'22'$	$4'22'1'^*$	(001)	$4'_z$	$4'_z\underline{2}'^*_x\underline{2}^*_{xy}$	$4'_z\underline{2}_x\underline{2}'_{xy}1'^*$		
		(100)	2_x	$2_x\underline{2}'^*_y\underline{2}'^*_z$	$2_x\underline{2}_y\underline{2}_z1'^*$	M	
		(110)	2_{xy}	$2'_{xy}\underline{2}^*_{x\bar{y}}\underline{2}'^*_z$	$2'_{xy}\underline{2}'_{x\bar{y}}\underline{2}_z1'^*$	M	
		$(hk0)$	1	$\underline{2}'^*_z$	$\underline{2}_z1'^*$	PM	P
		(hhl)	1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}'_{x\bar{y}}1'^*$	PM	P
		$(h0l)$	1	$\underline{2}'^*_y$	$\underline{2}_y1'^*$	PM	P
	12.6	(hkl)	1	1	$1'^*$	PM	P
222	$4'^*22'^*$	(001)	2_z	$\underline{2}'^*_{xy}\underline{2}'^*_{x\bar{y}}2_z$	$4'^*_z\underline{2}_x\underline{2}'^*_{xy}$	M	
		(100)	2_x	2_x	$2_x\underline{2}_y\underline{2}_z$	PM	
	*)	(110)	1	$\underline{2}'^*_{x\bar{y}}$	$2'^*_{xy}\underline{2}'^*_{x\bar{y}}\underline{2}_z$	PM	M
		$(hk0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	$\underline{2}'^*_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}$	PM	PM
		$(h0l)$	1	1	$\underline{2}_y$	PM	PM
	12.8	(hkl)	1	1	1	PM	PM
222	$\bar{4}^*2m^*$	(001)	2_z	$\underline{\bar{4}}^*_z$	$\underline{\bar{4}}^*_z\underline{2}_xm^*_{xy}$	M	
		(100)	2_x	2_x	$2_x\underline{2}_y\underline{2}_z$	PM	
	*)	(110)	1	$\underline{m}^*_{xy}$	$\underline{m}^*_{xy}m^*_{x\bar{y}}\underline{2}_z$	PM	P
		$(hk0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	1	$m^*_{x\bar{y}}$	PM	PM
		$(h0l)$	1	1	$\underline{2}_y$	PM	PM
	14.1	(hkl)	1	1	1	PM	PM

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{4}2m$	$\bar{4}2m1'^*$	(001)	$2_z m_{xy} m_{x\bar{y}}$	$\underline{\bar{4}}'^*_z \underline{2}'^*_x m_{xy}$	$\underline{\bar{4}}_z \underline{2}_x m_{xy} 1'^*$		
		(100)	2_x	$2_x \underline{2}'^*_y \underline{2}'^*_z$	$2_x \underline{2}_y \underline{2}_z 1'^*$	M	
		(110)	$m_{x\bar{y}}$	$\underline{m}'^*_{xy} m_{x\bar{y}} \underline{2}'^*_z$	$\underline{m}_{xy} m_{x\bar{y}} \underline{2}_z 1'^*$	PM	P
		$(hk0)$	1	$\underline{2}'^*_z$	$\underline{2}_z 1'^*$	PM	P
		(hhl)	$m_{x\bar{y}}$	$m_{x\bar{y}}$	$m_{x\bar{y}} 1'^*$	PM	P
		$(h0l)$	1	$\underline{2}'^*_y$	$\underline{2}_y 1'^*$	PM	P
	14.4	(hkl)	1	1	$1'^*$	PM	P
$\bar{4}'2m'$	$\bar{4}'2m'1'^*$	(001)	$2_z m'_{xy} m'_{x\bar{y}}$	$\underline{\bar{4}}^*_z \underline{2}'^*_x m'_{xy}$	$\underline{\bar{4}}'_z \underline{2}_x m'_{xy} 1'^*$	M	
		(100)	2_x	$2_x \underline{2}'^*_y \underline{2}'^*_z$	$2_x \underline{2}_y \underline{2}_z 1'^*$	M	
		(110)	$m'_{x\bar{y}}$	$\underline{m}^*_{xy} m'_{x\bar{y}} \underline{2}'^*_z$	$\underline{m}'_{xy} m'_{x\bar{y}} \underline{2}_z 1'^*$	PM	P
		$(hk0)$	1	$\underline{2}'^*_z$	$\underline{2}_z 1'^*$	PM	P
		(hhl)	$m'_{x\bar{y}}$	$m'_{x\bar{y}}$	$m'_{x\bar{y}} 1'^*$	PM	P
		$(h0l)$	1	$\underline{2}'^*_y$	$\underline{2}_y 1'^*$	PM	P
	14.8	(hkl)	1	1	$1'^*$	PM	P
$\bar{4}'2'm$	$\bar{4}'2'm1'^*$	(001)	$2_z m_{xy} m_{x\bar{y}}$	$\underline{\bar{4}}^*_z \underline{2}^*_x m_{xy}$	$\underline{\bar{4}}'_z \underline{2}'_x m_{xy} 1'^*$		
		(100)	$2'_x$	$2'_x \underline{2}^*_y \underline{2}'^*_z$	$2'_x \underline{2}'_y \underline{2}_z 1'^*$	M	
		(110)	$m_{x\bar{y}}$	$\underline{m}'^*_{xy} m_{x\bar{y}} \underline{2}'^*_z$	$\underline{m}_{xy} m_{x\bar{y}} \underline{2}_z 1'^*$	PM	P
		$(hk0)$	1	$\underline{2}'^*_z$	$\underline{2}_z 1'^*$	PM	P
		(hhl)	$m_{x\bar{y}}$	$m_{x\bar{y}}$	$m_{x\bar{y}} 1'^*$	PM	P
		$(h0l)$	1	$\underline{2}^*_y$	$\underline{2}'_y 1'^*$	PM	P
	14.9	(hkl)	1	1	$1'^*$	PM	P

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
222	$\bar{4}'^{*}2m'^{*}$	(001)	2_z	$\underline{\bar{4}}'^{*}_z$	$\underline{\bar{4}}'^{*}_z\underline{2}_x m'^{*}_{xy}$		
		(100)	2_x	2_x	$2_x\underline{2}_y\underline{2}_z$	PM	
	*)	(110)	1	$\underline{m}'^{*}_{xy}$	$\underline{m}'^{*}_{xy}m'^{*}_{x\bar{y}}\underline{2}_z$	PM	PM
		$(hk0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	1	$m'^{*}_{x\bar{y}}$	PM	PM
		$(h0l)$	1	1	$\underline{2}_y$	PM	PM
	14.11	(hkl)	1	1	1	PM	PM
$\bar{4}'$	$\bar{4}'2^{*}m'^{*}$	(001)	2_z	$\underline{2}^{*}_x\underline{2}^{*}_y 2_z$	$\underline{\bar{4}}'_z\underline{2}^{*}_x m'^{*}_{xy}$		
		(100)	1	$\underline{2}^{*}_y$	$2^{*}_x\underline{2}^{*}_y\underline{2}_z$	PM	
		(110)	1	$\underline{m}'^{*}_{xy}$	$\underline{m}'^{*}_{xy}m'^{*}_{x\bar{y}}\underline{2}_z$	PM	PM
		$(hk0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	1	$m'^{*}_{x\bar{y}}$	PM	PM
		$(h0l)$	1	$\underline{2}^{*}_y$	$\underline{2}^{*}_y$	PM	PM
	14.12	(hkl)	1	1	1	PM	PM
$\bar{4}'$	$\bar{4}'2'^{*}m^{*}$	(001)	2_z	$\underline{2}'^{*}_x\underline{2}'^{*}_y 2_z$	$\underline{\bar{4}}'_z\underline{2}'^{*}_x m^{*}_{xy}$	M	
		(100)	1	$\underline{2}'^{*}_y$	$2'^{*}_x\underline{2}'^{*}_y\underline{2}_z$	PM	M
		(110)	1	$\underline{m}^{*}_{xy}$	$\underline{m}^{*}_{xy}m^{*}_{x\bar{y}}\underline{2}_z$	PM	P
		$(hk0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	1	$m^{*}_{x\bar{y}}$	PM	PM
		$(h0l)$	1	$\underline{2}'^{*}_y$	$\underline{2}'^{*}_y$	PM	PM
	14.15	(hkl)	1	1	1	PM	PM

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$m'm'm$	$4^*/mm'm'^*$	(001)	$m'_x m'_y 2_z$	$\underline{\bar{4}}^*_z m'_x \underline{2}'^*_{xy}$	$4^*_z/\underline{m}_z\underline{2}'_x/m'_x\underline{2}'^*_{xy}/m'^*_{xy}$		
		(100)	$2'_x m'_y m_z$	$2'_x m'_y m_z$	$2'_x/\underline{m}'_x\underline{2}'_y/m'_y\underline{2}_z/m_z$	PM	M
	$*)$	(110)	m_z	$\underline{2}'^*_{x\bar{y}}\underline{m}'^*_{xy}m_z$	$2'^*_{xy}/\underline{m}'^*_{xy}\underline{2}'^*_{x\bar{y}}/m'^*_{x\bar{y}}\underline{2}_z/m_z$	PM	M
		$(hk0)$	m_z	m_z	$\underline{2}_z/m_z$	PM	M
		(hhl)	1	1	$\underline{\bar{1}}$	PM	M
		$(h0l)$	m'_y	m'_y	$\underline{2}'_y/m'_y$	PM	M
	15.18	(hkl)	1	1	$\underline{\bar{1}}$	PM	M
mmm	$4'^*/mmm'^*$	(001)	$m_x m_y 2_z$	$\underline{\bar{4}}'^*_z m_x \underline{2}'^*_{xy}$	$4'^*_z/\underline{m}_z\underline{2}_x/m_x\underline{2}'^*_{xy}/m'^*_{xy}$		
		(100)	$2_x m_y m_z$	$2_x m_y m_z$	$2_x/\underline{m}_x\underline{2}_y/m_y\underline{2}_z/m_z$	P	
	$*)$	(110)	m_z	$\underline{2}'^*_{x\bar{y}}\underline{m}'^*_{xy}m_z$	$2'^*_{xy}/\underline{m}'^*_{xy}\underline{2}'^*_{x\bar{y}}/m'^*_{x\bar{y}}\underline{2}_z/m_z$	PM	M
		$(hk0)$	m_z	m_z	$\underline{2}_z/m_z$	PM	M
		(hhl)	1	1	$\underline{\bar{1}}$	PM	M
		$(h0l)$	m_y	m_y	$\underline{2}_y/m_y$	PM	M
	15.22	(hkl)	1	1	$\underline{\bar{1}}$	PM	M
$4'/m$	$4'/mm^*m'^*$	(001)	$4'_z$	$4'_z\underline{2}^*_x\underline{2}'^*_{xy}$	$4'_z/\underline{m}_z\underline{2}^*_x/m^*_x\underline{2}'^*_{xy}/m'^*_{xy}$		
		(100)	m_z	$\underline{m}^*_x\underline{2}^*_y m_z$	$2^*_x/\underline{m}^*_x\underline{2}^*_y/m^*_y\underline{2}_z/m_z$	P	
	$+)$	(110)	m_z	$\underline{m}'^*_{xy}\underline{2}'^*_{x\bar{y}}m_z$	$2'^*_{xy}/\underline{m}'^*_{xy}\underline{2}'^*_{x\bar{y}}/m'^*_{x\bar{y}}\underline{2}_z/m_z$	PM	M
		$(hk0)$	m_z	m_z	$\underline{2}_z/m_z$	PM	M
		(hhl)	1	$\underline{2}'^*_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}/m'^*_{x\bar{y}}$	PM	M
		$(h0l)$	1	$\underline{2}^*_y$	$\underline{2}^*_y/m^*_y$	PM	M
	15.23	(hkl)	1	1	$\underline{\bar{1}}$	PM	M

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$4/m'$	$4/m'm'^{*}m'^{*}$	(001)	4_z	$4_z\underline{2}^*_x\underline{2}^*_{xy}$	$4_z/\underline{m}'_z\underline{2}^*_x/m'^*_x\underline{2}^*_{xy}/m'^*_{xy}$		
		(100)	m'_z	$\underline{m}'^*_x\underline{2}^*_y m'_z$	$2^*_x/\underline{m}'^*_x\underline{2}^*_y/m'^*_y\underline{2}_z/m'_z$	PM	
		(110)	m'_z	$\underline{m}'^*_{xy}\underline{2}^*_{x\bar{y}}m'_z$	$2^*_{xy}/\underline{m}'^*_{xy}\underline{2}^*_{x\bar{y}}/m'^*_{x\bar{y}}\underline{2}_z/m'_z$	PM	
		$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		(hhl)	1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m'^*_{x\bar{y}}$	PM	
		$(h0l)$	1	$\underline{2}^*_y$	$\underline{2}^*_y/m'^*_y$	PM	
	15.30	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
$m'm'm'$	$4^*/m'm'm'$	(001)	$m'_xm'_y2_z$	$\underline{\bar{4}}^*_z m'_x\underline{2}^*_{xy}$	$4^*_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}^*_{xy}/m'^*_{xy}$		
		(100)	$2_xm'_ym'_z$	$2_xm'_ym'_z$	$2_x/\underline{m}'_x\underline{2}_y/m'_y\underline{2}_z/m'_z$	PM	
	*)	(110)	m'_z	$\underline{2}^*_{x\bar{y}}\underline{m}'^*_{xy}m'_z$	$2^*_{xy}/\underline{m}'^*_{xy}\underline{2}^*_{x\bar{y}}/m'^*_{x\bar{y}}\underline{2}_z/m'_z$	PM	
		$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		(hhl)	1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}/m'^*_{x\bar{y}}$	PM	
		$(h0l)$	m'_y	m'_y	$\underline{2}_y/m'_y$	PM	
	15.31	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
$4/m'$	$4/m'm^*m^*$	(001)	4_z	$4_z\underline{2}'^*_x\underline{2}'^*_{xy}$	$4_z/\underline{m}'_z\underline{2}'^*_x/m^*_x\underline{2}'^*_{xy}/m^*_{xy}$	M	
		(100)	m'_z	$\underline{m}^*_x\underline{2}'^*_y m'_z$	$2'^*_x/\underline{m}^*_x\underline{2}'^*_y/m^*_y\underline{2}_z/m'_z$	PM	
		(110)	m'_z	$\underline{m}^*_{xy}\underline{2}'^*_{x\bar{y}}m'_z$	$2'^*_{xy}/\underline{m}^*_{xy}\underline{2}'^*_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m'_z$	PM	
		$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		(hhl)	1	$\underline{2}'^*_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}/m^*_{x\bar{y}}$	PM	
		$(h0l)$	1	$\underline{2}'^*_y$	$\underline{2}'^*_y/m^*_y$	PM	
	15.35	(hkl)	1	1	$\underline{\bar{1}}'$	PM	

F_1	J_{1j}	(hkl)	$\hat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
mmm'	$4^*/m'mm^*$	(001)	$m_xm_y2_z$	$\underline{\bar{4}}'^*_z m_x \underline{2}'^*_{xy}$	$4^*_z/\underline{m}'_z\underline{2}'_x/m_x\underline{2}'^*_{xy}/m^*_{xy}$		
		(100)	$2'_xm_ym'_z$	$2'_xm_ym'_z$	$2'_x/\underline{m}_x\underline{2}'_y/m_y\underline{2}_z/m'_z$	PM	
	$*)$	(110)	m'_z	$\underline{2}'^*_{x\bar{y}}\underline{m}^*_{xy}m'_z$	$2'^*_{xy}/\underline{m}^*_{xy}\underline{2}'^*_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m'_z$	PM	
		$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		(hhl)	1	$\underline{2}'^*_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}/m^*_{x\bar{y}}$	PM	
		$(h0l)$	m_y	m_y	$\underline{2}'_y/m_y$	PM	
	15.36	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
$4'/m'$	$4'/m'm'^*m^*$	(001)	$4'_z$	$4'_z\underline{2}^*_x\underline{2}'^*_{xy}$	$4'_z/\underline{m}'_z\underline{2}^*_x/m'^*_x\underline{2}'^*_{xy}/m^*_{xy}$		
		(100)	m'_z	$\underline{m}'^*_x\underline{2}^*_ym'_z$	$2^*_x/\underline{m}'^*_x\underline{2}^*_y/m'^*_y\underline{2}_z/m'_z$	PM	
		(110)	m'_z	$\underline{m}^*_{xy}\underline{2}'^*_{x\bar{y}}m'_z$	$2'^*_{xy}/\underline{m}^*_{xy}\underline{2}'^*_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m'_z$	PM	
		$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		$(h0l)$	1	$\underline{2}^*_y$	$\underline{2}^*_y/m'^*_y$	PM	
		(hhl)	1	$\underline{2}'^*_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}/m^*_{x\bar{y}}$	PM	
	15.40	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
$m'm'm'$	$4'^*/m'm'm^*$	(001)	$m'_xm'_y2_z$	$\underline{\bar{4}}^*_z m'_x\underline{2}'^*_{xy}$	$4'^*_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}'^*_{xy}/m^*_{xy}$		
		(100)	$2_xm'_ym'_z$	$2_xm'_ym'_z$	$2_x/\underline{m}'_x\underline{2}_y/m'_y\underline{2}_z/m'_z$	PM	
	$*)$	(110)	m'_z	$\underline{2}'^*_{x\bar{y}}\underline{m}^*_{xy}m'_z$	$2'^*_{xy}/\underline{m}^*_{xy}\underline{2}'^*_{x\bar{y}}/m^*_{x\bar{y}}\underline{2}_z/m'_z$	PM	
		$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		(hhl)	1	$\underline{2}'^*_{x\bar{y}}$	$\underline{2}'^*_{x\bar{y}}/m^*_{x\bar{y}}$	PM	
		$(h0l)$	m'_y	m'_y	$\underline{2}_y/m'_y$	PM	
	15.41	(hkl)	1	1	$\underline{\bar{1}}'$	PM	

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
mmm'	$4'^*/m'm'^*m$	(001)	$m_{xy}m_{x\bar{y}}2_z$	$\underline{\bar{4}}^*_z m_{xy}\underline{2}^*_x$	$4'^*_z/\underline{m}'_z\underline{2}^*_x/m'^*_x\underline{2}'_{xy}/m_{xy}$		
		(100)	m'_z	$\underline{2}^*_y\underline{m}'^*_x m'_z$	$2^*_x/\underline{m}'^*_x\underline{2}^*_y/m'^*_y\underline{2}_z/m'_z$	PM	
	$*)$	(110)	$2'_{xy}m_{x\bar{y}}m'_z$	$2'_{xy}m_{x\bar{y}}m'_z$	$2'_{xy}/\underline{m}_{xy}\underline{2}'_{x\bar{y}}/m_{x\bar{y}}\underline{2}_z/m'_z$	PM	
		$(hk0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		(hhl)	$m_{x\bar{y}}$	$m_{x\bar{y}}$	$\underline{2}'_{x\bar{y}}/m_{x\bar{y}}$	PM	
		$(h0l)$	1	$\underline{2}^*_y$	$\underline{2}^*_y/m'^*_y$	PM	
	15.42	(hkl)	1	1	$\underline{\bar{1}}'$	PM	
32	$321'^*$	(0001)	3_z	$3_z\underline{2}'^*_{10}$	$3_z\underline{2}_{10}1'^*$	M	
		$(2\bar{1}\bar{1}0)$	2_{10}	2_{10}	$2_{10}1'^*$	PM	P
		$(01\bar{1}0)$	1	$\underline{2}'^*_{10}$	$\underline{2}_{10}1'^*$	PM	P
		$(2h\bar{h}\bar{h}l)$	1	1	$1'^*$	PM	P
		$(0h\bar{h}l)$	1	$\underline{2}'^*_{10}$	$\underline{2}_{10}1'^*$	PM	P
		$(hki0)$	1	1	$1'^*$	PM	P
	18.2	$(hkil)$	1	1	$1'^*$	PM	P
$\bar{3}'$	$\bar{3}'m^*$	(0001)	3_z	$3_z\underline{2}'^*_{10}$	$\underline{\bar{3}}'_z\underline{2}'^*_{10}/m^*_{2\bar{1}}$	M	
		$(2\bar{1}\bar{1}0)$	1	$\underline{m}^*_{2\bar{1}}$	$2'^*_{10}/\underline{m}^*_{2\bar{1}}$	PM	
		$(01\bar{1}0)$	1	$\underline{2}'^*_{10}$	$\underline{2}'^*_{10}/m^*_{2\bar{1}}$	PM	
		$(2h\bar{h}\bar{h}l)$	1	1	$\underline{\bar{1}}'$	PM	
		$(0h\bar{h}\bar{l})$	1	$\underline{2}'^*_{10}$	$\underline{2}'^*_{10}/m^*_{2\bar{1}}$	PM	
		$(hki0)$	1	1	$\underline{\bar{1}}'$	PM	
	20.15	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	

F_1	J_{1j}	(hkl)	$\hat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{3}'$	$\bar{3}'m'^*$	(0001)	3_z	$3_z\underline{2}^*_{10}$	$\underline{\bar{3}}'_z\underline{2}^*_{10}/m'^*_{2\bar{1}}$		
		$(2\bar{1}\bar{1}0)$	1	$\underline{m}'^*_{2\bar{1}}$	$2^*_{10}/\underline{m}'^*_{2\bar{1}}$	PM	
		$(01\bar{1}0)$	1	$\underline{2}^*_{10}$	$\underline{2}^*_{10}/m'^*_{2\bar{1}}$	PM	
		$(2h\bar{h}\bar{h}l)$	1	1	$\underline{\bar{1}}'$	PM	
		$(0h\bar{h}\bar{l})$	1	$\underline{2}^*_{10}$	$\underline{2}^*_{10}/m'^*_{2\bar{1}}$	PM	
		$(hki0)$	1	1	$\underline{\bar{1}}'$	PM	
	20.18	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	
$\bar{6}'$	$\bar{6}'1'^*$	(0001)	3_z	$\underline{\bar{6}}^*_z$	$\underline{\bar{6}}'_z1'^*$	M	
		$(hki0)$	m'_z	m'_z	$m'_z1'^*$	PM	P
	22.4	$(hkil)$	1	1	$1'^*$	PM	P
$\bar{3}'$	$6^*/m'^*$	(0001)	3_z	$\underline{\bar{6}}'^*_z$	$6^*_z/\underline{m}'^*_z$		
	$\star)$	$(hki0)$	1	$\underline{2}^*_z$	$\underline{2}^*_z/m'^*_z$	PM	
	23.13	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	
$\bar{3}'$	$6'^*/m^*$	(0001)	3_z	$\underline{\bar{6}}'^*_z$	$6'^*_z/\underline{m}^*_z$	M	
		$(hki0)$	1	$\underline{2}'^*_z$	$\underline{2}'^*_z/m^*_z$	PM	
	23.16	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	
32	6^*22^*	(0001)	3_z	$3_z\underline{2}^*_{12}$	$6^*_z\underline{2}_{10}\underline{2}^*_{12}$		
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}\underline{2}^*_{12}\underline{2}^*_z$	$2_{10}\underline{2}^*_{12}\underline{2}^*_z$		
	$\star)$	$(01\bar{1}0)$	1	$\underline{2}^*_z$	$\underline{2}_{10}2^*_{12}\underline{2}^*_z$	PM	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	PM	PM
		$(0h\bar{h}l)$	1	1	$\underline{2}_{10}$	PM	PM
		$(hki0)$	1	$\underline{2}^*_z$	$\underline{2}^*_z$	PM	PM
	24.1	$(hkil)$	1	1	1	PM	PM

F_1	J_{1j}	(hkl)	$\hat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
622	$6221'^{*}$	(0001)	6_z	$6_z\underline{2}'^{*}_{10}\underline{2}'^{*}_{12}$	$6_z\underline{2}_{10}\underline{2}_{12}1'^{*}$	M	
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}\underline{2}'^{*}_{12}\underline{2}'^{*}_{z}$	$2_{10}\underline{2}_{12}\underline{2}_{z}1'^{*}$	M	
		$(01\bar{1}0)$	2_{12}	$\underline{2}'^{*}_{10}2_{12}\underline{2}'^{*}_{z}$	$\underline{2}_{10}2_{12}\underline{2}_{z}1'^{*}$	M	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}'^{*}_{12}$	$\underline{2}_{12}1'^{*}$	PM	P
		$(0h\bar{h}l)$	1	$\underline{2}'^{*}_{10}$	$\underline{2}_{10}1'^{*}$	PM	P
		$(hki0)$	1	$\underline{2}'^{*}_{z}$	$\underline{2}_{z}1'^{*}$	PM	P
	24.3	$(hkil)$	1	1	$1'^{*}$	PM	P
$6'22'$	$6'22'1'^{*}$	(0001)	$6'_z$	$6'_z\underline{2}'^{*}_{10}\underline{2}^{*}_{12}$	$6'_z\underline{2}_{10}\underline{2}'_{12}1'^{*}$		
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}\underline{2}^{*}_{12}\underline{2}^{*}_{z}$	$2_{10}\underline{2}'_{12}\underline{2}'_{z}1'^{*}$		
	+)	$(01\bar{1}0)$	$2'_{12}$	$\underline{2}'^{*}_{10}2'_{12}\underline{2}^{*}_{z}$	$\underline{2}_{10}2'_{12}\underline{2}'_{z}1'^{*}$	M	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^{*}_{12}$	$\underline{2}'_{12}1'^{*}$	PM	P
		$(0h\bar{h}l)$	1	$\underline{2}'^{*}_{10}$	$\underline{2}_{10}1'^{*}$	PM	P
		$(hki0)$	1	$\underline{2}'^{*}_{z}$	$\underline{2}_{z}1'^{*}$	PM	P
	24.6	$(hkil)$	1	1	$1'^{*}$	PM	P
32	$6'^{*}22'^{*}$	(0001)	3_z	$3_z\underline{2}'^{*}_{12}$	$6'^{*}_z\underline{2}_{10}\underline{2}'^{*}_{12}$	M	
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}\underline{2}'^{*}_{12}\underline{2}'^{*}_{z}$	$2_{10}\underline{2}'^{*}_{12}\underline{2}'^{*}_{z}$	M	M
		$(01\bar{1}0)$	1	$\underline{2}'^{*}_{z}$	$\underline{2}_{10}2'^{*}_{12}\underline{2}'^{*}_{z}$	PM	M
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}'^{*}_{12}$	$\underline{2}'^{*}_{12}$	PM	PM
		$(0h\bar{h}l)$	1	1	$\underline{2}_{10}$	PM	PM
		$(hki0)$	1	$\underline{2}'^{*}_{z}$	$\underline{2}'^{*}_{z}$	PM	PM
	24.8	$(hkil)$	1	1	1	PM	PM

F_1	J_{1j}	(hkl)	$\hat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
32	$\bar{6}^*m^*2$	(0001)	3_z	$\underline{\bar{6}}^*_z$	$\underline{\bar{6}}^*_z m^*_{2\bar{1}}\underline{2}_{12}$		
		$(2\bar{1}\bar{1}0)$	1	$\underline{m}^*_{2\bar{1}}$	$\underline{2}_{12}\underline{m}^*_{2\bar{1}}m^*_z$	PM	P
		$(01\bar{1}0)$	2_{12}	2_{12}	$2_{12}m^*_{2\bar{1}}m^*_z$	PM	P
		$(\bar{2}hhhl)$	1	1	$\underline{2}_{12}$	PM	PM
		$(0h\bar{h}l)$	1	1	$m^*_{2\bar{1}}$	PM	PM
		$(hki0)$	1	1	m^*_z	PM	PM
	26.1	$(hkil)$	1	1	1	PM	PM
$\bar{6}m2$	$\bar{6}m21'^*$	(0001)	$3_z m_{2\bar{1}}$	$\underline{\bar{6}}'^*_z m_{2\bar{1}}\underline{2}'^*_{12}$	$\underline{\bar{6}}_z m_{2\bar{1}}\underline{2}_{12}1'^*$		
		$(2\bar{1}\bar{1}0)$	m_z	$\underline{2}'^*_{12}\underline{m}'^*_{2\bar{1}}m_z$	$\underline{2}_{12}\underline{m}_{2\bar{1}}m_z1'^*$	PM	P
	+)	$(01\bar{1}0)$	$m_z m_{2\bar{1}}2_{12}$	$m_z m_{2\bar{1}}2_{12}$	$m_z m_{2\bar{1}}2_{12}1'^*$	P	P
		$(\bar{2}hhhl)$	1	$\underline{2}'^*_{12}$	$\underline{2}_{12}1'^*$	PM	P
		$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m_{2\bar{1}}$	$m_{2\bar{1}}1'^*$	PM	P
		$(hki0)$	m_z	m_z	$m_z1'^*$	PM	P
	26.4	$(hkil)$	1	1	$1'^*$	PM	P
$\bar{6}'m'2$	$\bar{6}'m'21'^*$	(0001)	$3_z m'_{2\bar{1}}$	$\underline{\bar{6}}^*_z m'_{2\bar{1}}\underline{2}'^*_{12}$	$\underline{\bar{6}}'_z m'_{2\bar{1}}\underline{2}_{12}1'^*$	M	
		$(2\bar{1}\bar{1}0)$	m'_z	$\underline{2}'^*_{12}\underline{m}^*_{2\bar{1}}m'_z$	$\underline{2}_{12}\underline{m}'_{2\bar{1}}m'_z1'^*$	PM	P
		$(01\bar{1}0)$	$m'_z m'_{2\bar{1}}2_{12}$	$m'_z m'_{2\bar{1}}2_{12}$	$m'_z m'_{2\bar{1}}2_{12}1'^*$	PM	P
		$(\bar{2}hhhl)$	1	$\underline{2}'^*_{12}$	$\underline{2}_{12}1'^*$	PM	P
		$(0h\bar{h}l)$	$m'_{2\bar{1}}$	$m'_{2\bar{1}}$	$m'_{2\bar{1}}1'^*$	PM	P
		$(hki0)$	m'_z	m'_z	$m'_z1'^*$	PM	P
	26.8	$(hkil)$	1	1	$1'^*$	PM	P

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{6}'m2'$	$\bar{6}'m2'1'^*$	(0001)	$3_z m_{2\bar{1}}$	$\underline{\bar{6}}^*_z m_{2\bar{1}}\underline{2}^*_{12}$	$\underline{\bar{6}}'_z m_{2\bar{1}}\underline{2}'_{12}1'^*$		
		$(2\bar{1}\bar{1}0)$	m'_z	$\underline{2}^*_{12}\underline{m}'^*_{2\bar{1}}m'_z$	$\underline{2}'_{12}\underline{m}_{2\bar{1}}m'_z1'^*$	PM	P
		$(01\bar{1}0)$	$m'_z m_{2\bar{1}}2'_{12}$	$m'_z m_{2\bar{1}}2'_{12}$	$m'_z m_{2\bar{1}}2'_{12}1'^*$	PM	P
		$(\overline{2h}hhl)$	1	$\underline{2}^*_{12}$	$\underline{2}'_{12}1'^*$	PM	P
		$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m_{2\bar{1}}$	$m_{2\bar{1}}1'^*$	PM	P
		$(hki0)$	m'_z	m'_z	$m'_z1'^*$	PM	P
	26.9	$(hkil)$	1	1	$1'^*$	PM	P
$\bar{6}'$	$\bar{6}'m^*2'^*$	(0001)	3_z	$3_z\underline{2}'^*_{12}$	$\underline{\bar{6}}'_z m^*_{2\bar{1}}\underline{2}'^*_{12}$	M	
		$(2\bar{1}\bar{1}0)$	m'_z	$\underline{2}'^*_{12}\underline{m}^*_{2\bar{1}}m'_z$	$\underline{2}'^*_{12}\underline{m}^*_{2\bar{1}}m'_z$	PM	PM
		$(01\bar{1}0)$	m'_z	m'_z	$2'^*_{12}m^*_{2\bar{1}}m'_z$	PM	PM
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}'^*_{12}$	$\underline{2}'^*_{12}$	PM	PM
		$(0\bar{h}hl)$	1	1	$m^*_{2\bar{1}}$	PM	PM
		$(hki0)$	m'_z	m'_z	m'_z	PM	PM
	26.13	$(hkil)$	1	1	1	PM	PM
32	$\bar{6}'^*m'^*2$	(0001)	3_z	$\underline{\bar{6}}'^*_z$	$\underline{\bar{6}}'^*_z m'^*_{2\bar{1}}\underline{2}_{12}$		
		$(2\bar{1}\bar{1}0)$	1	$\underline{m}'^*_{2\bar{1}}$	$\underline{2}_{12}\underline{m}'^*_{2\bar{1}}m'^*_z$	PM	PM
	$\star)$	$(01\bar{1}0)$	2_{12}	2_{12}	$2_{12}m'^*_{2\bar{1}}m'^*_z$	PM	PM
		$(\overline{2h}hhl)$	1	1	$\underline{2}_{12}$	PM	PM
		$(0h\bar{h}l)$	1	1	$m'^*_{2\bar{1}}$	PM	PM
		$(hki0)$	1	1	m'^*_z	PM	PM
	26.14	$(hkil)$	1	1	1	P	P

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{6}'$	$\bar{6}'m'^*2^*$	(0001)	3_z	$3_z\underline{2}^*_{12}$	$\underline{\bar{6}}'_z m'^*_{2\bar{1}}\underline{2}^*_{12}$		
		$(2\bar{1}\bar{1}0)$	m'_z	$\underline{2}^*_{12}\underline{m}'^*_{2\bar{1}}m'_z$	$\underline{2}^*_{12}\underline{m}'^*_{2\bar{1}}m'_z$	PM	PM
		$(01\bar{1}0)$	m'_z	m'_z	$2^*_{12}m'^*_{2\bar{1}}m'_z$	PM	PM
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}$	PM	PM
		$(0\bar{h}hl)$	1	1	$m'^*_{2\bar{1}}$	PM	PM
		$(hki0)$	m'_z	m'_z	m'_z	PM	PM
	26.16	$(hkil)$	1	1	1	PM	PM
$\bar{3}m$	$6^*/m^*mm^*$	(0001)	$3_z m_{2\bar{1}}$	$\underline{\bar{6}}^*_z m_{2\bar{1}}\underline{2}^*_{12}$	$6^*_z/\underline{m}^*_z\underline{2}_{10}/m_{2\bar{1}}\underline{2}^*_{12}/m^*_{01}$		
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}\underline{2}^*_{12}\underline{2}^*_z$	$2_{10}/\underline{m}_{2\bar{1}}\underline{2}^*_{12}/m^*_{01}\underline{2}^*_z/m^*_z$		
	+)	$(01\bar{1}0)$	$m_{2\bar{1}}$	$\underline{m}^*_{01}m_{2\bar{1}}\underline{2}^*_z$	$\underline{2}_{10}/m_{2\bar{1}}2^*_{12}/\underline{m}^*_{01}\underline{2}^*_z/m^*_z$	P	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m^*_{01}$	PM	M
		$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m_{2\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}}$	PM	M
		$(hki0)$	1	$\underline{2}^*_z$	$\underline{2}^*_z/m^*_z$	PM	M
	27.1	$(hkil)$	1	1	$\underline{\bar{1}}$	PM	M
$\bar{3}m$	$6'^*/m'^*mm'^*$	(0001)	$3_z m_{2\bar{1}}$	$\underline{\bar{6}}'^*_z m_{2\bar{1}}\underline{2}'^*_{12}$	$6'^*_z/\underline{m}'^*_z\underline{2}_{10}/m_{2\bar{1}}\underline{2}'^*_{12}/m'^*_{01}$		
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}\underline{2}'^*_{12}\underline{2}'^*_z$	$2_{10}/\underline{m}_{2\bar{1}}\underline{2}'^*_{12}/m'^*_{01}\underline{2}'^*_z/m'^*_z$	M	M
	+)	$(01\bar{1}0)$	$m_{2\bar{1}}$	$\underline{m}'^*_{01}m_{2\bar{1}}\underline{2}'^*_z$	$\underline{2}_{10}/m_{2\bar{1}}\underline{2}'^*_{12}/\underline{m}'^*_{01}\underline{2}'^*_z/m'^*_z$	PM	M
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}'^*_{12}$	$\underline{2}'^*_{12}/m'^*_{01}$	PM	M
		$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m_{2\bar{1}}$	$\underline{2}_{10}/m_{2\bar{1}}$	PM	M
		$(hki0)$	1	$\underline{2}'^*_z$	$\underline{2}'^*_z/m'^*_z$	PM	M
	27.17	$(hkil)$	1	1	$\underline{\bar{1}}$	PM	M

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$6'/m'$	$6'/m'm^*m^*$	(0001)	$6'_z$	$6'_z\underline{2}^*_{10}\underline{2}'^*_{12}$	$6'_z/\underline{m}'_z\underline{2}^*_{10}/m^*_{2\bar{1}}\underline{2}'^*_{12}/m'^*_{01}$		
		$(2\bar{1}\bar{1}0)$	m'_z	$\underline{2}'^*_{12}\underline{m}^*_{2\bar{1}}m'_z$	$2^*_{10}/\underline{m}^*_{2\bar{1}}\underline{2}'^*_{12}/m'^*_{01}\underline{2}'_z/m'_z$	PM	M
	+)	$(01\bar{1}0)$	m'_z	$\underline{2}^*_{10}\underline{m}'^*_{01}m'_z$	$\underline{2}^*_{10}/m^*_{2\bar{1}}2'^*_{12}/\underline{m}'^*_{01}\underline{2}'_z/m'_z$	PM	M
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}'^*_{12}$	$\underline{2}'^*_{12}/m'^*_{01}$	PM	M
		$(0h\bar{h}l)$	1	$\underline{2}^*_{10}$	$\underline{2}^*_{10}/m^*_{2\bar{1}}$	PM	M
		$(hki0)$	m'_z	m'_z	$\underline{2}'_z/m'_z$	PM	M
	27.19	$(hkil)$	1	1	$\underline{\bar{1}}$	PM	M
$\bar{3}'m'$	$6^*/m'^*m'm'^*$	(0001)	$3_zm'_{2\bar{1}}$	$\underline{\bar{6}}'^*_z m'_{2\bar{1}}\underline{2}^*_{12}$	$6^*_z/\underline{m}'^*_z 2_{10}/m'_{2\bar{1}}\underline{2}^*_{12}/m'^*_{01}$		
		$(2\bar{1}\bar{1}0)$	2_{10}	$2_{10}\underline{2}^*_{12}\underline{2}^*_z$	$2_{10}/\underline{m}'_{2\bar{1}}\underline{2}^*_{12}/m'^*_{01}\underline{2}^*_z/m'^*_z$		
	⋆)	$(01\bar{1}0)$	$m'_{2\bar{1}}$	$\underline{m}'^*_{01}m'_{2\bar{1}}\underline{2}^*_z$	$\underline{2}_{10}/m'_{2\bar{1}}\underline{2}^*_{12}/\underline{m}'^*_{01}\underline{2}^*_z/m'^*_z$	PM	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m'^*_{01}$	PM	
		$(0h\bar{h}l)$	$m'_{2\bar{1}}$	$m'_{2\bar{1}}$	$\underline{2}_{10}/m'_{2\bar{1}}$	PM	
		$(hki0)$	1	$\underline{2}^*_z$	$\underline{2}^*_z/m'^*_z$	PM	
	27.30	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	
$6/m'$	$6/m'm'^*m'^*$	(0001)	6_z	$6_z\underline{2}^*_{10}\underline{2}^*_{12}$	$6_z/\underline{m}'_z\underline{2}^*_{10}/m'^*_{2\bar{1}}\underline{2}^*_{12}/m'^*_{01}$		
		$(2\bar{1}\bar{1}0)$	m'_z	$\underline{2}^*_{12}\underline{m}'^*_{2\bar{1}}m'_z$	$2^*_{10}/\underline{m}'^*_{2\bar{1}}\underline{2}^*_{12}/m'^*_{01}\underline{2}_z/m'_z$	PM	
		$(01\bar{1}0)$	m'_z	$\underline{2}^*_{10}\underline{m}'^*_{01}m'_z$	$\underline{2}^*_{10}/m'^*_{2\bar{1}}2^*_{12}/\underline{m}'^*_{01}\underline{2}_z/m'_z$	PM	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m'^*_{01}$	PM	
		$(0h\bar{h}l)$	1	$\underline{2}^*_{10}$	$\underline{2}^*_{10}/m'^*_{2\bar{1}}$	PM	
		$(hki0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
	27.31	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{3}'m$	$6^*/m'^*mm^*$	(0001)	$3_z m_{2\bar{1}}$	$\underline{\bar{6}}'^*_z m_{2\bar{1}}\underline{2}'^*_{12}$	$6^*_z/\underline{m}'^*_z 2'_{10}/m_{2\bar{1}}\underline{2}'^*_{12}/m^*_{01}$		
		$(2\bar{1}\bar{1}0)$	$2'_{10}$	$2'_{10}\underline{2}'^*_{12}\underline{2}^*_z$	$2'_{10}/\underline{m}_{2\bar{1}}\underline{2}'^*_{12}/m^*_{01}\underline{2}^*_z/m'^*_z$	M	
	$\star)$	$(01\bar{1}0)$	$m_{2\bar{1}}$	$\underline{m}^*_{01}m_{2\bar{1}}\underline{2}^*_z$	$\underline{2}'_{10}/m'_{2\bar{1}}\underline{2}'^*_{12}/\underline{m}^*_{01}\underline{2}^*_z/m'^*_z$	P	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}'^*_{12}$	$\underline{2}'^*_{12}/m^*_{01}$	PM	
		$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m_{2\bar{1}}$	$\underline{2}'_{10}/m_{2\bar{1}}$	PM	
		$(hki0)$	1	$\underline{2}^*_z$	$\underline{2}^*_z/m'^*_z$	PM	
	27.35	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	
$6/m'$	$6/m'm^*m^*$	(0001)	6_z	$6_z\underline{2}'^*_{10}\underline{2}'^*_{12}$	$6_z/\underline{m}'_z\underline{2}'^*_{10}/m^*_{2\bar{1}}\underline{2}'^*_{12}/m^*_{01}$	M	
		$(2\bar{1}\bar{1}0)$	m'_z	$\underline{2}'^*_{12}\underline{m}^*_{2\bar{1}}m'_z$	$\underline{2}'^*_{10}/\underline{m}^*_{2\bar{1}}\underline{2}'^*_{12}/m^*_{01}\underline{2}_z/m'_z$	PM	
		$(01\bar{1}0)$	m'_z	$\underline{2}'^*_{10}\underline{m}^*_{01}m'_z$	$\underline{2}'^*_{10}/m^*_{2\bar{1}}\underline{2}'^*_{12}/\underline{m}^*_{01}\underline{2}_z/m'_z$	PM	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}'^*_{12}$	$\underline{2}'^*_{12}/m^*_{01}$	PM	
		$(0h\bar{h}l)$	1	$\underline{2}'^*_{10}$	$\underline{2}'^*_{10}/m^*_{2\bar{1}}$	PM	
		$(hki0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
	27.36	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	
$\bar{3}'m$	$6'^*/m^*mm'^*$	(0001)	$3_z m_{2\bar{1}}$	$\underline{\bar{6}}^*_z m_{2\bar{1}}\underline{2}^*_{12}$	$6'^*_z/\underline{m}^*_z 2'_{10}/m_{2\bar{1}}\underline{2}^*_{12}/m'^*_{01}$		
		$(2\bar{1}\bar{1}0)$	$2'_{10}$	$2'_{10}\underline{2}^*_{12}\underline{2}^*_z$	$2'_{10}/\underline{m}_{2\bar{1}}\underline{2}^*_{12}/m'^*_{01}\underline{2}'^*_z/m^*_z$	M	
		$(01\bar{1}0)$	$m_{2\bar{1}}$	$\underline{m}'^*_{01}m_{2\bar{1}}\underline{2}'^*_z$	$\underline{2}'_{10}/m_{2\bar{1}}\underline{2}^*_{12}/\underline{m}'^*_{01}\underline{2}'^*_z/m^*_z$	PM	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m'^*_{01}$	PM	
		$(0h\bar{h}l)$	$m_{2\bar{1}}$	$m_{2\bar{1}}$	$\underline{2}'_{10}/m_{2\bar{1}}$	PM	
		$(hki0)$	1	$\underline{2}'^*_z$	$\underline{2}'^*_z/m^*_z$	PM	
	27.40	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{3}'m'$	$6'^*/m^*m^*m'$	(0001)	$3_z m'_{01}$	$\underline{\bar{6}}^*_z m'_{01}\underline{2}'^*_{10}$	$6'^*_z/\underline{m}^*_z\underline{2}'^*_{10}/m^*_{2\bar{1}}\underline{2}_{12}/m'_{01}$	M	
		$(2\bar{1}\bar{1}0)$	m'_{01}	$m'_{01}\underline{m}^*_{2\bar{1}}\underline{2}'^*_z$	$2'^*_{10}/\underline{m}^*_{2\bar{1}}\underline{2}_{12}/m'_{01}\underline{2}'^*_z/m^*_z$	PM	
		$(01\bar{1}0)$	2_{12}	$2_{12}\underline{2}'^*_{10}\underline{2}'^*_z$	$\underline{2}'^*_{10}/m^*_{2\bar{1}}2_{12}/\underline{m}'_{01}\underline{2}'^*_z/m^*_z$	M	
		$(2h\bar{h}\bar{h}l)$	m'_{01}	m'_{01}	$\underline{2}_{12}/m'_{01}$	PM	
		$(0h\bar{h}l)$	1	$\underline{2}'^*_{10}$	$\underline{2}'^*_{10}/m^*_{2\bar{1}}$	PM	
		$(hki0)$	1	$\underline{2}'^*_z$	$\underline{2}'^*_z/m^*_z$	PM	
	27.41	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	
$6'/m$	$6'/mm^*m'^*$	(0001)	$6'_z$	$6'_z\underline{2}'^*_{10}\underline{2}^*_{12}$	$6'_z/\underline{m}_z\underline{2}'^*_{10}/m^*_{2\bar{1}}\underline{2}^*_{12}/m'^*_{01}$		
		$(2\bar{1}\bar{1}0)$	m_z	$\underline{2}^*_{12}\underline{m}^*_{2\bar{1}}m_z$	$2'^*_{10}/\underline{m}^*_{2\bar{1}}\underline{2}^*_{12}/m'^*_{01}\underline{2}'_z/m_z$	P	
	+)	$(01\bar{1}0)$	m_z	$\underline{2}'^*_{10}\underline{m}'^*_{01}m_z$	$\underline{2}'^*_{10}/m^*_{2\bar{1}}2^*_{12}/\underline{m}'^*_{01}\underline{2}'_z/m_z$	PM	
		$(2h\bar{h}\bar{h}l)$	1	$\underline{2}^*_{12}$	$\underline{2}^*_{12}/m'^*_{01}$	PM	
		$(0h\bar{h}l)$	1	$\underline{2}'^*_{10}$	$\underline{2}'^*_{10}/m^*_{2\bar{1}}$	PM	
		$(hki0)$	m_z	m_z	$\underline{2}'_z/m_z$	PM	
	27.42	$(hkil)$	1	1	$\underline{\bar{1}}'$	PM	
23	$231'^*$	(001)	2_z	$\underline{2}'^*_x\underline{2}'^*_y2_z$	$\underline{2}_x\underline{2}_y2_z1'^*$	M	
		(110)	1	$\underline{2}'^*_z$	$\underline{2}_z1'^*$	PM	P
		$(kl0)$	1	$\underline{2}'^*_z$	$\underline{2}_z1'^*$	PM	P
		(hhl)	1	1	$1'^*$	PM	P
		(hkl)	1	1	$1'^*$	PM	P
	28.1	(111)	3_p	3_p	$3_p1'^*$	PM	P

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
23	4^*32^*	(001)	2_z	$\underline{2}^*_{xy}\underline{2}^*_{x\bar{y}}2_z$	$4^*_z\underline{2}_x\underline{2}^*_{xy}$		
		(110)	1	$\underline{2}^*_{x\bar{y}}$	$2^*_{xy}\underline{2}^*_{x\bar{y}}\underline{2}_z$	PM	
	$\star)$	$(kl0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}^*_{x\bar{y}}$	PM	PM
		(hkl)	1	1	1	PM	PM
	30.1	(111)	3_p	$3_p\underline{2}^*_{x\bar{y}}$	$3_p\underline{2}^*_{x\bar{y}}$		
432	$4321'^*$	(001)	4_z	$4_z\underline{2}'^*_x\underline{2}'^*_{xy}$	$4_z\underline{2}_x\underline{2}_{xy}1'^*$	M	
		(110)	2_{xy}	$2_{xy}\underline{2}'^*_{x\bar{y}}\underline{2}'^*_z$	$2_{xy}\underline{2}_{x\bar{y}}\underline{2}_z1'^*$	M	
		$(kl0)$	1	$\underline{2}'^*_z$	$\underline{2}_z1'^*$	PM	P
		(hhl)	1	$\underline{2}'^*_{x\bar{y}}$	$\underline{2}_{x\bar{y}}1'^*$	PM	P
		(hkl)	1	1	$1'^*$	PM	P
	30.2	(111)	3_p	$3_p\underline{2}'^*_{x\bar{y}}$	$3_p\underline{2}_{x\bar{y}}1'^*$	M	
$4'32'$	$4'32'1'^*$	(001)	$4'_z$	$4'_z\underline{2}'^*_x\underline{2}^*_{xy}$	$4'_z\underline{2}_x\underline{2}'_{xy}1'^*$		
		(110)	$2'_{xy}$	$2'_{xy}\underline{2}^*_{x\bar{y}}\underline{2}'^*_z$	$2'_{xy}\underline{2}'_{x\bar{y}}\underline{2}_z1'^*$	M	
	$+)$	$(kl0)$	1	$\underline{2}'^*_z$	$\underline{2}_z1'^*$	PM	P
		(hhl)	1	$\underline{2}^*_{x\bar{y}}$	$\underline{2}'_{x\bar{y}}1'^*$	PM	P
		(hkl)	1	1	$1'^*$	PM	P
	30.4	(111)	3_p	$3_p\underline{2}^*_{x\bar{y}}$	$3_p\underline{2}'_{x\bar{y}}1'^*$		

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
23	$4'^{*}32'^{*}$	(001)	2_z	$\underline{2}'^{*}_{xy}\underline{2}'^{*}_{x\bar{y}}2_z$	$4'^{*}_z\underline{2}_x\underline{2}'^{*}_{xy}$	M	
		(110)	1	$\underline{2}'^{*}_{x\bar{y}}$	$2'^{*}_{xy}\underline{2}'^{*}_{x\bar{y}}\underline{2}_z$	PM	M
		$(kl0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	$\underline{2}'^{*}_{x\bar{y}}$	$\underline{2}'^{*}_{x\bar{y}}$	PM	PM
		(hkl)	1	1	1	PM	PM
	30.5	(111)	3_p	$3_p\underline{2}'^{*}_{x\bar{y}}$	$3_p\underline{2}'^{*}_{x\bar{y}}$	M	M
23	$\bar{4}^{*}3m^{*}$	(001)	2_z	$\underline{\bar{4}}^{*}_z$	$\underline{\bar{4}}^{*}_z\underline{2}_x m^{*}_{xy}$	M	
		(110)	1	$\underline{m}^{*}_{xy}$	$\underline{m}^{*}_{xy}m^{*}_{x\bar{y}}\underline{2}_z$	PM	P
		$(kl0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	1	$m^{*}_{x\bar{y}}$	PM	PM
		(hkl)	1	1	1	PM	PM
	31.1	(111)	3_p	3_p	$3_p m^{*}_{x\bar{y}}$	PM	P
$\bar{4}3m$	$\bar{4}3m1'^{*}$	(001)	$2_z m_{xy} m_{x\bar{y}}$	$\underline{\bar{4}}'^{*}_z\underline{2}^{*}_x m_{xy}$	$\underline{\bar{4}}_z\underline{2}_x m_{xy}1'^{*}$		
		(110)	$m_{x\bar{y}}$	$\underline{m}'^{*}_{xy}m_{x\bar{y}}\underline{2}'^{*}_z$	$\underline{m}_{xy}m_{x\bar{y}}\underline{2}_z1'^{*}$	PM	P
	+)	$(kl0)$	1	$\underline{2}'^{*}_z$	$\underline{2}_z1'^{*}$	PM	P
		(hhl)	$m_{x\bar{y}}$	$m_{x\bar{y}}$	$m_{x\bar{y}}1'^{*}$	PM	P
		(hkl)	1	1	$1'^{*}$	PM	P
	31.2	(111)	$3_p m_{x\bar{y}}$	$3_p m_{x\bar{y}}$	$3_p m_{x\bar{y}}1'^{*}$	P	P

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$\bar{4}'3m'$	$\bar{4}'3m'1'^{*}$	(001)	$2_z m'_{xy} m'_{x\bar{y}}$	$\underline{\bar{4}}^{*}_z \underline{2}'^{*}_x m'_{xy}$	$\underline{\bar{4}}'_z \underline{2}_x m'_{xy} 1'^{*}$	M	
		(110)	$m'_{x\bar{y}}$	$\underline{m}^{*}_{xy} m'_{x\bar{y}} \underline{2}'^{*}_z$	$\underline{m}'_{xy} m'_{x\bar{y}} \underline{2}_z 1'^{*}$	PM	P
		$(kl0)$	1	$\underline{2}'^{*}_z$	$\underline{2}_z 1'^{*}$	PM	P
		(hhl)	$m'_{x\bar{y}}$	$m'_{x\bar{y}}$	$m'_{x\bar{y}} 1'^{*}$	PM	P
		(hkl)	1	1	$1'^{*}$	PM	P
	31.4	(111)	$3_p m'_{x\bar{y}}$	$3_p m'_{x\bar{y}}$	$3_p m'_{x\bar{y}} 1'^{*}$	PM	P
23	$\bar{4}'^{*}3m'^{*}$	(001)	2_z	$\underline{\bar{4}}'^{*}_z$	$\underline{\bar{4}}'^{*}_z \underline{2}_x m'^{*}_{xy}$		
		(110)	1	$\underline{m}'^{*}_{xy}$	$\underline{m}'^{*}_{xy} m'^{*}_{x\bar{y}} \underline{2}_z$	PM	PM
	$\star$)	$(kl0)$	1	1	$\underline{2}_z$	PM	PM
		(hhl)	1	1	$m'^{*}_{x\bar{y}}$	PM	PM
		(hkl)	1	1	1	PM	PM
	31.5	(111)	3_p	3_p	$3_p m'^{*}_{x\bar{y}}$	PM	PM
$m\bar{3}$	$m\bar{3}m^{*}$	(001)	$2_z m_x m_y$	$\underline{\bar{4}}^{*}_z m_x \underline{2}^{*}_{xy}$	$4^{*}_z/\underline{m}_z \underline{2}_x/m_x \underline{2}^{*}_{xy}/m^{*}_{xy}$		
		(110)	m_z	$\underline{2}^{*}_{x\bar{y}} \underline{m}^{*}_{xy} m_z$	$2^{*}_{xy}/\underline{m}^{*}_{xy} \underline{2}^{*}_{x\bar{y}}/m^{*}_{x\bar{y}} \underline{2}_z/m_z$	P	
	+)	$(kl0)$	m_z	m_z	$\underline{2}_z/m_z$	PM	M
		(hhl)	1	$\underline{2}^{*}_{x\bar{y}}$	$\underline{2}^{*}_{x\bar{y}}/m^{*}_{x\bar{y}}$	PM	M
		(hkl)	1	1	$\underline{\bar{1}}$	PM	M
	32.1	(111)	3_p	$3_p \underline{2}^{*}_{x\bar{y}}$	$\underline{\bar{3}}_p \underline{2}^{*}_{x\bar{y}}/m^{*}_{x\bar{y}}$		

F_1	J_{1j}	(hkl)	$\widehat{F}_1$	T_{1j}	$\overline{J}_{1j}$	T	$\overline{J}$
$m\bar{3}$	$m\bar{3}m'^{*}$	(001)	$2_z m_x m_y$	$\underline{\bar{4}}{}'^{*}_z m_x \underline{2}'^{*}_{xy}$	$4'^{*}_z/\underline{m}_z\underline{2}_x/m_x\underline{2}'^{*}_{xy}/m'^{*}_{xy}$		
		(110)	m_z	$\underline{2}'^{*}_{x\bar{y}}\underline{m}'^{*}_{xy}m_z$	$2'^{*}_{xy}/\underline{m}'^{*}_{xy}\underline{2}'^{*}_{x\bar{y}}/m'^{*}_{x\bar{y}}\underline{2}_z/m_z$	PM	M
	$+)$	$(kl0)$	m_z	m_z	$\underline{2}_z/m_z$	PM	M
		(hhl)	1	$\underline{2}'^{*}_{x\bar{y}}$	$\underline{2}'^{*}_{x\bar{y}}/m'^{*}_{x\bar{y}}$	PM	M
		(hkl)	1	1	$\underline{\bar{1}}$	PM	M
	32.11	(111)	3_p	$3_p\underline{2}'^{*}_{x\bar{y}}$	$\underline{\bar{3}}_p\underline{2}'^{*}_{x\bar{y}}/m'^{*}_{x\bar{y}}$	M	M
$m'\bar{3}'$	$m'\bar{3}'m'^{*}$	(001)	$2_z m'_x m'_y$	$\underline{\bar{4}}{}'^{*}_z m'_x \underline{2}^{*}_{xy}$	$4^{*}_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}^{*}_{xy}/m'^{*}_{xy}$		
		(110)	m'_z	$\underline{2}^{*}_{x\bar{y}}\underline{m}'^{*}_{xy}m'_z$	$2^{*}_{xy}/\underline{m}'^{*}_{xy}\underline{2}^{*}_{x\bar{y}}/m'^{*}_{x\bar{y}}\underline{2}_z/m'_z$	PM	
	$\star)$	$(kl0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		(hhl)	1	$\underline{2}^{*}_{x\bar{y}}$	$\underline{2}^{*}_{x\bar{y}}/m'^{*}_{x\bar{y}}$	PM	
		(hkl)	1	1	$\underline{\bar{1}}'$	PM	
	32.15	(111)	3_p	$3_p\underline{2}^{*}_{x\bar{y}}$	$\underline{\bar{3}}'_p\underline{2}^{*}_{x\bar{y}}/m'^{*}_{x\bar{y}}$		
$m'\bar{3}'$	$m'\bar{3}'m^{*}$	(001)	$2_z m'_x m'_y$	$\underline{\bar{4}}{}^{*}_z m'_x \underline{2}'^{*}_{xy}$	$4'^{*}_z/\underline{m}'_z\underline{2}_x/m'_x\underline{2}'^{*}_{xy}/m^{*}_{xy}$	M	
		(110)	m'_z	$\underline{2}'^{*}_{x\bar{y}}\underline{m}^{*}_{xy}m'_z$	$2'^{*}_{xy}/\underline{m}^{*}_{xy}\underline{2}'^{*}_{x\bar{y}}/m^{*}_{x\bar{y}}\underline{2}_z/m'_z$	PM	
		$(kl0)$	m'_z	m'_z	$\underline{2}_z/m'_z$	PM	
		(hhl)	1	$\underline{2}'^{*}_{x\bar{y}}$	$\underline{2}'^{*}_{x\bar{y}}/m^{*}_{x\bar{y}}$	PM	
		(hkl)	1	1	$\underline{\bar{1}}'$	PM	
	32.18	(111)	3_p	$3_p\underline{2}'^{*}_{x\bar{y}}$	$\underline{\bar{3}}'_p\underline{2}'^{*}_{x\bar{y}}/m^{*}_{x\bar{y}}$	M	

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JANA PŘÍVRATSKÁ

VLASTNOSTI SYMETRIE DOMÉNOVÝCH STĚN SPOJUJÍCÍCH DOMÉNOVÉ STAVY V PLNĚ TRANSPONOVATELNÝCH PÁRECH

ABSTRAKT HABILITAČNÍ PRÁCE

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Úvod

Nehomogenity mohou vytvořit podmínky pro vznik efektů, které jsou zakázané v homogenních systémech, např. nehomogenní teplota nebo nehomogenní deformace může způsobit vznik elektrické polarizace v izotropních nebo centrosymetrických pevných nebo kapalných krystalech. Tento jev je způsoben tím, že nehomogenita obvykle snižuje symetrii a tím umožňuje existenci některých vlastností, které se nemohly vyskytovat v systémech s vyšší symetrií.

Doménové stěny, které tvoří tenkou přechodovou vrstvu mezi dvěma doménami, reprezentují zvláštní případ nehomogenity. V této oblasti dochází ke snížení translační symetrie ze tří na dva nezávislé směry. Je zde tedy možné očekávat výskyt nových efektů.

K popisu symetrie doménové stěny se používají tzv. *vrstvové grupy*, které obsahují pouze ty operace symetrie, vůči nimž je vrstva, reprezentující doménovou stěnu, invariantní. V těchto grupách jsou nutně zakázány některé operace symetrie, které mohou existovat uvnitř domén, jako jsou rotační a inverzní osy nebo roviny symetrie, které nejsou kolmé nebo rovnoběžné s vrstvou. Na druhé straně, rovinná doménová stěna může být invariantní vůči operacím, které vyměňují doménové stavy na protilehlých stranách stěny. Protože tyto operace nemohou existovat uvnitř samotných domén, může ve stěně naopak dojít i ke zvýšení počtu prvků symetrie. Proto rozdíl mezi symetrií doménového stavu a symetrií stěny není obecně jen prosté snížení symetrie, které umožňuje vznik nových efektů v doménové stěně, ale také zvýšení symetrie, následkem čehož mohou vymizet některé vlastnosti, které naopak v samotných doménách existují.

V této souvislosti je zajímavé vyšetřit především ty případy, kdy doménové stěny mají vlastnosti, které se nevyskytují v doménách. Barjachtar *a kol.* [1, 2] teoreticky studovali výskyt elektrické polarizace v doménových stěnách v magneticky uspořádaných krystalech. Walker a Gooding [7], na základě teoretických úvah, předpověděli existenci elektrické polarizace ve stěně, která spojuje dvě nepolární domény v α-fázi křemene a odvodili rovnice popisující průběh polarizace při průchodu touto stěnou. Podobné kvalitativní výsledky získali i Saint-Grégoire a Janovec [3] pouze na základě symetrické analýzy.

Cílem této práci bylo prostudovat možnost výskytu spontánně polarizovaných a/nebo spontánně zmagnetovaných doménových stěn oddělujících domény, v nichž je střední hodnota polarizace a magnetizace nulová. Práce se omezila pouze na 380 magnetických plně transponovatelných doménových párů, zvláštní pozornost byla věnována nemagnetickým neferoelastickým párům, tj. párům, kde obě domény vykazují stejnou spontánní deformaci.

Ke studiu daného problému bylo využito přiblížení spojitého prostředí, kdy translační část T_G prostorové grupy $\mathcal{G}$ je spojitá na celém euklidovském prostoru $\mathbf{E_3}$, resp. $\mathbf{E_2}$ v případě vrstvové grupy. V obou případech je to tedy krystalografická bodová grupa (v $\mathbf{E_3}$ či v $\mathbf{E_2}$), která určuje vlastnosti prostředí z hlediska symetrie. Z tohoto důvodu bylo možné studium grup bodového charakteru (*point-like groups*) nahradit studiem bodových grup, takže i ve výsledných tabulkách se z důvodu jednoduchosti používají jen odpovídající bodové grupy.

Symetrie stěn a vrstvové grupy řezu

Pro rovinnou doménovou stěnu, která je rovnoběžná s rovinou určenou Millerovými indexy (hkl) a spojuje dvě domény s doménovými stavy S_1 a S_2 byl zvolen symbol $[S_1(hkl)S_2] = [S_1(\mathbf{n})S_2]$, protože rovinná stěna může být určena také svou normálou $\mathbf{n}$. V těchto symbolech odpovídá kladná orientace normály přechodu z doménového stavu uváděného na prvním místě do doménového stavu uváděného na místě druhém. Z této úmluvy pak vyplývá identita $[S_1(-\mathbf{n})S_2] = [S_2(\mathbf{n})S_1]$.

Jak již bylo řečeno v úvodu, symetrie doménové stěny je popsána vrstvovou grupou. Tato grupa bude značena

$T[S_1(hkl)S_2] = T[S_1|S_2] = T_{12}(hkl) = T_{12}(\mathbf{n}) = T_{12}$,

aby se vyjádřilo, že rovinná stěna spojuje doménové stavy S_1 a S_2 a je rovnoběžná s rovinou (hkl), resp. kolmá na její normálový vektor $\mathbf{n}$.

Operace $u \in T_{12}(\mathbf{n})$ musí splňovat dvě nutné podmínky:

1. Jako operace vrstvové grupy, u musí buď zachovat orientaci normály $\mathbf{n}$ nebo ji změnit v opačný vektor -$\mathbf{n}$. Operace druhého typu jsou značeny podtržením příslušného symbolu.
2. Jako operace grupy symetrie J_{12} doménového páru $\{S_1, S_2\}$, u musí zachovat oba stavy S_1 a S_2 nezměněné, nebo je musí zaměnit, tj. $uS_1 = S_2$ a $uS_2 = S_1$. Operace, které zaměňují doménové stavy, budou označeny hvězdičkou u příslušného symbolu, (u^*) .

Tyto dvě podmínky mohou být splněny čtyřmi způsoby.

1. *Operace zachovávající orientaci normály:*
   (a) Operace $u = f_{12}$, která nemění ani normálu $\mathbf{n}$ ani doménové stavy S_1 a S_2. Tyto operace samozřejmě nemění doménovou stěnu $[S_1|S_2]$ a jsou nazývány *triviálními operacemi symetrie doménové stěny.*

(b) Operace $u = \underline{r}^{\star}_{12}$, která zaměňuje doménové stavy S_1 a S_2, ale nepřevrací normálu $\mathbf{n}$, tj. zachovává poloprostory na obou stranách stěny v původních polohách. Tyto operace mění původní doménovou stěnu ve *stěnu převrácenou* $[S_2|S_1]$ vzhledem k $[S_1|S_2]$.

2. *Operace měnící orientaci normály.*

(a) Operace $u = \underline{s}_{12}$, která mění orientaci normály $\mathbf{n}$, a tím vyměňuje poloprostory na stranách stěny. Protože tyto poloprostory jsou spojeny s doménovými stavy S_1 a S_2, je tato výměna doprovázena i výměnou doménových stavů na obou stranách stěny. Operace $\underline{s}_{12}$ tak převádí původní stěnu na *stěnu převrácenou* $[S_2|S_1]$.

(b) Operace $u = \underline{t}^{\star}_{12}$, která zaměňuje poloprostory (převrací normálu $\mathbf{n}$) a současně i doménové stavy S_1 a S_2. Operace $\underline{t}^{\star}_{12}$ nechává tedy doménovou stěnu beze změny, je proto *netriviální operací symetrie doménové stěny.*

Tabulka 1 shrnuje vliv všech čtyř typů operací symetrie na normálu $\mathbf{n}$, na oba doménové stavy S_1 a S_2, a na doménovou stěnu $[S_1(\mathbf{n})S_2]$.

Tabulka 1: Vliv čtyř typů operace symetrie u na doménovou stěnu

u	$u\mathbf{n}$	uS_1	uS_2	$u[S_1(\mathbf{n})S_2]$	stěna
$\underline{f}_{12}$	$\mathbf{n}$	S_1	S_2	$[S_1(\mathbf{n})S_2] \equiv [S_2(-\mathbf{n})S_1]$	*původní stěna*
$\underline{r}^{\star}_{12}$	$\mathbf{n}$	S_2	S_1	$[S_2(\mathbf{n})S_1] \equiv [S_1(-\mathbf{n})S_2]$	*převrácená stěna*
$\underline{s}_{12}$	$-\mathbf{n}$	S_1	S_2	$[S_1(-\mathbf{n})S_2] \equiv [S_2(\mathbf{n})S_1]$	*převrácená stěna*
$\underline{t}^{\star}_{12}$	$-\mathbf{n}$	S_2	S_1	$[S_2(-\mathbf{n})S_1] \equiv [S_1(\mathbf{n})S_2]$	*původní stěna*

Z tohoto rozboru vyplývá, že vrstvová grupa $T_{12}(hkl)$, která popisuje symetrii doménové stěny $[S_1(hkl)S_2]$, obsahuje všechny triviální a netriviální operace symetrie stěny.

K nalezení této grupy lze použít standardního postupu [4, 5, 6], kdy se nejprve určí grupa J_{12} neuspořádaného doménového páru $\{S_1, S_2\}$

$$J_{12} = F_1 + \underline{j}^{\star}_{12} F_1, \qquad (0.1)$$

kde F_1 reprezentuje symetrii doménových stavů S_1, S_2 a $\underline{j}^{\star}_{12}$ je operace, která tyto stavy zaměňuje, $\underline{j}^{\star}_{12} S_1 = S_2$ a $\underline{j}^{\star}_{12} S_2 = S_1$. Vrstvová grupa řezu $\overline{J}_{12}$ grupy J_{12} podél

roviny určené Millerovými indexy (hkl) má tvar

$$\overline{J}_{12} = \widehat{F}_1 + \underline{t}^{\star}_{12}\widehat{F}_1 + \underline{r}^{\star}_{12}\widehat{F}_1 + \underline{s}_{12}\widehat{F}_1, \qquad (0.2)$$

kde operace $\underline{t}^{\star}_{12}, \underline{r}^{\star}_{12}, \underline{s}_{12}$ jsou definovány v tabulce 1 a podgrupa $\widehat{F}_1$ je ta část vrstvové grupy řezu F_1, která nemění orientaci normály roviny (hkl).

V tomto rozvoji do levých tříd

$\underline{t}^{\star}_{12}\widehat{F}_1$ obsahuje všechny operace měnící současně stranu i doménový stav,
$\underline{r}^{\star}_{12}\widehat{F}_1$ všechny operace měnící pouze doménový stav a
$\underline{s}_{12}\widehat{F}_1$ všechny operace měnící pouze orientaci normály.
Pak grupa symetrie stěny T_{12} může být odvozena z vrstvové grupy řezu $\overline{J}_{12}$ grupy symetrie J_{12} doménového páru $\{S_1, S_2\}$ sjednocením prvních dvou levých tříd

$$T_{12} = \widehat{F}_1 + \underline{t}^{\star}_{12}\widehat{F}_1. \qquad (0.3)$$

První dvě levé třídy v rovnici (0.2) zahrnují všechny operace, které nechávají doménovou stěnu beze změny. Lze tedy rozlišit

- *symetrickou stěnu*, pro kterou je $T_{12} > \widehat{F}_1$
- *asymetrickou stěnu*, pro kterou je $T_{12} = \widehat{F}_1$.

Poslední dvě levé třídy v rovnici (0.2) zahrnují všechny operace, které mění danou stěnu $W_{12} = S_1(hkl)S_2]$ ve stěnu převrácenou $W_{21} = [S_2(hkl)S_1]$ s opčným pořadím doménových stavů. Lze tedy definovat

- *reverzibilní stěnu*, pro kterou je $\overline{J}_{12} > T_{12}$
- *ireverzibilní stěnu*, pro kterou je $\overline{J}_{12} = T_{12}$.

Výsledky

Vzhledem k tomu, že cílem práce bylo především studium existence nenulové spontánní polarizace ($\mathbf{P} \neq \mathbf{0}$) a spontánní magnetizace ($\mathbf{M} \neq \mathbf{0}$) v doménových stěnách, bylo v první řadě nalezeno 155 magnetických vrstvových grup. Ty je možno rozdělit do čtyř skupin.

a) 22 z nich je pyromagnetických $\mathbf{M} \neq \mathbf{0}$ nepyroelektrických $\mathbf{P} = \mathbf{0}$,

b) 22 z nich je nepyromagnetických $\mathbf{M} = \mathbf{0}$ pyroelectrických $\mathbf{P} \neq \mathbf{0}$,

c) 20 z nich je pyromagnetických $\mathbf{M} \neq \mathbf{0}$ pyroelektrických $\mathbf{P} \neq \mathbf{0}$,

d) 91 z nich je nepyromagnetických $\mathbf{M} = \mathbf{0}$ nepyroelektrických $\mathbf{P} = \mathbf{0}$.

Pro všech 64 vrstvových grup z a), b), c) byl proveden rozbor možných směrů polarizace a magnetizace navzájem i vzhledem ke směru vnější normály vrstvy. Výsledky jsou souhrnně zachyceny v tabulkách 2,3 a 4. Symbol $\mathbf{M} \leftrightarrow \mathbf{n}$ nebo $\mathbf{P} \leftrightarrow \mathbf{n}$ je užit pro vzájemnou polohu magnetizace (nebo polarizace) a normály vrstvy. V tabulkách jsou zachyceny pouze dvě limitní polohy $\mathbf{M}$, $\mathbf{P}$, $\mathbf{n}$, tj. rovnoběžnost $\|$ a kolmost $\perp$. Prázdné místo znamená, že symetrie vrstvy nepreferuje žádný význačný směr.

Tyto tabulky ukazují, že symetrie určuje směr polarizace a magnetizace

- *jednoznačně* pro 51 vrstvových grup, z toho ve 30 případech jsou polarizace a magnetizace kolmé k vrstvě,
- *částečně* pro 10 vrstvových grup
- a pouze pro 3 vrstvové grupy *není* vzájemná poloha polarizace a magnetizace symetrií nijak ovlivněna. Je to v případě triviálních grup $1, 1'$ a $\bar{1}$.

Tabulka 2: **Pyromagnetické ($\mathbf{M} \neq 0$) nepyroelektrické ($\mathbf{P} = 0$) vrstvy.**

M je určena symetrií	$\mathbf{M} \leftrightarrow \mathbf{n}$	počet grup
jednoznačně	$\|$	16
	$\perp$	3
částečně	$\perp$	1
		1
není		1

Tabulka 3: **Pyroelektrické ($\mathbf{P} \neq 0$) nepyromagnetické ($\mathbf{M} = 0$) vrstvy.**

P je určena symetrií	$\mathbf{P} \leftrightarrow \mathbf{n}$	počet grup
úplně	$\|$	16
	$\perp$	3
částečně	$\perp$	1
		1
není		1

Tabulka 4: **Pyromagnetické ($\mathbf{M} \neq 0$) pyroelektrické ($\mathbf{P} \neq 0$) vrstvy.**

P a M jsou určeny symetrií	$\mathbf{P} \leftrightarrow \mathbf{M}$	$\mathbf{P} \leftrightarrow \mathbf{n}$	$\mathbf{M} \leftrightarrow \mathbf{n}$	počet grup
úplně	$\|$	$\|$	$\|$	8
		$\perp$	$\perp$	2
	$\perp$	$\perp$	$\|$	1
		$\|$	$\perp$	1
		$\perp$		1
P - úplně **M** - částečně	$\perp$	$\|$	$\perp$	1
		$\perp$		1
P - částečně **M** - úplně	$\perp$	$\perp$	$\|$	1
			$\perp$	1
částečně		$\perp$	$\perp$	1
				1
nejsou				1

Prodalší analýzu byly sestaveny tabulky vrstvových grup řezu pro všech 380 magnetických plně transponovatelných doménových párů, řezy byly prováděny pro všechny krystalografické rovinné orbity. Následně byly určeny grupy symetrie doménových stěn ve všech výše zmíněných případech. Výsledky tohoto rozboru umožňují rozdělit doménové páry opět do čtyř skupin:

Typ $\mathcal{MP}$ - grupa $J_{12}[F_1]$ popisuje doménové páry, v nichž některé stěny mohou být současně pyromagnetické i pyroelektrické.

Těchto doménových párů je 184.

Typ $\mathcal{M}$ - grupa $J_{12}[F_1]$ popisuje doménové páry, v nichž některé stěny mohou být pyromagnetické, ale *všechny* doménové stěny jsou nepyroelektrické.
Těchto doménových párů je 69.

Typ $\mathcal{P}$ - grupa $J_{12}[F_1]$ popisuje doménové páry, v nichž některé stěny mohou být pyroelektrické, ale *všechny* doménové stěny jsou nepyromagnetické.
Těchto doménových párů je 37.

Typ $\mathcal{O}$ - grupa $J_{12}[F_1]$ popisující doménové páry, v nichž *všechny* stěny jsou současně nepyromagnetické a nepyroelektrické.
Těchto doménových párů je 90.

Následující tabulka 5 uvádí přehled počtu jednotlivých typů doménových párů v závislosti na:

a) typu grup J_{12} a F_1

b) na struktuře těchto grup.

Tabulka 5: **Počet a typ doménových párů v závislosti na struktuře grupy $J_{12}[F_1]$**

stěna	J_{12} $[F_1]$	MP $[MP]$	M $[MP]$	P $[MP]$	0 $[MP]$	M $[M]$	0 $[M]$	P $[P]$	0 $[P]$	0 $[0]$	$\sum$
$\mathcal{PM}$	$q[q]$	16	9	9	8	6	6	6	6	16	82
	$q1'[q]$			13			8	8		19	48
	$i[i]$					11	8			8	27
	$i'[i']$									27	27
$\mathcal{M}$	$i[q]$		13			8			8	19	48
	$i1'[i']$									21	21
$\mathcal{P}$	$q1'[q1']$							11	8	8	27
	$i1'[i1']$									10	10
$\mathcal{O}$	$i'[q]$				13		8		8	19	48
	$i1'[q1']$								10	11	21
	$i1'[i]$						10			11	21
	$\sum$	16	22	22	21	25	40	25	40	169	380

V tabulce byla použita následující symbolika

- q - magnetická grupa obsahující čárkované nebo nečárkované operace kromě $1', \bar{1}$ a $\bar{1}'$,

- i - magnetická grupa obsahující čárkované nebo nečárkované operace kromě $1'$ a $\bar{1}'$, tj. $\bar{1}$ je její generátor,
- i' - magnetická grupa obsahující čárkované nebo nečárkované operace kromě $1'$ a $\bar{1}$, tj. $\bar{1}'$ je její generátor.

Další analýza se soustředila na doménové páry, které jsou tvořeny antiferomagnetickými a neferoelektrickými doménovými stavy, tj. stavy s nulovou střední magnetizací a polarizací. Tato situace nastává v 169 případech:

- typ $\mathcal{MP}$ nastává v 70 případech,
- typ $\mathcal{M}$ nastává v 40 případech,
- typ $\mathcal{P}$ nastává v 18 případech,
- typ $\mathcal{O}$ nastává v 41 případech.

Pro doménové páry patřící k typům $\mathcal{MP}, \mathcal{M}$ a $\mathcal{P}$, tj. pro 128 případů, jsou konkrétní vrstvové grupy T_{12} a $\overline{J}_{12}$ uvedeny v dodatcích G, H a I. Analýza těchto tabulek umožnila určit všechny kombinace grup symetrie doménové stěny T_{12} a vrstvové grupy řezu $\overline{J}_{12}$ a studovat je nezávisle na řezné rovině (hkl).

Následující tabulka 6 zachycuje výsledky získané pro polární stěnu. Symbol $\mathbf{P}(W_{12})$ byl použit pro vektor polarizace ve stěně $[S_1(hkl)S_2]$ a $\mathbf{P}(W_{21})$ v převrácené stěně $[S_2(hkl)S_1]$. Písmena S, A, R a I reprezentují stěnu symetrickou, asymetrickou, reverzibilní a ireverzibilní.

Pro ilustraci rozdílu mezi symetrickou (i)reverzibilní stěnou W_{12} a převrácenou stěnou W_{21} je možno použít výsledky získané pro křemen. Z tabulek v dodatku C (str. 81), případně v dodatku G (str.113), vyplývá pro $J_{12} = 622$ a $F_1 = 32$ (tedy nepolární doménový stav), že např. stěny $[S_1, S_2]$ rovnoběžné s osou z jsou *symetrické*, mají symetrii:

a) $T_{12} = 2^{\star}_{\underline{z}}$, a stěna tedy může být spontánně polarizovaná ve směru osy z,

b) $T_{12} = 2_{10}2^{\star}_{\underline{12}}2^{\star}_{\underline{z}}$ ve speciálním případě orbity určené Bravisovými-Millerovými indexy $(2\bar{1}\bar{1}0)$, kdy symetrie stěny vyžaduje nulovou spontánní polarizaci. Stěny rovnoběžné s osou z jsou:

a) *reverzibilní* ve speciálním případě orbity určené rovinou $(01\bar{1}0)$,

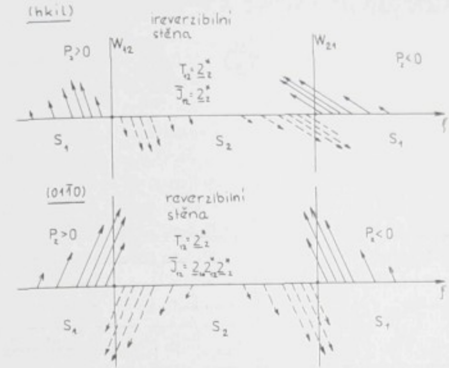
b) *ireverzibilní* v ostatních případech.

Tabulka 6: **Pyroelektrické nepyromagnetické stěny**

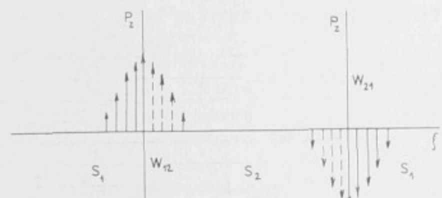
(fa ... feroelastický doménový pár , na ... neferoelastický doménový pár)

T_{12}	**P**	$\mathcal{J}_{12}$	fa, na	typ $\mathcal{J}_{12}$	sym. stěny	$\mathbf{P}(W_{12})$	$\mathbf{P}(W_{21})$
$11'$		$11'$	fa, na	P	AI	$\mathbf{P}_1$	$\mathbf{P}_2$
		$\underline{1}1'$	na	0	AR	$\mathbf{P}$	$-\mathbf{P}$
		$\underline{2}1'$	fa, na	P	AR	$\mathbf{P}_{\perp\underline{2}}$	$-\mathbf{P}_{\perp\underline{2}}$
						$\mathbf{P}_{\Vert\underline{2}}$	$\mathbf{P}_{\Vert\underline{2}}$
		m^*1'	fa, na	P	AR	$\mathbf{P}_{\perp m^*}$	$-\mathbf{P}_{\perp m^*}$
						$\mathbf{P}_{\Vert m^*}$	$\mathbf{P}_{\Vert m^*}$
$21'$	$\mathbf{P}\Vert 2$	$2m^*m^*1'$	na	P	AR	$\mathbf{P}$	$\mathbf{P}$
		$2\underline{22}1'$	fa	0	AR	$\mathbf{P}$	$-\mathbf{P}$
$\underline{2}^*1'$	$\mathbf{P}\Vert\underline{2}^*$	$\underline{2}^*1'$	fa, na	P	SI	$\mathbf{P}_1$	$\mathbf{P}_2$
		$\underline{2}^*/m^*1'$	fa, na	0	SR	$\mathbf{P}$	$-\mathbf{P}$
		$\underline{2}^*\underline{2}2^*1'$	fa, na	0	SR	$\mathbf{P}$	$-\mathbf{P}$
$m1'$	$\mathbf{P}\Vert m$	$m1'$	fa, na	P	AI	$\mathbf{P}_1$	$\mathbf{P}_2$
		$\underline{2}/m1'$	fa, na	0	AR	$\mathbf{P}$	$-\mathbf{P}$
		$2^*m^*m1'$	na	P	AR	$\mathbf{P}_{\perp 2^*}$	$-\mathbf{P}_{\perp 2^*}$
						$\mathbf{P}_{\Vert 2^*}$	$\mathbf{P}_{\Vert 2^*}$
$\underline{m}^*1'$	$\mathbf{P}\Vert\underline{m}^*$	$2^*/\underline{m}^*1'$	fa, na	0	SR	$\mathbf{P}$	$-\mathbf{P}$
		$\underline{m}^*m^*\underline{2}1'$	fa, na	P	SR	$\mathbf{P}_{\perp\underline{2}}$	$-\mathbf{P}_{\perp\underline{2}}$
						$\mathbf{P}_{\Vert\underline{2}}$	$\mathbf{P}_{\Vert\underline{2}}$
$mm21'$	$\mathbf{P}\Vert 2$	$\underline{m}mm1'$	fa	0	AR	$\mathbf{P}$	$-\mathbf{P}$
$\underline{m}^*m\underline{2}^*1'$	$\mathbf{P}\Vert\underline{2}^*$	$\underline{m}^*m\underline{2}^*1'$	na	P	SI	$\mathbf{P}_1$	$\mathbf{P}_2$
		$\underline{m}^*m^*m1'$	fa, na	0	SR	$\mathbf{P}$	$-\mathbf{P}$
$31'$	$\mathbf{P}\Vert 3$	$3m^*1'$	na	P	AR	$\mathbf{P}$	$\mathbf{P}$

Na obrázku 1 je schématicky znázorněn průběh polarizace $\mathbf{P}_{xy}$ ve stěně, tj. složky kolmé na osu z, jejíž globální příspěvek je nulový. Tento průběh je naznačen jak pro obecnou ireverzibilní stěnu $(hkil)$, tak pro speciální reverzibilní stěnu $(01\bar{1}0)$. Současně je zde uveden i případ převrácené stěny. Obrázek 2 reprezentuje průběh z-ové složky $\mathbf{P}_z$ polarizace ve stěně $[S_1, S_2]$, kdy globální příspěvek této složky polarizace je nenulový.



Obr.1



Obr.2

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Slovníček

n-dimensional Euclidean space	n-rozměrný euklidovský prostor
space group	prostorová grupa
point group	bodová grupa
translational group	translační grupa
pointlike line group	přímková grupa bodového charakteru
pointlike plane group	rovinná grupa bodového charakteru
pointlike space group	prostorová grupa bodového charakteru
layer group	vrstvová grupa
pointlike layer group	vrstvová grupa bodového charakteru
sectional layer group	vrstvová grupa řezu
face orbit	orbita stěn
plane orbit	orbita rovin
family	rodina
holohedral	holohedrický
domain	doména
domain bulk	vnitřek domény
domain state	doménový stav
ordered domain pair	uspořádaný doménový pár
unordered domain pair	neuspořádaný doménový pár
domain wall	doménová stěna
symmetry group of a domain wall	grupa symetrie doménové stěny
reversed wall	převrácená stěna
simple form	prostý (krystalografický) tvar
the time inversion	časová inverze
the space inversion	prostorová inverze
the space and time inversion	prostoročasová inverze
magnetic group	magnetická grupa
magnetization	magnetizace
polarization	polarizace
pyromagnetic	pyromagnetický
pyroelectric	pyroelektrický
asymmetric	asymetrický
symmetric	symetrický
irreversible	ireverzibilní
reversible	reverzibilní

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