COLOR PERCEPTION ESTIMATIONS OF METAMERIC PAIRS UNDER DIFFERENT ILLUMINANCE LEVELS

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Abstract: *LEDs or light emitting diodes of the lighting class dominate both the indoor and outdoor lighting industries today due to their accuracy and consumer-friendly color temperature. In the context of color science, it is necessary to analyze both the spectral power distribution of lighting and the human characteristics of color perception under these lights. In this article, we provide estimates of the appearance of eleven metameric pairs under LEDs with four correlated color temperatures and six illuminance levels, using color difference formulas based on the CIELAB, CAM02-UCS, and CAM16-UCS models to verify our estimates. We followed ASTM D4086 standard visual methods for detecting metamerism and for estimating the magnitude of a metameric color difference. Our investigations found that color appearance models are more reliable than CIELAB in evaluating color difference under various LED conditions. CAM16-UCS more accurately predicted the color difference estimates between all three formulas. Our comparative study confirms that the variation in the estimates with the CCT and illuminance levels of the LED sources depends on the color appearance model used. The results also showed that in order to determine the color difference of metameric pairs, optimal conditions regarding the colorimetric properties of the samples and the variability of the observer should be considered separately. We noticed an increasing correlation trend with increasing illuminance. However, there was no such increase or decrease trend in CCTs. The trend of the STRESS change in the color appearance models showed the influence of the chromatic adaptation, but the establishment of adaptation patterns is far beyond the scope of this work. Although our research has had limitations on correlated color temperature and illuminance, we believe that it can be beneficial for the lighting application to ensure correct lighting decisions when assessing the color differences of metameric pairs.*

Keywords: *Correlated color temperature, illuminance, CAM02-UCS, CAM16-UCS, color difference formula.*

1 INTRODUCTION

To see color, you have to have light. Our eyes only can see the colors that are bounced off or reflected from the objects. With the change of light sources, colored objects appear differently. The emerging market of lighting industries and addition of new light sources increase the complexity of the scenario for color related field like dyeing and printing industries, paper industries, graphics, art, painting and architecture etc. A wide variety of electric lighting is in use today including incandescent, fluorescent, metal halide and light emitting diodes (LEDs). Recent progresses in illuminant technology, LED lights have taken the market by storm over the last few years, largely due to their high efficiency and affordability. According to 2018 data from The National Electrical Manufacturers Association (NEMA), LED bulbs account for about 65% of the consumer lamp market, followed by halogen incandescent, which account for about 28% of the market [1].

After almost three decades of the acceptance of the CIE (The International Commission on Illumination known as the Commission Internationale de l'Elcairage), in 1963 CIE defined new series of daylight illuminants (D55, D65, and D75) based on Judd et al. [2] approach and Simonds' [3] method. Recent CIE publication document 15.4 defined the standard spectral power distributions (SPD) of LED sources for color specification [4]. Since there is not a single LED lighting standard for color matching that map directly to available LED lamps on the market today, they are best utilized as an additional light source to gauge how the product may appear in an environment illuminated by a similar LED source. Nonetheless, it is perfectly possible to obtain different SPD's for light sources with the same CCT and thanks to metamerism [5-6].

Knowledge about the lighting quality such as spectral power distribution (SPD), chromaticity coordinates (*x, y* or *u', v'*), correlated color temperature (CCT), color rendering index (CRI), luminance, illuminance, and luminance and illuminance efficacy will allow the user to ensure the optimum lighting decisions. Different color difference formulas and color tolerance systems are widely utilized in industrial acceptability applications for that purpose. Color differences between pairs of colored samples depend on whether one or another light source is used and the capacity of the observers to make judgments of color difference. After the formulation of CIE L*a*b* color space in 1976, various advanced color difference formulas such as CMC [7], CIE94 [8], CIEDE2000 [9] has been formulated based on some experimental data sets namely RIT-DuPont [10], Witt [11], Leeds [12], BFD [13], and so on. In 2002, CIE adopted CIECAM02 color appearance model which was a revised version of CIECAM97s [14]. One of the major parts of the model is its chromatic adaptation transform, CIECAT02. Luo et al. later derived three uniform color spaces based on CIECAM02 namely CAM02-SCD, CAM02-LCD, and CAM02-UCS.However, the appearance correlates of CIECAM02,the lightness (*J*), colorfulness (*M*), and hue angle (*h*) form a uniform color space that can be used to calculate color differences, as long as a viewing condition is fixed. Later, Li et al. [15] developed, a new color appearance model called CAM16 based on a conical space to overcome the mathematical problems associated with CIE CAM02. Based on CAM16, they also developed a new chromatic adaptation transformation, CAT16 and a new uniform color space, CAM16-UCS.

According to CIE recommendations, a set of 'reference' viewing conditions must be fulfilled for color difference assessments e.g. D65 simulator at 1000 lux, normal color vision observers, object viewing mode, stimulus size of more than 4° subtended visual angle, color-difference magnitude of 0 to 5 CIELAB units and visually homogeneous sample structure [16]. The question then is the current color space and color appearance models optimum to predict absolute color, color difference and color changes as a result of changes in lighting conditions. The best way to visualize how color will react in different lighting conditions is to use a light booth. For our present work, we have used a lighting booth equipped with four LEDs of four different correlated color temperatures e.g. 6500K, 5000K, 4000K and 2700K. We have set up six luminance levels: 50lx, 100lx, 200lx, 500lx, 1000lx and 1500lx by control switch attached to the lighting booth.

The objective of the present work was to carry out a visual experiment involving color difference formula based on CIELAB, CAM02-UCS and CAM16-UCS in order to study the possible relationships between the correlated colour temperature or the illuminance level with the discriminatory capacity of a set of observers. The initial hypothesis is that there is no relationship between the CCT or the illuminance level of a light source with the color difference formulas itself. Another hypothesis is that the CCT or the illuminance level of a light source has no influence on color difference variability of the observers.

2 DATA ANALYSIS

2.1 Observer's variability assessment

An individual's color difference ability undoubtedly depends on the individual conditions. There are two forms of observer variability (i) intra-observer and (ii) inter-observer variability. The standardized residual sum of squares (*STRESS*) metric [17] is a statistical measure widely used in color research field to indicate intra or inter-variability of observers. The *STRESS* value can be calculated by using (1):

$$STRESS = 100.\sqrt{\frac{\sum(\Delta E_i - F_i \Delta V_i)^2}{(F_1^2 \Delta V_1^2)}}$$

and (1)

$$F_1 = \frac{\sum \Delta E_i^2}{\sum \Delta E_i \Delta V_i}$$

where: ΔE_i and ΔV_i are the computed and the perceived color difference for the i =1,…n sample pair respectively and F_1 is an adjusting factor between ΔE_i and ΔV_i .

The percent *STRESS* values are always between 0 and 100. Values of *STRESS* near to zero indicate better agreement between two sets of data. In color-difference studies, a *STRESS* value exceeding 35 is typically an indicator of the poor performance of the color difference formula [18].

After visual assessments, the grey scale number (GS) for each pair was transformed to the corresponding visual color difference (ΔV) by (2):

$$\Delta V = 26.36. e^{-GS/1.659} - 0.9532 \quad (2)$$

2.2 Color difference calculation with color appearance models

The equation for calculating color difference in color appearance models is represented as (3):

$$\Delta E = \sqrt{(\Delta J')^2 + (\Delta a')^2 + (\Delta b')^2} \quad (3)$$

With $J'= ((1+100C_1)*J)/ (1+C_1*J)$; $M'=43.86* \ln (1+C_2*M)$; $a'=M' \cos(h)$ and $b'=M' \sin(h)$ and $M=C.F_L^{1/4}$

Here *J*, *M*, *h* and *C are* the lightness, colorfulness, hue angle, and chroma values respectively. $\Delta J'$, $\Delta a'$ and $\Delta b'$ are the *J'*, *a'* and *b'* differences between the samples in a pair in color appearance model. The C_1 and C_2 coefficients based upon color appearance model having values 0.007 and 0.0228 respectively.

2.3 Metamerism index

The Metamerism index (MI) is a single number index that indicates how well two samples that match under one illuminant will match under another illuminant. The MI is calculated with the *ΔL, Δa*, and *Δb* values of a sample and a standard under a reference illuminant and a test illuminant. Usually the D65 illuminant (daylight) is used as the reference illuminant. Metameric differences (*ME*) for CIELAB color space and metameric differences by visual evaluation (*MV*) can be calculated by using (4) and (5) respectively:

$$\Delta ME = \sqrt{(\Delta L^*_{65} - \Delta L^*_{50})^2 + (\Delta a^*_{65} - \Delta a^*_{50})^2 + \\ +(\Delta b^*_{65} - \Delta b^*_{50})^2} \quad (4)$$

and

$$\Delta MV = \sqrt{(\Delta V_{65} - \Delta V_{50})^2} \quad (5)$$

where: ΔL_{65}^* = lightness difference at daylight source D65; ΔL_{50}^* = lightness difference at light source D50; Δa_{65}^* = red/green difference at daylight source D65; Δa_{50}^* = red/green difference at light source D50; Δb_{65}^* = yellow/blue difference at daylight source D65; Δb_{50}^* = yellow/blue difference at light source D50; ΔV_{65} = visual color difference at daylight source D65; and ΔV_{50}= visual color difference at light source D50.

3 EXPERIMENTAL METHODS

In order to conduct a psychophysical experiment, four LEDs with six different illuminance levels from dark to very bright and eleven metameric sample pairs were selected. Figure 1 is representing the distribution of eleven metameric pairs in the *a*b** and *L*a** plane of CIELAB color space under Illuminant D65/2°.

As we can see from both figures, sample pairs 1 to 7 are having almost constant lightness value (64.5 approximately). Sample pairs 8, 9 and 11 have similar chroma and hue whereas sample pairs 3 and 10 have high chroma value with nearly similar lightness. The mean color difference of the 11 metamers calculated under standard D65/2° were 3.8 *ΔE*ab* units.

The experiment was performed in viewing cabinet, whose interior painted with Munsell N7 spectrally neutral paint and has dimensions of 42 cm (width) × 74 cm (depth) × 74 cm (height). The cabinet had four LEDs with nominal CCT 2700 K, 4000 K, 5000 K and 6500 K. The experiment was divided into six phases to investigate the color appearance of the samples at different illuminance levels from dark to very bright (50 lx, 100 lx, 200 lx, 500 lx, 1000 lx and 1500 lx). The spectral power distribution and the luminance of the different configurations was measured with a Photo Research PR-740 spectroradiometer over a plaque containing pressed Barium Sulphate white standard produced by Merck placed in the centre of the bottom surface of the lighting cabinet. The SPDs and position of all light sources at six illuminance levels in *xy* diagram are shown in Figure 2 and in Table 1.

From the position of light sources in CIE *xy* chromaticity diagram in Figure 2, it can be seen that three LEDs such as 6500 K, 4000 K and 2700 K CCTs were located on the planckian locus whereas LED at 5000 K CCT was located on the CIE daylight locus. All light sources have sufficiently high CIE Color Rendering Index (CRI) as shown in Table 1. Also it is noticeable from Table 1 that for all lighting conditions, maximum CCTs were found at 200lux.

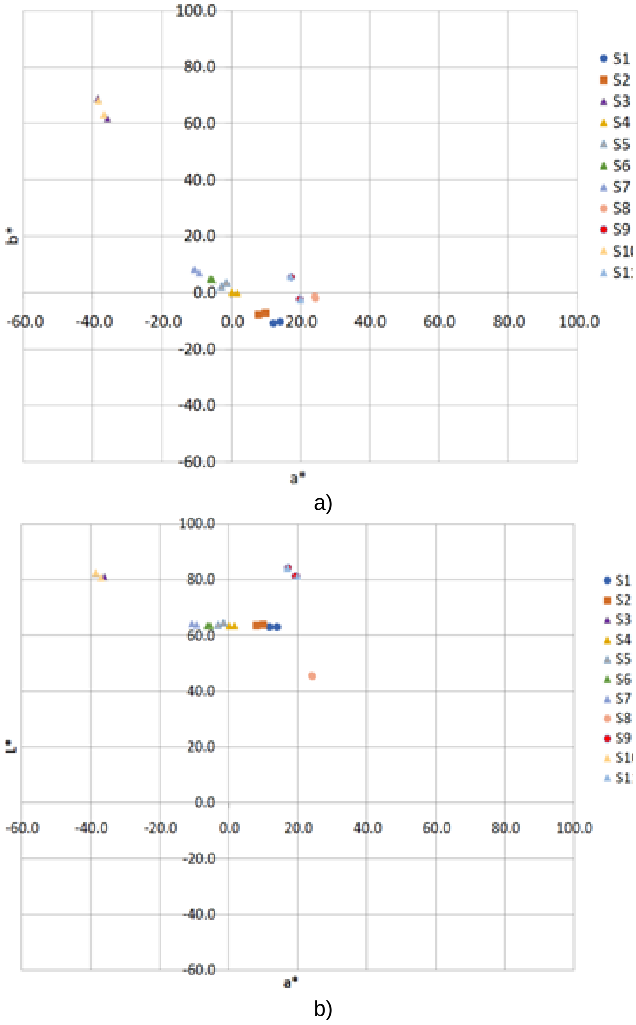


Figure 1 Distribution of 11 sample pairs on *a*b** plane (a) and *L*a** plane (b) under standard D65/2°

Seven observers between 20 and 56 years of age were asked for the experiment. All 11 metameric pairs were presented to all observers in five consecutive sessions under four light sources and six illuminance levels. The observers were asked to adapt to the mid-gray interior of the cabinet for 2 minutes after each new lighting condition and illuminance levels. After adaptation, they were provided with the grey scale and sample pairs. Due to the determination method used in the experiment; the participants were required to evaluate and compare the sample pair with grey scale. Each participant was asked to determine a closest grey scale value according to his/her own perception. The distance between observers and sample was 50 cm.

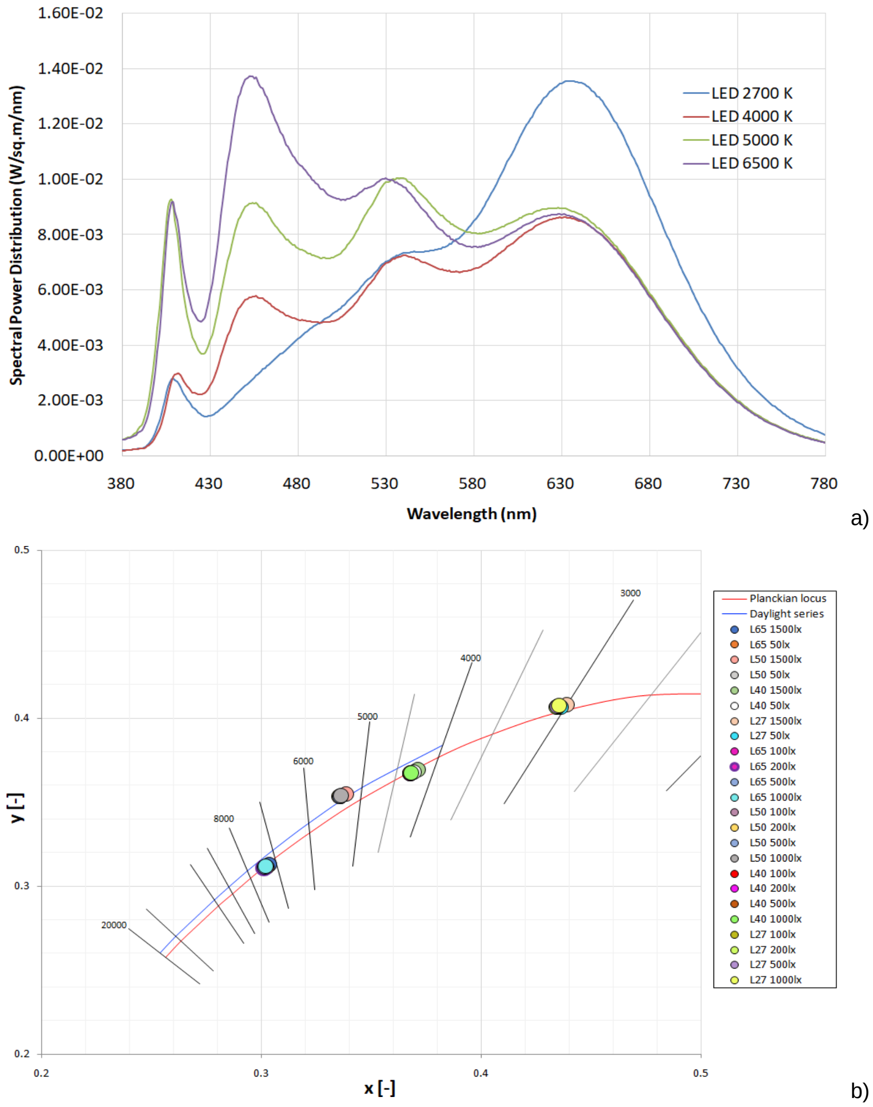


Figure 2 SPD of four LEDs (a) and position of light sources in CIE xy diagram at different conditions (b)

Table 1 The parameters of the light sources

Nominal CCT [K]	Measured values	Illuminance level [lux]					
		50	***100***	***200***	***500***	***1000***	***1500***
6500	*x*	0.3023	0.3015	0.3014	0.3016	0.3020	0.3036
	y	0.3116	0.3112	0.3111	0.3113	0.3117	0.3128
	CCT [K]	7225	7279	7406	7388	7351	7233
	Luminance [cd/m^2]	6.64	11.98	7.53	17.06	29.48	178.41
	CRI	93	93	93	93	93	93
5000	*x*	0.3361	0.3357	0.3356	0.3359	0.3363	0.3387
	y	0.3537	0.3535	0.3534	0.3536	0.354	0.3550
	CCT [K]	5254	5370	5374	5363	5345	5258
	Luminance [cd/m^2]	8.18	14.47	8.97	19.92	33.67	198.22
	CRI	98	98	98	98	98	98
4000	*x*	0.3693	0.3677	0.3676	0.3678	0.3682	0.3712
	y	0.3678	0.3671	0.3671	0.3673	0.3676	0.3693
	CCT [K]	4176	4300	4304	4299	4290	4211
	Luminance [cd/m^2]	4.82	8.53	5.33	12.19	21.18	136.23
	CRI	97	97	97	97	97	97
2700	*x*	0.4361	0.4346	0.4344	0.4347	0.4355	0.4390
	y	0.4066	0.4067	0.4068	0.4071	0.4075	0.4082
	CCT [K]	3034	3061	3064	3061	3050	2998
	Luminance [cd/m^2]	4.72	10.34	6.53	14.78	25.67	157.56
	CRI	95	95	95	95	95	95

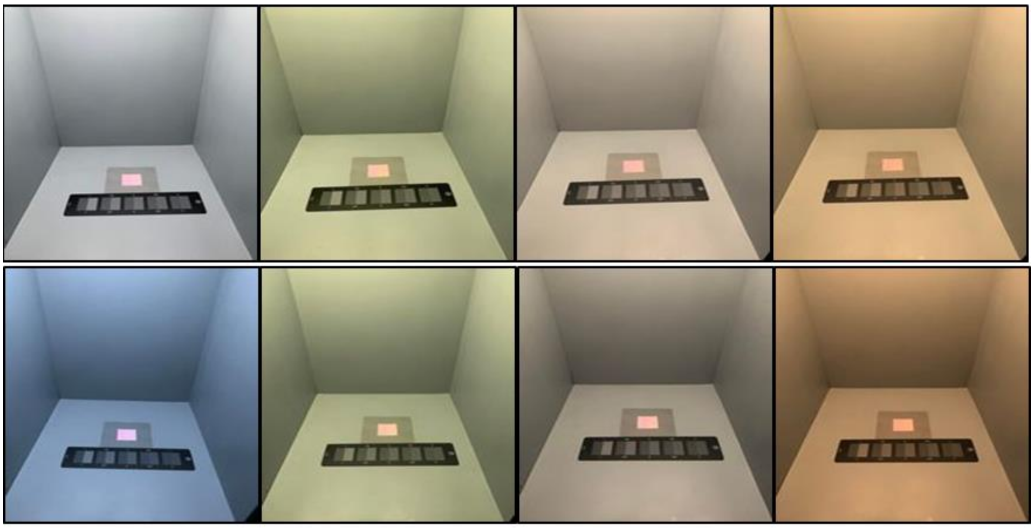


Figure 3 Position of a sample pair and the grey scale in the viewing cabinet at maximum (top) and minimum (bottom) luminance levels

The illumination: viewing geometry was always approximately 0°:45°. All the observers had normal color vision. All of the visual evaluations were conducted in a completely darkened room. In total 1200 assessment data were collected. Figure 3 is demonstrating the position of sample pair along with grey scale during visual assessment at different conditions.

4 RESULTS AND DISCUSSION

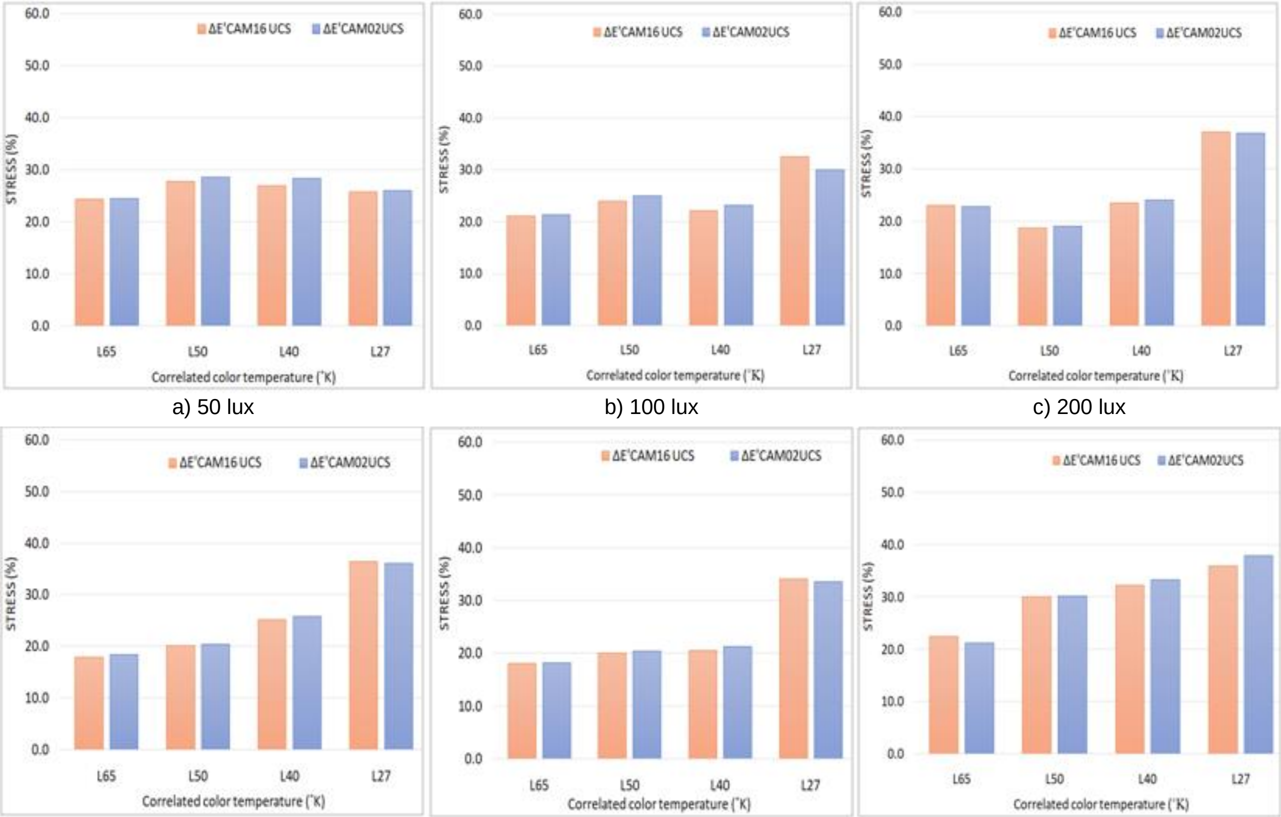
4.1 Performance of different color difference formula in terms of STRESS

Figure 4 shows the inter-observer variability in term *STRESS* as a function of the CCT for six illuminance levels: 50lx, 100lx, 200lx, 500lx, 1000lx and 1500 lx. *STRESS* and correlation coefficient values for CIELAB color difference formula are listed in Table 2.

Table 2 STRESS and correlation coefficient (COQ) for CIELAB color difference formula

Illuminance [lux]	Parameter	6500K	5000K	4000K	2700K
50	COQ	0.87	0.68	0.70	0.79
	STRESS	28.72	47.20	42.88	29.54
100	COQ	0.82	0.72	0.79	0.82
	STRESS	34.06	42.54	34.15	32.94
200	COQ	0.83	0.81	0.83	0.83
	STRESS	34.36	35.36	32.77	36.13
500	COQ	0.89	0.83	0.83	0.84
	STRESS	27.48	34.51	33.17	34.99
1000	COQ	0.86	0.83	0.85	0.85
	STRESS	30.52	34.64	28.95	32.39
1500	COQ	0.72	0.64	0.70	0.73
	STRESS	41.33	48.88	42.51	32.97

Figure 4 and Table 2 illustrate that for all CCTs and for all illuminance levels, the inter-observer variability with regard to the *STRESS* units was lowest for the CAM16-UCS models than for the CIELAB color space. CAM02-UCS also showed a better correlation compared to CIELAB. This suggests that the CIELAB estimate of the color difference at different color temperatures and illuminance levels is not reliable when compared to color appearance models. Also, CAM16-UCS has shown better performance compared to CAM02-UCS due to the associated new chromatic adaptation transformation called CAT16, which is in line with the researchers' earlier results [15]. The trend of the *STRESS* change in the color appearance models shows the influence of the chromatic adaptation of the observers during the evaluation (the observers were asked to adapt to the medium gray interior of the cabinet for 2 minutes after each new lighting condition and illuminance), but the adaptation pattern is neither specific nor known. This comparative study confirms that the variation with the CCT of the LED sources depends on the color model used [19]. Since the applicable range of von Kries-type chromatic adaptation in terms of color temperature change is unknown or there is no particular trend of the effect of the degree of adaptation contained in the chromatic adaptation transformation (CAT) matrices on the estimate [19], it is necessary to validate the results for other datasets under LED sources. It is also worth mentioning that both color appearance models have the lowest *STRESS* values at illuminance 1000lx.The lighting position at different illuminance levels for different CCTs remains almost the same and all LEDs have a very pure and sharp spectral power distribution, as can be seen in Figure 2. We consider our estimate to be reasonable, which can be justified or corrected by testing with other data sets or with multiple observers. In addition, the adaptation time of 2 minutes should be validated by future studies.



a) 50 lux b) 100 lux c) 200 lux
d) 500 lux e) 1000 lux f) 1500 lux

Figure 4 STRESS values for CAM16-UCS and CAM02-UCS at four CCTs: 6500 K (L65), 5000K (L50), 4000K (L40) and 2700K (L27) for six illuminance levels: 50-1500 lux (a-f)

4.2 Effects of illuminance on visual difference and intra-observer's variability

Figure 5 shows the visual color difference within the samples as a function of illuminance, while Table 3 shows the mean intra-observer variability using *STRESS*.

It can be seen that samples 8 and 10 were highly variable with changes in illuminance and CCTs, while samples 4, 9 and 11 showed an average change trend with changes in conditions. Samples 1, 2, 3, 5, 6 and 7 showed negligible influence of the change in conditions. It can also be seen from Figure 5 that samples showed maximum variability in the ranking of visual differences at 2700 K within observers.

As can be seen from Table 3, the variability of the intra-observer gradually decreases with increasing illuminance up to 1000 lux. At 1500 lx the variability of the observer begins to increase.

In order to validate this increasing trend, it is necessary to continue experimenting with current samples with illuminance levels above 1500 lx. From Figure 5 and Table 3, it can be concluded that in order to determine the color difference of metameric pairs, optimal conditions with regard to the colorimetric properties of the samples and the variability of the observer should be considered separately.

Table 3 Average intra-observer's variability using *STRESS*

Illuminance [lux]	STRESS			
	6500 K	*5000 K*	*4000 K*	*2700 K*
50	18.57	16.02	18.13	13.77
100	16.01	13.83	13.57	12.15
200	11.34	8.74	13.13	7.06
500	7.67	8.69	9.02	9.68
1000	6.13	6.21	10.60	5.70
1500	14.21	12.28	11.59	10.87

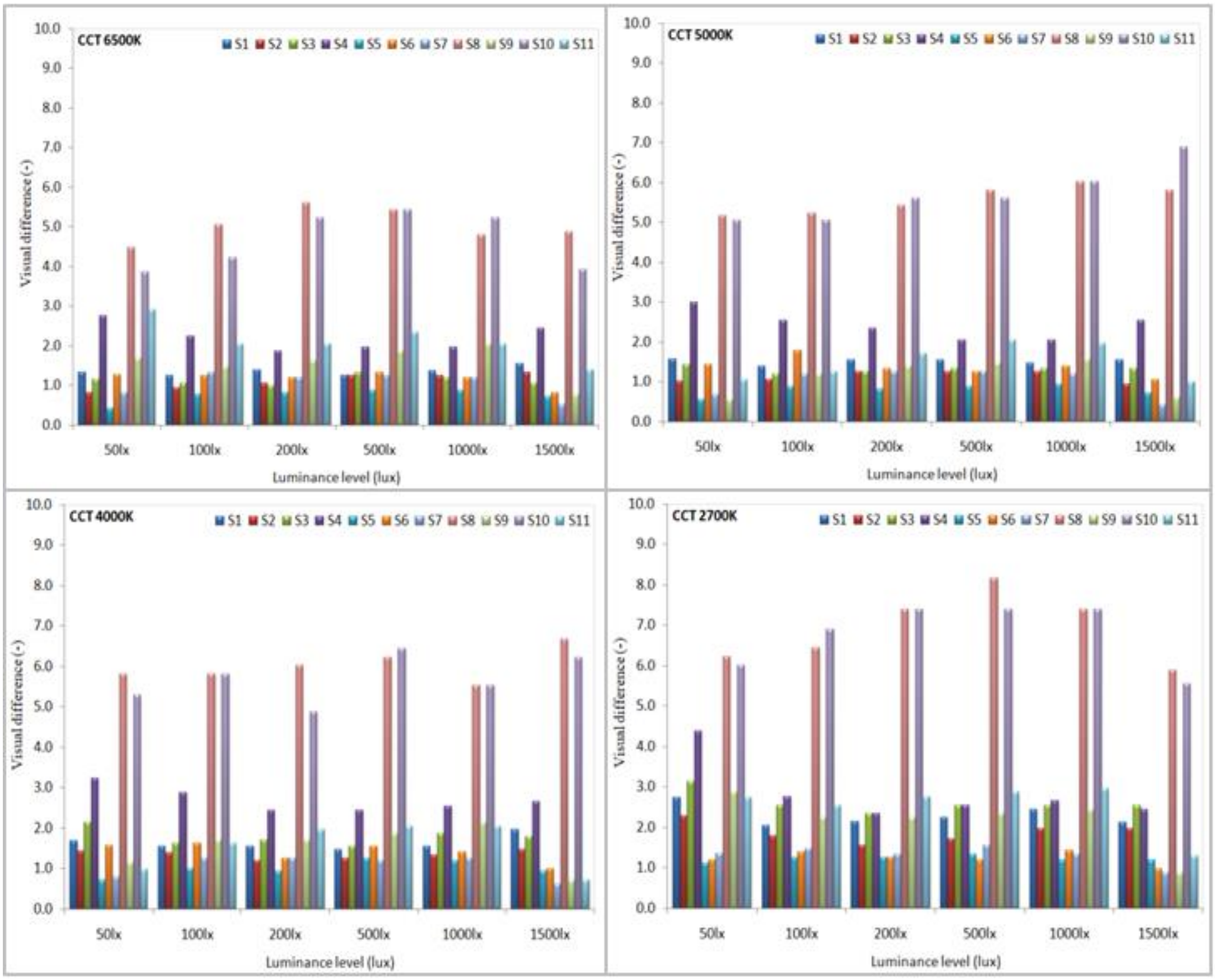


Figure 5 Visual color difference within samples at six illuminance level under four CCTs

4.3 Comparison of visual scales and colorimetric magnitudes

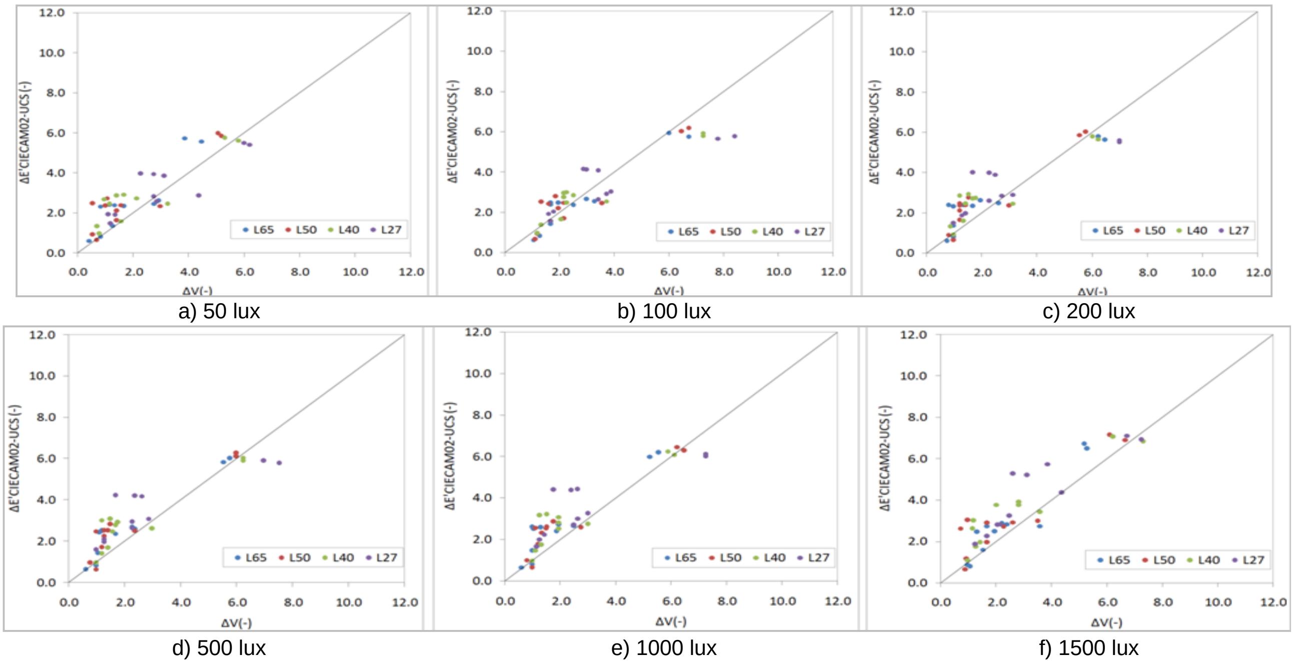
Figure 6 shows the correlation between the visual and calculated color difference in the CAM02-UCS and CAM16-UCS model as a function of the CCT for six illuminance levels: 50lx, 100lx, 200lx, 500lx, 1000lx and 1500lx. The correlation coefficient values for these two formulas are shown in Table 4.

Figure 6 (A) and (B) show that the CAM16-UCS formula performed better than the CAM02-UCS formula as expected. The central diagonal line with an inclination of 45° represents the ideal correlation between visual assessment and calculated color difference estimates. As can be seen from Table 4 together with Figure 6, the correlation increased with increasing illuminance levels. However, there is no such increase or decrease trend in CCTs. All estimates are on or near the ideal correlation line for both color appearance models.

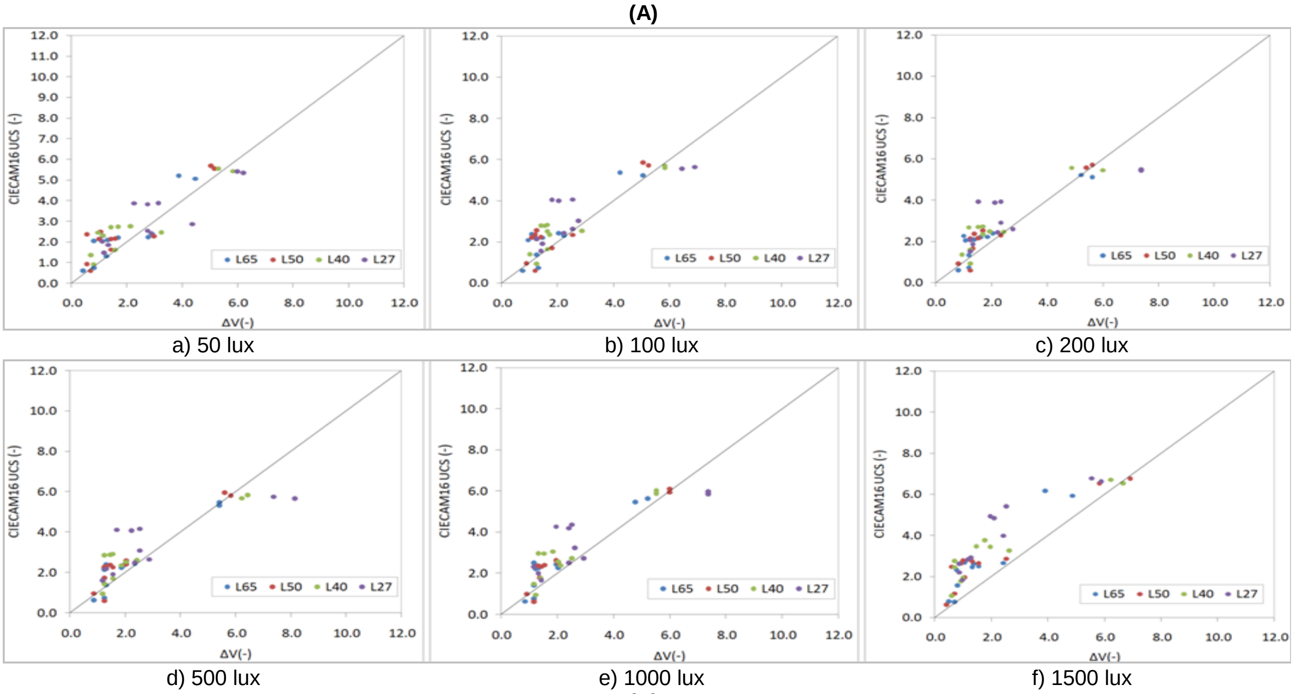
Also, the calculated sizes of the color differences examined here were typically linear with the visual scales and excellent performance was found at 1000 lx for all CCTs that support the previous results from Sections 4.1 and 4.2.

Table 4 Correlation coefficient for CAM02-UCS and CAM16-UCS

Illuminance [lux]	CAM02-UCS				CAM16-UCS			
	6500 K	*5000 K*	*4000 K*	*2700 K*	*6500 K*	*5000 K*	*4000 K*	*2700 K*
50	0.894	0.900	0.900	0.854	0.895	0.910	0.920	0.851
100	0.932	0.910	0.920	0.883	0.934	0.920	0.930	0.841
200	0.938	0.951	0.934	0.840	0.939	0.954	0.941	0.835
500	0.947	0.960	0.920	0.836	0.951	0.960	0.930	0.829
1000	0.948	0.957	0.931	0.850	0.949	0.961	0.938	0.841
1500	0.942	0.951	0.933	0.820	0.937	0.953	0.945	0.858



(A)

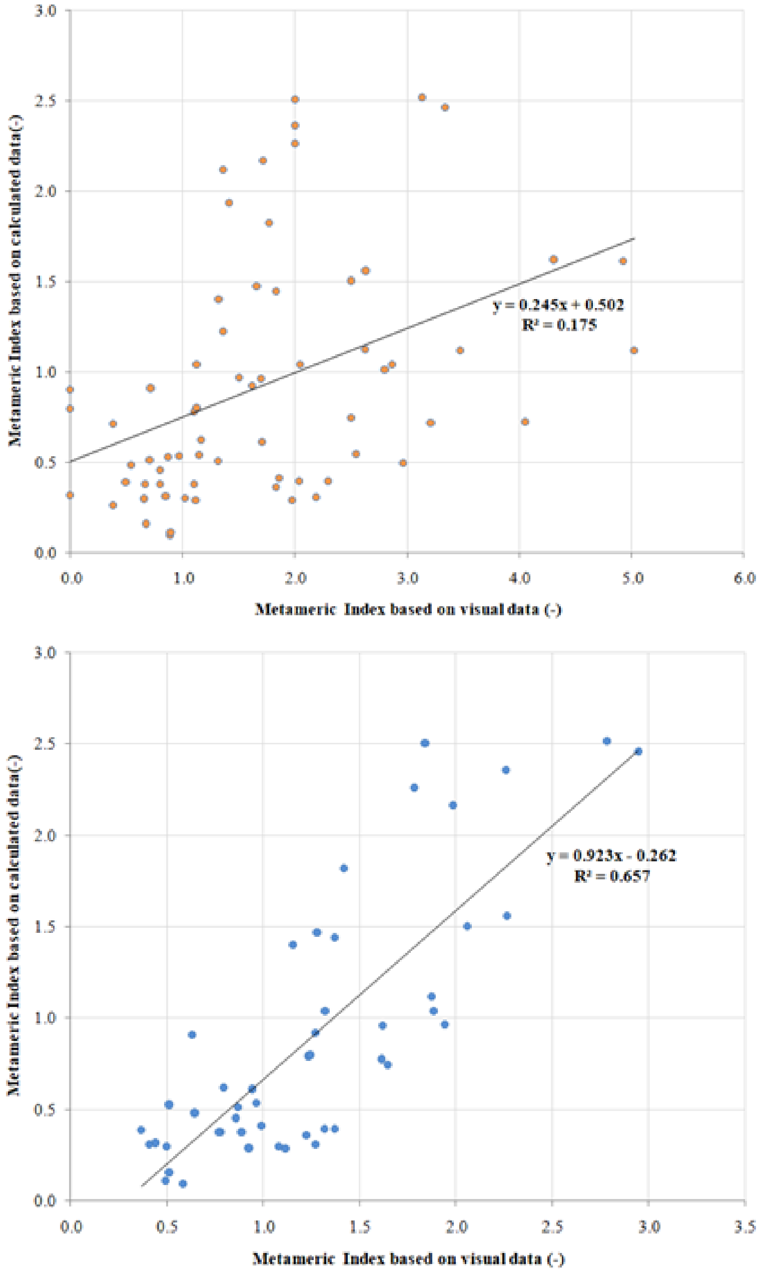


(B)

Figure 6 Comparison of visual assessment and calculated color difference in the (A) CAM02-UCS and (B) CAM16-UCS

4.4 Correlation between metameric color difference in visual evaluation and CAM02-UCS color space

Figure 7 shows the correlation between metameric color differences in visual assessment and the CAM02-UCS color space under all light sources and an illuminance of 1500 lx. Using data obtained from equations (4) and (5) we drew the graph in Figure 7. From Figure 7 it can be seen that when outlier data are taken into account, there is a very poor correlation. This may be due to difficulties in detecting low levels of metamerism of samples 8, 9 and 11 as they contain optical brighteners. However, when the data excluded outliers, the correlation increased by a unit of 0.657, as can be seen from the right graph in Figure 7. This justifies that the selected light sources and illuminance levels were correct and meaningful to detect all degrees of metamerism.



a)

b)

Figure 7 Correlation between metameric color differences based on visual data and calculated data: a) all samples and b) excluding samples visually judged to be outliers. Samples were measured under all CCTs and an illuminance of 1500 lx

5 CONCLUSION

Four different LED sources with six illuminance levels were used to assess the appearance of the eleven metameric pairs. To test the validity of our estimates, we used three color difference formulas based on the CIELAB, CAM02-UCS and CAM16-UCS models. Based on our estimates, we have proven that color appearance models are more reliable than CIELAB in evaluating color differences under different LED conditions. CAM16-UCS more accurately predicted the color difference estimates between all three formulas.

Our comparative study confirms that the variation in the estimates with the CCT and the illuminance levels of the LED sources depends on the color model used. The results also suggested that in order to determine the color difference of metameric pairs, optimal conditions regarding the colorimetric properties of the samples and the variability of the observer should be considered separately. We were also able to substantiate that our chosen light sources and illuminance levels were correct and meaningful to detect all degrees of metamerism. We noticed an increasing correlation trend with increasing illuminance.

Although the optimization or modification of the model based on estimates is not analyzed in this work, we believe that this current work shows the importance of correct lighting decisions in evaluating the color differences of metameric pairs.

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