

TECHNICKÁ UNIVERZITA V LIBERCI

FAKULTA STROJNÍ

KATEDRA VOZIDEL A MOTORŮ



**IDENTIFIKACE A PREDIKCE ZDROJŮ VIBRACÍ A
HLUKU BRZDY NA VOZIDLE**

**IDENTIFICATION AND PREDICTION SOURCE
VIBRATION AND NOISE OF DISC BRAKE ON CAR**

Doktorská disertační práce

Studijní program:	P2302 Stroje a zařízení
Studijní obor:	2302V010 Konstrukce strojů a zařízení
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Školitel:	Doc. Dr. Ing. Pavel Němeček

Liberec - 2014

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Rozsah práce	Počet stran	150
	Počet obrázků	114
	Počet tabulek	13

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ACKNOWLEDGMENT

I wish to express my deep gratitude and special thanks to my supervisor and his family, Associate Professor Pavel Němeček for all guidance, tremendous support and encouragement throughout my research.

I wish to thank Faculty of Mechanical Engineering of Technical University of Liberec and the Czech government for its provided fellowship. I also wish to thank the Nha Trang University, Vietnam for support me to study at the Czech Republic.

I wish to express many thanks to all the members of the Department of Vehicles and Engines (KVM-TUL). Especially, Mr. Voženílek Robert, Mr. Blažek Josef, Mr. Brabec Pavel and Mrs. Baudyšová Jitka for their valuable comments during my work and for their kind and friendly attitude. I also wish to thanks my colleague Dittrich Aleš and Dráb Ondřej who help me for experiment during my research.

I would like to thank Dr. Abd. Rahim Bin Abu Bakar at Faculty of Mechanical Engineering, University Teknologi Malaysia for discussions and support simulation in my thesis, Mr. Radek Holubec at the TRW Company for recommendation and provide experimental data. I also would like to thank the Department of Material Science (TUL) for helping in my experiment.

Finally, I wish to thank my family: My father – Quat who died when I was studying here, my mother – Sang, my sister's families – Nhanh & Mai, Duc & Hien, and my wife – Tram for their unfailing support and their incredible patience during the research. I would like to thank all of my friends in Vietnam and Czech Republic who encouraged me in my graduate studies and gave me a lot of help and support.

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ABSTRACT

The vibration and noise of disc brake system is among the most important priorities for today's vehicle manufacturers that are known as namely brake squeal. Overall brake squeal can be annoying to the driver, vehicle passengers, passers-by, pedestrians, etc. Therefore, identification and prediction vibration and noise of disc brake system is necessary to understand brake squeal for the purpose of vehicle designs become quieter and make the car more high quality. It is also the purpose of this thesis.

Brake squeal is very complex in both phenomenon and mechanism, it is very difficult to identify and calculation because it depend on more factors. In this work, a 3D model of disc brake system of real car was developed for simulation and prediction. By using software Abaqus with the complex eigenvalue analysis was shown that nature frequencies of disc brake components plays an important role in creating noise. Predict stable of system helps find out the unstable modes and can be forecast noise in various operation of disc brake system in real conditions. The results shown that friction coefficient at interface surface of pad and disc is most important then Young's modulus and pressure in contributing noise occurrence.

The brake squeal experiment was performed on chassis dynamometer system at TUL with full disc brake system suspension of real car by standard SAE J2521. This helps simulation is approach to various operations of disc brake system in real conditions. Beside, the experiment also confirms the simulation results as natural frequencies, mode shape and frequencies occur noise by using measurement equipments as microphone and accelerometer. Furthermore, an analysis of SEM and EDS for pads with different worn were shown that the percentage noise occurrence of pad that high worn is higher than pad with low worn. Simultaneously, it is also explained effecting characteristic of disc and pad surfaces to contribute creating noise.

Keywords: Brake squeal, Vibration, Noise, Prediction, Disc brake

ANOTACE

V dnešní době je pro výrobce automobilů jednou z hlavních priorit řešení problematiky hluku a vibrací kotoučkových brzd vznikajících při brzdění (často označováno jako „skřípění brzd“). Takto vzniklé vibrace a hluk snižují komfort posádky vozu a zároveň mohou být velmi nepříjemné pro ostatní účastníky silničního provozu (chodci, cyklisti a ostatní účastníci silničního provozu). Schopnost identifikace a predikce hluku a vibrací vznikajících při brzdění (což je také cílem této práce) je velice důležitá pro vývoj nových vozidel především z hlediska zvyšování jízdního komfortu posádky vozu.

Skřípění brzd je velmi komplexní problém a je velmi obtížné ho identifikovat (případně modelovat pomocí výpočtových rovnic), jelikož závisí na mnoha faktorech. 3D model systému kotoučové brzdy reálného vozu vytvořený v této práci byl použit pro simulace vznikajícího hluku a vibrací a následnou predikci. K tomu byl použit software ABAQUS, díky kterému bylo prokázáno, že vlastní frekvence jednotlivých komponent kotoučové brzdy mají zásadní vliv na vznik hluku a vibrací vznikajících při brzdění. Díky tomu lze určit hluk a vibrace při různých režimech brzdění za reálných podmínek. Výsledky také ukazují, že koeficient tření mezi brzdovou destičkou a brzdovým kotoučem má na vznik hluku a vibrací při brzdění větší vliv než Youngův modul a tlak.

Experimentální měření bylo provedeno na válcovém dynamometru na Katedře vozidel a motorů Technické univerzity v Liberci. K experimentu bylo použito vozidlo určené pro běžný provoz. Experiment byl proveden dle normy SAE J2521. To napomohlo simulaci dosáhnout rozdílných stavů brzdového systému odpovídajících reálným podmínkám. Provedený experiment taktéž potvrzuje výsledky simulace (vlastní frekvence, vlastní tvar). Pomocí SEM a EDS analýzy brzdových destiček různého opotřebení je ukázáno, že míra hluku vzniklého brzděním je u více opotřebované brzdové destičky vyšší než u brzdové destičky nové. Zároveň je také popsáno vzájemné ovlivňování brzdového kotouče a destiček na vytváření hluku.

Klíčová slova: Brzdy, vibrace, hluk, predikce, kotoučové brzdy

LIST OF SYMBOLS

μ		Friction coefficient
Ω	[rad/s]]	Angular velocity
P	[bar]	Pressure of piston line
Re, α		Real part
Im, ω		Imaginary part
ξ		Damping ratio
λ		Eigenvalue
E	[MPa]	Young modulus
ν		Poisson's ratio
F	[Hz]	Frequency
a	[m/s ²]	Vehicle acceleration
g	[m/s ²]	Gravitational acceleration
I_a	[kg.m.s ²]	Available inertia
P_m	[KW]	Maximum motor power
R_a	[μ m]	Arithmetic mean surface roughness
R_q	[μ m]	Root mean squeal
R_t	[μ m]	Total height of the roughness profile

INTRODUCTION

1.1 GENERAL

The problem Noise, Vibration, and Harshness (NVH) are among the most important priorities for today's vehicle manufacturers. Studies about NVH of car are including the engine, driveline, tire contact patch and road surface, wind and brake, noise from cooling fans etc. Many noise problems are due either vibration or noise transmitted through a variety of paths, and then radiated acoustically into the cabin. Statistics show that more than \$100 million is spent annually on brake NVH warranty work in North America [1, 2]. This is an excellent financial incentive to drive noise control and research activities to reduce or eliminate brake noise [1]

The problem NVH can occur over a wide range of vehicle operating condition from fractional speeds of less than a km/h to highway speeds. One of components generate unwanted vibration and noise and unpleasant noise is the brake system. The vibration and noise of brake are known as namely **brake squeal**. It affects the passenger and driver and reduces quality of the car. Therefore, brake squeal plays an important role, as brake noise has been a problem since the earliest of vehicles. Most brake squeal is produced by vibration of the brake components, especially the pads and disc is known as force-coupled excitation.

Since 1930's, Brake squeal have been studied by many researchers and companies including investigator, theories, experiments and numerical analysis method, which involves multiple disciplines such as nonlinear dynamics, contact mechanics and tribology. Most researches have focused on gaining physical insight into brake squeal phenomenon. However, disc brake squeal remains a concern of the customer's perception of quality and an elusive problem in the automotive industry, indicating that much work is still needed to further the understanding of brake squeal [3, 4]. To understand more about brake NVH, we can

consider the frequency range it covers. Figure 1.1 is a good overview of the spectrum of common brake noise and vibration issues.

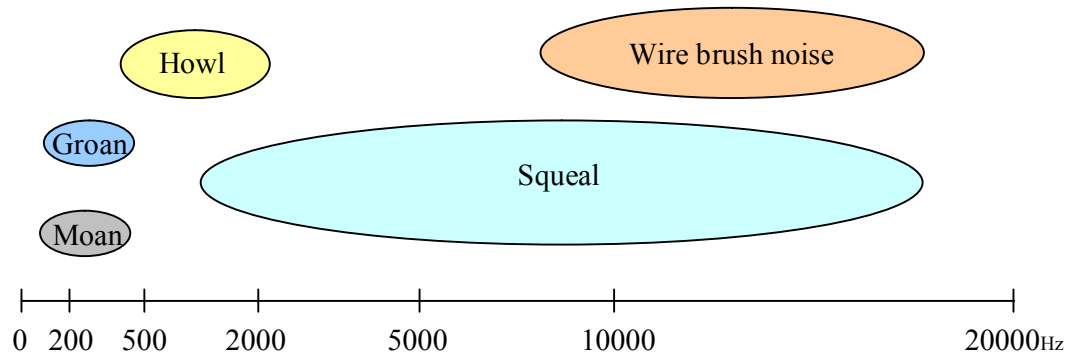


Fig 1.1 The frequency range of common brake noise [1]

There are several distinct categories of brake noise including brake moan, brake creep groan, brake judder in range low frequencies less than 1000 kHz while brake squeal in high frequencies range more than 1000 kHz.

At the lower end of the frequency range, well below the squeal frequencies, moan and groan occurs. Brake moan occurring at moderate speeds and characterised by frequency components from 100 to 400 Hz and brake creep-groan occurring at speeds less than walking pace and characterised by frequency components below 400 Hz. For the most part, moan and groan tend to happen at low speeds, generally less than 20 km/h [1, 5]. On figure 1.2 is an example of a low speed moan at waveform and figure 1.3 a frequency analysis has been performed.

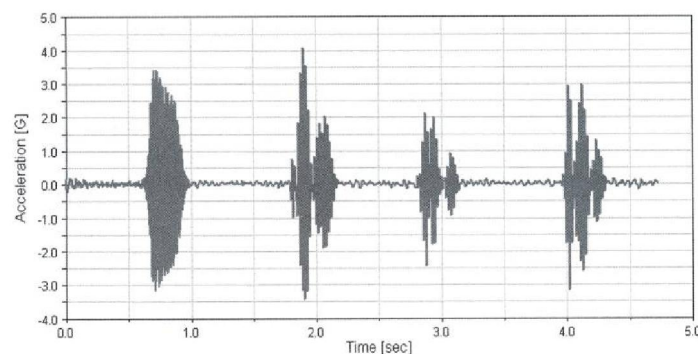


Fig 1.2 Time waveform of a moan noise [1]

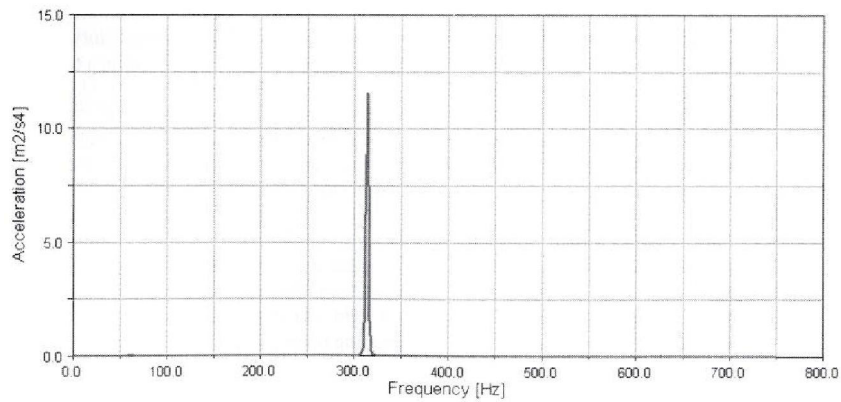


Fig 1.3 Frequency spectrum of the example moan vibration [1]

Groan is a classical example of the development of friction relaxation autovibration [6], or stick-slip [7, 8]. Brake groan arise as the vehicle is stopping, with the brake gradually released, while torque is applied to the wheel at the same time. The pressure on the brake lining decreases, the wheel torque surpassing that of the braking force. As a result, the wheel might rotate jerkily with slippage.

Squeal brake is defined as a phenomenon of dynamic instability that occurs at one or more of the natural frequencies of the brake system. Brake squeal in general is caused by friction induced, self excited and self sustained vibration in a short time via the rotating disc and is often simply called self excited vibration [4]. Unstable brake leads to vibration of structure and noise occurs. The main of noise caused by friction material, the couple of pads and rotor, beside have more components of the brake system like caliper, anchor bracket, knuckle and suspension as figure 1.4. Pads and rotor coupling has major impact on mid to high frequency squeal from 4 to 16 kHz. Low frequency squeal from 1 to 3 kHz typically involves caliper, anchor bracket, knuckle and suspension, in addition to pads and rotor [5]. Squeal is generated in brakes by either the high frequency free bending oscillations of the blocks or the rotating brake disc excited by frictional micro oscillations. High frequency squeal may also be a result of resonant phenomena in the case of forced oscillations of thin-walled elements of drag torque. The main source of squeal is the metal brake disc, the surface of which generates sound frequency waves. A lower degree of squeal in the 4-10 kHz range, occurs due to vibration in the friction lining [9].

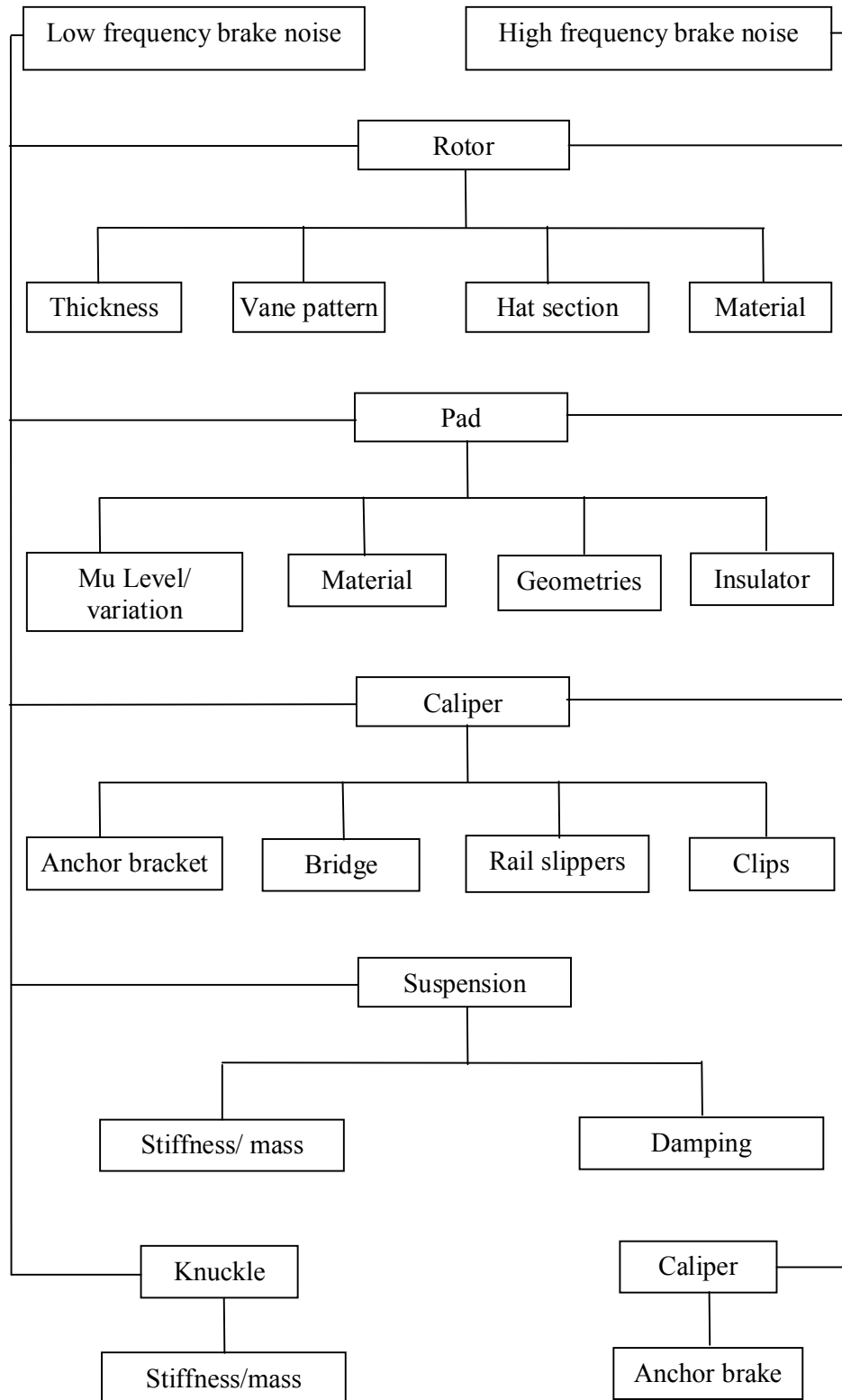


Fig 1.4 Factors influencing low and high frequency brake noise [5]

The most frequently encountered type of brake noise is squeal. Therefore, study brake squeal characteristics not only contribute to the luxury level of the car, it is an issue that affects customer satisfaction, warranty cost and brand image and company's efforts to gain a competitive edge in the automotive market [5].

1.2 AIMS OF THE RESEARCH

The aims of the research presents identification and prediction brake squeal problem by performed a simulation and experiment. The results are compared to find out unstable frequencies of system that may lead to occurrence of noise and vibration of disc brake. It includes objectives below:

- Consider contribution of the disc brake components like as rotor, pad, caliper, carrier, bolt, etc. to the brake squeal.
- Consider the effect of factors such as friction coefficient, Young's modulus (pad and disc), pressure and speed to the brake squeal.
- Analysis interface between disc and pad and its contribution to the brake squeal
- Consider effect of interface contact, damping and hardness of pad surface to the brake squeal
- Predict percentage occurrence noise on disc brake system in various operating conditions and with pads that have different worn.

Based on these objectives, unstable frequency of noise was simulated, experimental method performed and results compared. The main objectives of the research and methods used in this research include:

- Summaries background of brake squeals
- Theory about modal analysis and prediction brake squeal including a complex eigenvalue analysis.
- A simulation of modal analysis disc brake by software Abaqus was performed to find out natural frequencies and mode shape of all brake system components and disc brake assembly. After that, they was compared with the experimental results.

- Simulation stability of disc brake system in operation cases of disc brake for purpose prediction vibration and noise of disc brake.
- Predict vibration and noise of brake system by comparing a series four pads with different worn will be performed by microscopy surfaces (SEM) and Energy dispersive X-ray spectroscopy (EDS) analysis, then analysis contribution of them to brake squeal.
- Setup experimental method to measure vibration and noise of brake system following standard SAE J2521 on the real brake system of a car, through a chassis dynamometer at the brake laboratory of TUL.

Summary, the aims of research will help identify and prediction brake squeal in various operation conditions of a disc brake system on real car. Through this research provides solutions for reducing vibration and noise of disc brake.

1.3 OUTLINE OF THE THESIS

The thesis comprises researches of author from results of three international journals, and nine international conferences. The construction of thesis is divided into eight chapters as below:

Chapter 1. Introduction

Introduce the problem of vibration and noise of car, especially the noise of brake system known as namely brake squeal. Some components of brake system effect to brake squeal. Introduce objective of the dissertation and structure, content of the thesis.

Chapter 2. Review of recent researches

Presentation summaries of recent researches about brake squeal problem to see a wide range of problem. Besides, an analysis of theories and experiment in recent studies related to objectives of this dissertation.

Chapter 3. Finite element model and modal analysis

In this chapter, introduce the way to build a finite element model of the real disc brake system. Theory about modal analysis is presented for simulation modal analysis of components disc brake and disc assembly. This analysis was performed in two stages. Firstly, components of disc brake at free-free boundaries condition and secondly, for full boundaries conditions of disc assembly. The result of simulation shows natural frequencies and mode shape of the components disc brake. It will be rectified through experimental results.

Chapter 4. Simulation stability of disc brake system

In chapter four, a theory of complex eigenvalue analysis is presented. The simulation helps for prediction brake squeal based on the stability of a disc brake system. Besides a complex eigenvalue analysis to find out eigenvalue and eigenvector, unstable and stable of disc brake. This is the reason of explaining why the brake squeal occurs. In this chapter, sensitivity analysis was performed for considering changing of factors such as friction coefficient, pressure, velocity and Young modulus at different conditions, and their contribution of them to brake squeal. The simulation has done by software Abaqus 6.10.

Chapter 5. Brake Squeal Experiment

In this chapter present procedure for the experiment as equipments, prepare samples and processing experimental data. An experimental method will be described for prediction of brake squeal by standard SAE J2521 of real disc brake on a chassis dynamometer at the brake laboratory of TUL. The main is identifying and prediction vibration and noise of disc brake in some cases different about pressure, speed, and regime brake. The experiment was performed with full of a real disc brake system on the Renault Trafic car with two pads that have various worn.

Chapter 6. Effect of the characteristic of pad surfaces on brake squeal

The brake squeal occur due to vibration and noise of rotor and pads. Therefore, a study about interface of two surfaces of disc and pad was performed. The methods using for analysis are included: Scanning electron microscopy (SEM) and the Energy-dispersive X-

ray spectroscopy (EDS) for a series including four pads with 0% worn (new pad), 10% worn, 60% worn, and 70% worn will be microscopy surfaces to analysis characteristics of pad surfaces. In addition, experiments were performed to identify percentage occurrence noise due to worn of pads.

Chapter 7. Conclusion

Conclusions of thesis, discussions and recommendation for future perspectives are given in this chapter.

REVIEW OF RECENT RESEARCHES

2.1. INTRODUCTION

Brake system plays an importance of the vehicle, which are the most important safety and performance of the car. Since the vehicle was manufactured and developed. The engineers try to design for improving brake quality satisfies standards and environmental concerns. On of standards reduced noise and vibration (brake squeal) of the brake system. A number of theories have been presented to explain the mechanisms of brake squeal and apply them to the dynamics of disc brake. There are many models for brake squeal, and many prediction methods for explaining why occur noise in the brake system. More than 15 different models found in the literature [10], however brake squeal is a complicated problem, effected by a lot of factors. This has led to none of these models, which capture some features of brake, squeal well and ignore many others. In this chapter, a review of researches about brake squeal is presented including techniques and tool, methodology and analysis, models used for the simulation brake squeal.

2.2. HISTORICAL BACKGROUND RESEARCH

Brake squeal has been one of the most difficult concerns associated with automotive brake systems since their inception. Research into predicting and eliminating brake squeal has been conducted since the 1930s [4, 11]. First of all, drum brake system was studied due to their extensive use in the early automotive brake system. Disc brake system was studied because it is used extensively in modern vehicles and has become the focus of brake squeal research.

A century ago, the British engineer patented a disc brake in 1902 [10, 12] and he described a disc brake consisting of a disc of sheet metal which is rigidly connected to one of the rear wheels of the vehicle. To slow the vehicle, the disc is pinched at its edge by a pair of jaws.

This period also evidenced many of the early developments in brake technology. Some companies as Mercedes and Renault both introduced variants of the modern drum brake in 1903 [13]. The early brake designs of Lanchester and companies were substantially modified during the twentieth century. In particular, both the materials and actuation methods used have been improved. The widespread use of disc brakes on the front wheels of passenger vehicles can be partially attributed to increasingly stringent regulations throughout the world on vehicle braking. So engineers and companies try to research and development system of disc brake with new features for increasing quality of the car, one of them is studied noise and vibration of disc brake.

Since 1930s to 2012, brake squeal has been discussed in depth along with empirical remedies for squealing disc brakes. Major features of theories of disc brake squeal have been explained and examined. The existing experimental methods for finding the noise and vibration characteristic of brake allow for obtaining plausible design and material science solutions for decreasing noise and vibration in vehicle brake. However, these experimental methods are time and labor intensive, and their costs often exceed those of the standard resource bench and service tests of friction materials and units [9].

2.3. THEORIES OF BRAKE SQUEAL

There are theories have been introduced in the literature to explain the brake squeal phenomenon. Disc brake squeal occurs when a system experiences a very large amplitude vibration. There are two theories that attempt to explain why noise occurs. The first theory states that a negative friction-velocity gradient leads to stick-slip vibration, which has been identified as one of the major causes of brake squeal, and it occurs due to nonlinearity of the frictional force-velocity characteristics. The second theory believes that the high level of vibration results from geometric instabilities in the brake assembly. Both theories attribute the brake system vibration and the accompanying audible noise to variable friction forces at the lining-rotor interface [14]. For more understanding, this section will focus on this topic.

2.3.1 Background on vibration and waves of rotor

2.3.1.1 Modeling the brake rotor

Theory of modeling a disc brake assembly with the aim of understanding its vibrations and dynamics is complicated by the fact that the brake disc is rotating and the pads and caliper are fixed. On figure 2.1 is the schematic of a rotating disc and the angular coordinate θ and ϕ . The disc is assumed to be rotating counterclockwise at a constant speed Ω . In this figure, the vectors E_x , E_y and $E_z = E_x \times E_y$ form an orthonormal, right-handed basis.

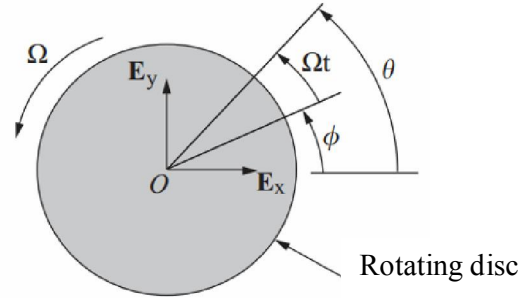


Fig 2.1 Schematic of a rotating disc and the angular coordinate θ and ϕ [10].

Some researchers modeled this rotor as a single-mode likes in [15], the beam model in [16] and the plate models see in [17-20] and the finite element models in [21-23]. To understand some of the issues associated with the vibration of the brake rotor, it is convenient to start with a discussion of a particular plate model for this body. In this model, the rotor is modelled as a uniform circular plate of thickness h , inner radius r_i and outer radius r_o . The inner radius is assumed to be fixed to a rigid axle. For any point of the disc, the displacement vector u can be represented as:

$$u = u_R(R, \phi, z, t)e_r + u_\phi(R, \phi, z, t)e_\phi + u_z(R, \phi, z, t)E_z \quad (2.1)$$

Where R , ϕ and z are a cylindrical polar co ordinate system. In Eq.(2.1) he has used the unit vectors, $e_r = \cos(\phi)E_x + \sin(\phi)E_y$, and $e_\phi = -\sin(\phi)E_x + \cos(\phi)E_y$ and assumed that E_z points along the rigid axle. The displacements u_R and u_ϕ are known as the in-plane or longitudinal displacements of the disc. The plate model assumes that the transverse out-of-plane vibration of the rotor is independent of z , $u_z = u_z(R, \phi, t)$ and is superposed on the steady state radial and tangential deformations that are induced by rotating the axle at a constant rotational speed Ω . The resulting equations of motion in [24] are

$$\rho h \frac{\partial^2 u_z}{\partial t^2} = \frac{h}{R} \frac{\partial}{\partial R} \left(\sigma_{RR} R \frac{\partial u_z}{\partial R} \right) + \frac{h \sigma_{\phi\phi}}{R^2} \frac{\partial^2 u_z}{\partial \phi^2} - \frac{E h^3}{12(1-\nu^2)} \nabla^4 u_z + q \quad (2.2)$$

Here $q = q(R, \phi, t)$ represents the contribution of external forces and moments, E is Young's modulus, ρ is the mass density, ν is the Poisson ratio, and $\nabla^4 = (\nabla^2)^2$ with

$$\nabla^2 = \frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{1}{R^2} \frac{\partial^2}{\partial \phi^2} \quad (2.3)$$

The stresses σ_{RR} and $\sigma_{\phi\phi}$ which are sometimes known as membrane stresses, are the stresses induced in the plate by the rotation. Although the rotation at angular speed Ω does induce stresses in the plane of the disc, the resulting strains are assumed to be sufficiently small that ρ , r_o , r_i , and h are negligibly different from those of the stationary disc.

According to the author of [10] the plate model has two limiting cases. First, setting σ_{RR} and $\sigma_{\phi\phi}$ to zero in Eq. (2.2) is known as ignoring the centrifugal effects. On the other hand, setting h to zero in Eq. (2.2) produces a model for the spinning disc, which ignores bending effects. Clearly, such a model for a brake rotor would be poor to say the least. We have written the equations of motion of the plate in terms of the Lagrangian co-ordinates R and ϕ . Owing to the rotation of the disc, and because the radial and tangential displacements induced by this rotation are assumed negligible, it is common to use a set of Eulerian co-ordinates r and θ to describe the equations of motion of the disc on figure 2.1. Based on the aforementioned assumptions, the spatial and material coordinates are related.

$$r \approx R, \quad \theta = \phi + \Omega t \quad (2.4)$$

Any function of R , θ and t can then be written as a different function of r , θ , and t :

$$f(R, \phi, t) = f(r, \theta - \Omega t, t) = \tilde{f}(r, \theta, t) \quad (2.5)$$

Well-known identities are then used to transform Eq. (2.2) to its Eulerian form. Most of the experimental results in the literature for the vibration of the brake rotor are expressed as functions of r and θ [10].

2.3.1.2 Standing waves and traveling wave

In model of the brake in figure 2.1, if ignoring loadings due to brake pad and centrifugal effect, the brake rotor is usually considered to be axisymmetric, we find that

$$\rho h \frac{\partial u_z}{\partial t^2} + \frac{Eh^3}{12(1-\nu^2)} \nabla^4 u_z = 0 \quad (2.6)$$

As is well known, the general solution of this equation is a doubly infinite sum of eigenmodes.

$$\begin{aligned} u_z(R, \phi, t) &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{mn}(R) \sin(\omega_{mn} t) (A_{mn} \sin(n\phi + \Psi_n)) \\ &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{mn}(R) \sin(\omega_{mn} t) (B_{mn} \sin(n\phi) + C_{mn} \cos(n\phi)) \end{aligned} \quad (2.7)$$

Where, A_{mn} , $B_{mn} = A_{mn} \sin(\Psi_n)$, $C_{mn} = A_{mn} \cos(\Psi_n)$ are constants, and ω_{mn} is the natural frequency of the mode which has n nodal diameters and m nodal circle. A nodal circle occurs at points of the disc where the function $F_{mn}(R) = 0$. A mode has m modal circles if $F_{mn}(R)$ has m non-trivial zeros, see on figure 2.2.

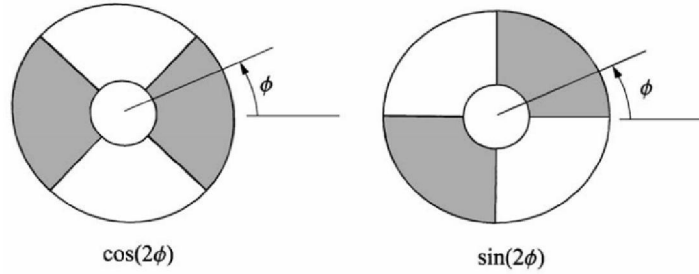


Fig 2.2 Two doublet modes of u_z for an annular disc[10].

The shaded parts of these figures have upward displacement $u_z > 0$ and have $m = 0$ nodal circles and $n = 2$ diameters. We can also write the Eulerian representations of Eq. (2.7) as

$$\begin{aligned} \tilde{u}_z(r, \theta, t) &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{mn}(r) \sin(\omega_{mn} t) (A_{mn} \sin(n\theta - n\Omega t + \Psi_n)) \\ &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_{mn}(r) \sin(\omega_{mn} t) (B_{mn} \sin(n\theta - n\Omega t) + C_{mn} \cos(n\theta - n\Omega t)) \end{aligned} \quad (2.8)$$

The both Eqs. (2.7) and (2.8) express the vibration of the plate as a linear superposition of standing waves. The solutions involving B_{mn} and C_{mn} can also be interpreted as the infinite sum of singlet modes and doublet modes. A singlet mode is a mode which has no nodal

diameters ($n=0$), while a doublet mode is a member of a pair of modes having the same frequency ω_{mn} and the same number of non-zero n nodal diameters and m nodal circles.

For the plate model of a circular disc, where $q = 0$ and the centrifugal effects are ignored, the frequencies ω_{mn} are independent of Ω . Tabulations of ω_{mn} as functions of the dimensions of the plate and its elastic properties are easily found in the literature. When the centrifugal effects are included and q due to the brake pad assemblies are incorporated, an exact solution to Eq. (2.2) is not available. However, it is common to use modes (2.7) or (2.8) in conjunction with various approximation and perturbation methods to analyze the dynamics of the disc [10, 25]. For axisymmetric discs, a topic of considerable interest is how the natural frequencies of the doublet modes might split apart when the symmetry of the disc is broken see in ref [25-28]. This splitting can occur either when a $q \neq 0$ is applied to the disc, the disc is rotated or when certain features, such as bolt assemblies or geometric imperfections, destroy the rotational symmetry of the disc.

In ref [29] the author pointed out that either of the solutions in Eq.(2.7) and Eq. (2.8) can also be considered as the superposition of travelling waves. To see this, one uses trigonometric identities to find that

$$2 \sin(\omega_{mn} t) \sin(n\theta + \Psi_n) = \sin(n\theta - \omega_{mn} t + \Psi_n) + \sin(n\theta + \omega_{mn} t + \Psi_n) \quad (2.9)$$

$$2 \sin(\omega_{mn} t) \sin(n\theta - n\Omega t + \Psi_n) = \sin(n\theta - (\omega_{mn} + n\Omega)t + \Psi_n) + \sin(n\theta + (\omega_{mn} - n\Omega)t + \Psi_n)$$

In other words, each of the eigenmodes in Eqs. (2.7) and (2.8) can be considered as the sum of two travelling waves. Viewing the motion of the disc as the superposition of travelling waves has the advantage that one immediately perceives the symmetry breaking effect of the rotation of the disc. It should be noted from Eq. (2.9) that backward traveling waves when viewed by an observer fixed in space, will, for the same natural frequency ω_{mn} , travel slower than forward traveling waves [10].

2.3.2 Models of disc brake squeal

To find out the solution for the disc squeal problem, a number of models about 16 models were used for explaining. The first model for analysis vibration and noise of brake squeal was presented by Jarvis and Mills in 1963 [10]. This analysis used a model of three degrees of freedom (DOF) subject to a single holonomic constrain, which a sprag-slip theory. Since

1963, researchers have tried to increase more DOF of model and combine more complex friction model. El-Butch & Ibrahim in 1999 consider seven DOF multi-body model of brake system including disc, pad, caliper and piston, where the first three components have both translational and rotational DOF, whereas the latter only translational DOF. Equations of motion are obtained using the Lagrangian approach with generalized forces to include follower forces, which are the only potential source of the instability in the model [30]. Also there are some researchers who have bucked the trend towards complexity and developed simple model. To have more review of these models, it is convenient for summarize a list of models including: The model of Jarvis and Mills (1963); The pin on disc system of Earles and coworkers (1971-1987); North's models (1972,1976); Millner's model (1978); Murakami, Tsunada and Kitamura's with seven DOF (1984); Lies's finite element model (1989); Matsui and Murakami with model of three DOF (1992); Brooks and his co-worker with model of twelve DOF (1993); Nishiwaki's models for brake noise (1993); Chargin and Dunne and Herting's finite element model (1997); Hulten and Flint's model (1999); El-Butch & Ibrahim with model of three DOF (1999); Nack's finite elements model (1999-2000); Chowdhary and Bajaj and Krousgrill's model (2001); McDaniel, Li, Moore and Chen's with model of three DOF (2001); Rudolph and Popp's with model of fourteen DOF (2001); Vadems Milovs study a model of two DOF (circumferential and transverse) to describe the rotating mass-spring-damper system of the disc brake (2003); Model of Finite element is developed in recent years like as Jame (2003); Abu-Bakar and Ouyang (2004, 2005, 2008); Mario Triches Junior and co-worker (2007); Franck Renaud (2012). Because a detail of the entire model above is not possible, this section contains only reviews of some models that have relationship and relates directly to my thesis.

2.3.2.1 The Model of Jarvis

To understand noise and vibration of a disc brake, Jarvis and Mills have performed an experiment (see in figure 2.3) to develop a theory, called sprag-slip. In sprag-slip, the oscillations occur due to the constrained interaction of various DOF in the system. That mean, unstable oscillation in the system could occur even with constant friction coefficient. In figure 2.3, he developed a model that has a rigid and a massless rod that pivoted at one end at O . An external force L at its free end and contacting a rigid is moving plane. At the point contact between the plane and the rod, a friction force F_f is presented. To illustrate this point, Spurr shows that equilibrium for the system above by equations.

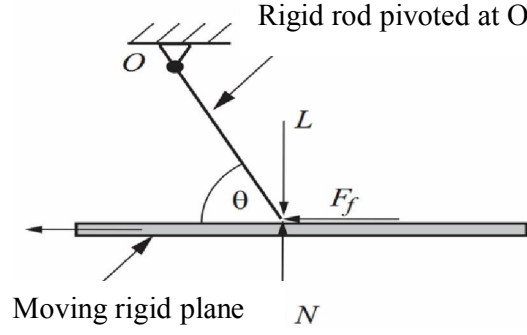


Fig 2.3 Schematic diagram of a sprag-slip theory [31]

$$F_f = \mu_k N$$

$$N = \frac{L}{(1 - u_k \tan(\theta))} , F_f = \frac{\mu_k L}{(1 - u_k \tan(\theta))} \quad (2.10)$$

In equation above, show that if $\theta \rightarrow \tan^{-1}(1/\mu_k)$, then $F_f \rightarrow \infty$. This critical case is what he termed *spragging*. The detail of this theory can be seen in reference [10, 31]. According to the author, he found that the instability of the system depended upon the friction coefficient, the magnitude of the friction force F_f and the normal force N .

As mention above, Jarvis and Mills have developed Spurr's idea, have progressively refined his approach by bringing the system closer and closer to a real brake. Because, Spurr's model is still far away from a real brake. A model is considered in figure 2.4.

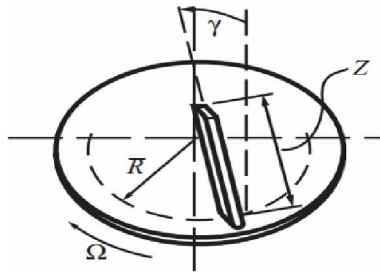


Fig 2.4 Schematic of Jarvis and Mill's cantilevered beam on disc [32]

To experiment Jarvis and Mills considered a cantilevered beam of length Z whose free end contacted the face of a rotating disc at a radius $r = \bar{R}$. Applying Lagrange's equations and using the appropriate normal functions for cantilevers and discs as generalized coordinates, their non-linear analysis using the method of slowly varying parameters showed that a

decreasing friction velocity relationship, $d\mu_k/dv_s=0$, had a negligible effect in generating unstable vibrations in such a system. In particular, the transverse vibration of the disc of disc is assumed to be.

$$W_z(r, \theta, t) = f(r)(-q_1(t)\cos(n\theta) + q_2(t)\sin(n\theta)) \quad (2.11)$$

While some parameters u_k and v_s are removed the equations of motion simplified considerably, the detail solution of this equation can be seen in reference [10, 32]

2.3.2.2 Models of Pin-on- disc

The models presented here as a part of the family of pin-on-disc models. From the modeling point of view, this means that the contact zone is so small that it can be assumed to be a point. Pin-on-disc systems also have the advantage of bringing to the fore the importance of the angle between the pin and the disc for the stability of the system. The first Earles and Lee introduce a model of pin on disc as in figure 2.5.

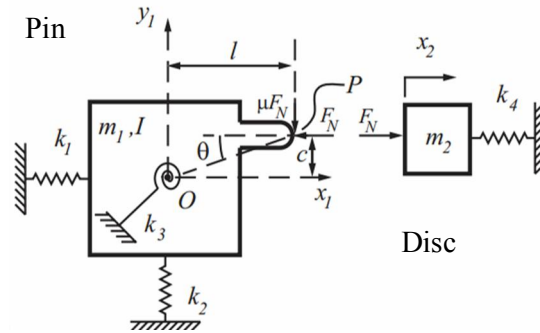


Fig 2.5 Model of Pin-disc oscillatory system [10, 33, 34]

The pin m_1 and disc m_2 are meant to represent the pad and rotor in a disc brake. The kinematic constraint is provided by the assumed persistent contact between the pin and disc at point P. From this model, a linear stability analysis was performed on lumped parameter models of pin-disc systems in order to find the flutter boundaries in parameter area. The model was expressed in equation of the form

$$M\ddot{x} + D\dot{x} + Kx = 0 \quad (2.12)$$

The result of this analysis indicated that at less one pin in the system must have a configuration that causes it to dig into the disc for squeal to occur. By following the model

in figure 2.5, Earles and Lee were able to find an equation for the eigenvalue λ of equilibrium in the form.

$$a_2 \lambda^2 + a_1 \lambda + a_0 = 0 \quad (2.13)$$

It is easy to see that the equilibrium state is unstable if any of the coefficients of Eq 2.13 are negative [10, 35]. In this work, they also concluded that, the stability of equilibrium in a model for a disc brake depends on μ_k and an angle characteristic of the contact geometry.

In 1971 Earles and Soar successively modelled a pin on disc system, talking about pin with considering its compressive and torsion. The braking system is eventually modelled as a two DOF system, as shown in figure 2.6. The top mass spring system is a disc. The bottom mass is the pin. They set up a foundation of finite element method in the early days by ways pin was sliced into five elements, and the system is divided into two linear systems, pin & disc and link by contact point.

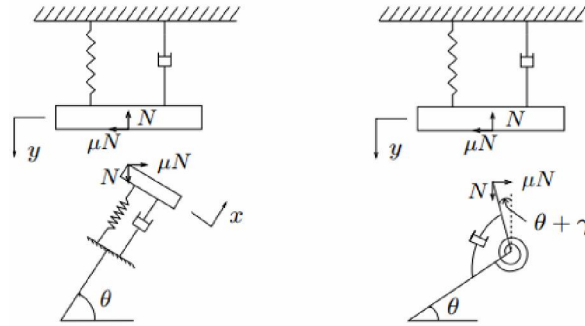


Fig 2.6 The compressive model (left) and torsional model for pin [36].

Continue with the pin on disc models, North (1972) followed a similar way, but started with a model of eight DOF shown in figure 2.7 [36]. The model is including four rigid bodies as pad, disc and caliper. Each of them allowed moving in direction y and rotation with eight DOF. From the model, he considers the stability of brake squeals.

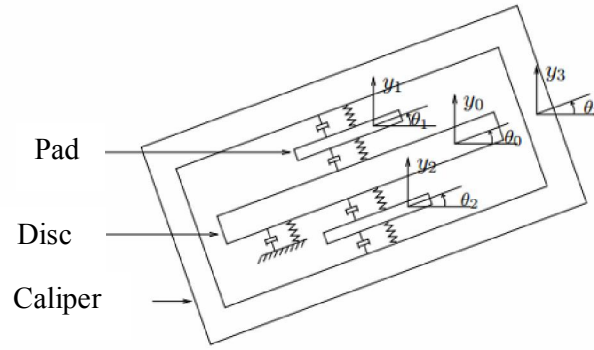


Fig 2.7 Model of North for disc brake assembly with eight DOF [36].

The next model of pin on disc of D. Philippe has presented in 2002. A model with two DOF in figure 2.8 is taken into account, the transverse motion of the disc and the fundamental bending mode of the pin. This kind of modeling is sometimes called a lumped parameter model because it proceeds by replacing the details of a mode by an equivalent discrete mass-spring-damper system. The friction law used for the coupling between the friction force F and the normal force N at the contact point is represented in figure 2.8. It is Coulomb's law with a constant coefficient of friction [36].

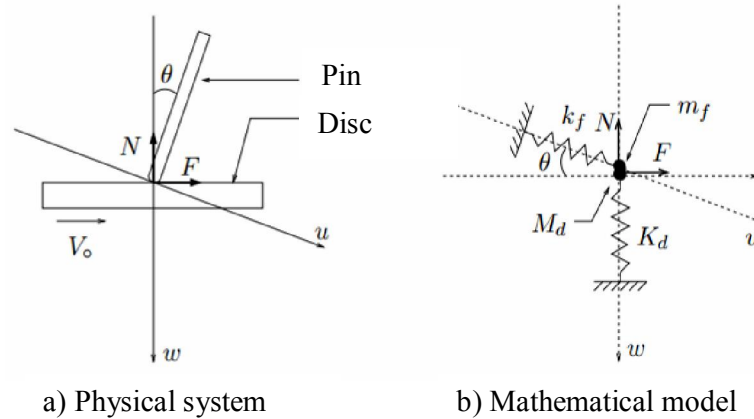


Fig 2.8 Model Pin on Disc two DOF [36]

In this section, an inclined pin sliding on a disc was modelled as a model two DOF system and will be calculated in linear and nonlinear analysis with cases damping and without damping. The result leads to in relation to the brakes, the most interesting case is to investigate the stability of the pin originally in steady sliding. It was shown that this motion

could become unstable for certain values of the pin angle. Whenever the pin tangential velocity reaches that of the disc, sticking occurs and for this model, the system becomes locked in that configuration [36].

Other models of D. Philippe with three DOF are presented in figure 2.9. Where u and w are respectively the transverse displacements of the pin and the disc and v is the longitudinal displacement for the pin. When the stick occurs, the set of equations falls within the scope of linear theory and have formed.

$$\begin{aligned} m_f \ddot{u} + c_f \dot{u} + k_f u &= -N' (\sin \theta - \mu_0 \cos \theta) + \mu_0 \cos \theta N_0 \\ m_c \ddot{v} + c_c \dot{v} + k_c v &= -N' (\cos \theta + \mu_0 \sin \theta) - \mu_0 \cos \theta N_0 \\ M_d \ddot{w} + C_d \dot{w} + K_d w &= N' \end{aligned} \quad (2.14)$$

The permanent contact condition becomes

$$w = u \sin \theta + v \cos \theta \quad (2.15)$$

From these equations, an analysis of linear and nonlinear was performed to find out the stability of the structure through the real part of the roots of the determinant equation:

$$a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0 = 0 \quad (2.16)$$

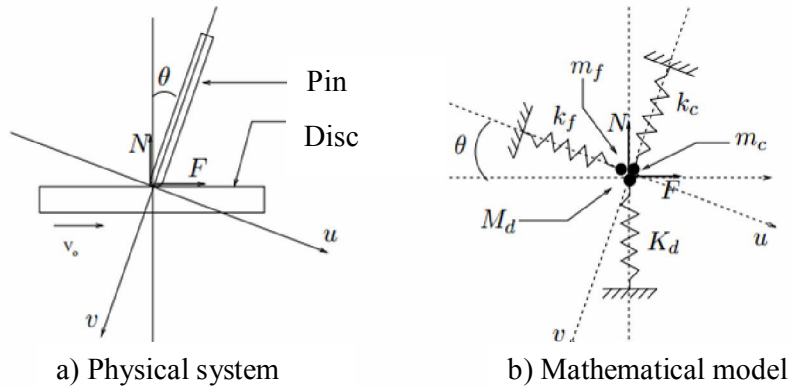


Fig 2.9 Model pin on disc three DOF [36]

The result of analysis show that when the frequency of the compressive mode is above that of the disc mode, the system is greatly stabilized as none of the previous regions of instability could be observed. When the compressive modal frequency lies between the pin and disc bending frequencies, the system again behaved in a completely different way from

what could be observed without the compressive mode. Instability can then arise for a narrow range of negative angles, close to -90° [36].

2.3.2.3 Mass-Spring-Damper Models

These models are considered to investigate the basic mechanisms of an instability that is one of the causes of disc brake noise. The damping of disc and pad are of equal importance as it reduces the chances of instability. It was also used to determine the conditions necessary to prevent this instability. Model assessments how are parameters as initial velocity, the dynamic coefficient of friction, damping of the disc and pad and the magnitude of the force contribute to the stability of the disc brake system. A number of models in this form are presented below.

To more explain about the brake squeal, North [37] considers the two DOF with the disc is modelled as a rigid body of mass. The disc is assumed to be sandwiched between two layers of friction material, see in figure 2.10.

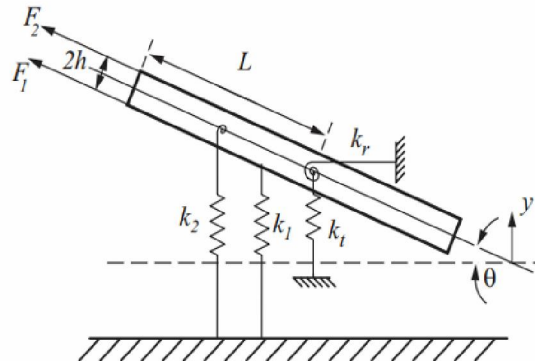


Fig 2.10 Model two DOF of a disc brake system [37]

This model features a rigid body model of the brake rotor and two friction forces F_1 and F_2 for the friction forces between the pad and rotor. The equation governing the linear vibration of the disc brake assembly is

$$M\ddot{x} + Kx = 0 \quad (2.17)$$

The author has established a criterion for instability of equilibrium and has a result in which squeal propensity increases with increasing friction coefficient. This is a part of North

theory that some researchers continue developing with multi degree of freedom model for disc brake systems. Therefore, equations of motion are rewrite in damping as

$$M\ddot{x} + C\dot{x} + Kx = 0 \quad (2.18)$$

For example, North [38] developed more complicated model for the disc brake assembly with eight DOF or model of Brooks [39] with twelve DOF. Another model of Rudolph and Popp [40, 41] includes 18 coupling elements between the rigid bodies. The forces and moments in these couplings depend on the relative displacement and relative velocity of two connected rigid bodies, as given in figure 2.11.

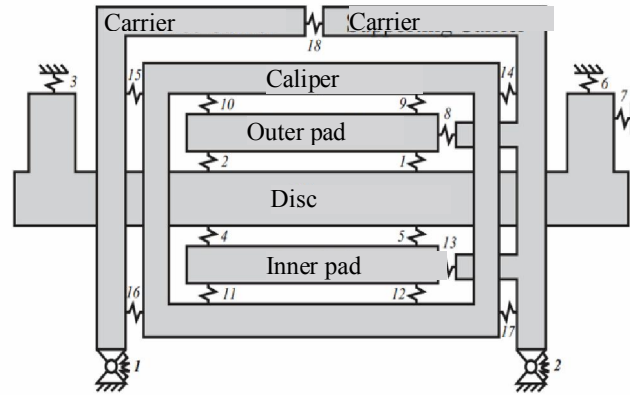


Fig 2.11 Model of Rudolph and Popp with 14 DOF [10, 40, 41]

Vadims Melons (2003) considered a model with two DOF including circumferential and transverse to describe the rotating mass-spring-damper system of the disc brake. Full of model can be seen in figure 2.12.

From this model, he writes the equations in circumferential and transverse direction, with system's equations for circumferential direction of motion

$$\begin{aligned} m_d \ddot{x}_d &= -k_{dx} x_d - c_{dx} \dot{x}_d - F_f \\ m_p \ddot{x}_p &= -k_{px} x_p - c_{px} \dot{x}_p + F_f \end{aligned} \quad (2.19)$$

and system's equation for transverse direction of motion

$$\begin{aligned} m_d \ddot{y}_d &= -k_{dy} y_d - c_{dy} \dot{y}_d - F_y \\ m_p \ddot{y}_p &= F_y - N \end{aligned} \quad (2.20)$$

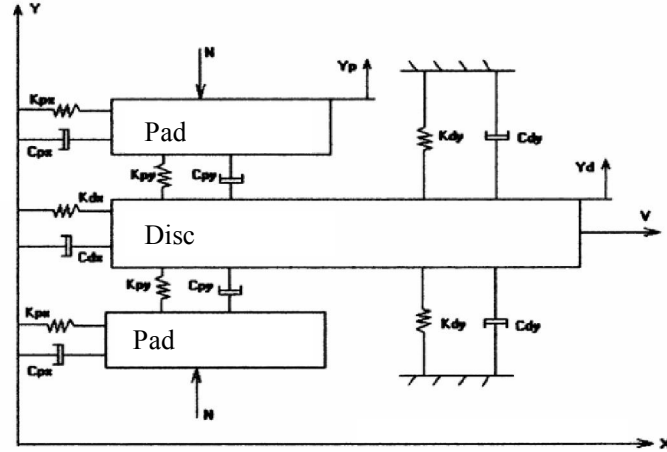


Fig 2.12 Model with two DOF of the disc brake system [30]

Where m is mass, c is damping and k is the stiffness of brake system, detail of model and solution for this case, can be found in reference [30]. The result shows simulation variable parameters such as wheel velocity, pad pressure force, damping and friction coefficients on disc squeal development.

In 2005, Nitesh Ahuja with a mathematical model with four DOF was presented, as seen in figure 2.13. For this model, an analysis stability of the disc brake to determine under what parametric condition the system becomes unstable.

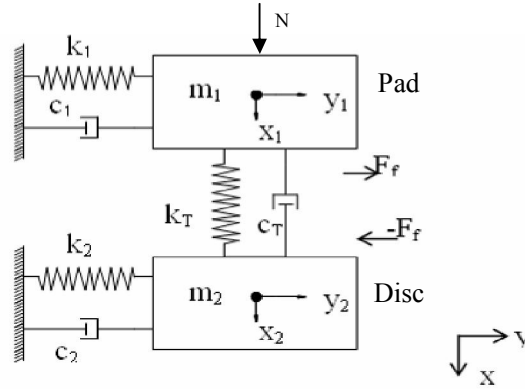


Fig 2.13 Model of four DOF [42]

The equations of motion are written in the form.

$$\begin{aligned}
m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 &= F_f \\
m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 &= -F_f \\
m_1 \ddot{y}_1 + c_T (\dot{y}_1 - \dot{y}_2) + k_T (y_1 - y_2) + k_T (y_1 - y_2)^3 &= N \\
m_2 \ddot{y}_2 - c_T (\dot{y}_1 - \dot{y}_2) - k_T (y_1 - y_2) - k_T (y_1 - y_2)^3 &= 0
\end{aligned} \tag{2.21}$$

From the equation, the author solves the coupled differential equation is known as Runge-Kutta's method, detail can be seen in [42] and a simulation with various parameters are used to predict the behavior. For this study, various simulations were performed to obtain the stability and instability under various conditions. The numerical analysis suggests that when the natural frequency of disc and pad are close, then the brake is more likely to be noisy. A non-linear analysis of the coupled equation was conducted to investigate the dynamical behavior of model for various system and friction parameters.

2.3.2.4 Finite Element Models (FE Model)

With the advances in the computer technology recent years, more complicated and complete FE models can be easily built as well as analysis in simulation time. Advantage in numerical method and algorithm much help engineers and researchers to obtain more reliable and accurate representation of stable and unstable brake system also brake squeal.

In 2005, Nitesh Ahuja used finite difference discretization for an annular disc brake. He divides the region into smaller regions and assigning each region a reference index will be provide very accurate results. Figure 2.14 shows a nodal point at the center of a small shaded region that has the index (i, j) and its neighbouring points have node indices $(i+1, j)$, $(i-1, j)$, $(i, j+1)$, $(i, j-1)$, $(i+1, j+1)$ and so on. These lines and circles are called grid lines and their intersections are referred to as mesh points of the grid or nodal points. The distance between two nodal points is the grid size. In some cases, the accuracy of the solution can be improved by reducing the mesh size [42]. From the equation, the free vibration of the disc without damping can be expanded using the central difference method. Detail of calculation see in reference [42], the result shows that natural frequencies and mode shapes of a disc brake obtained by the finite difference method. The author also simulated a couple disc and pad of disc brake system by software Ansys for comparing results.

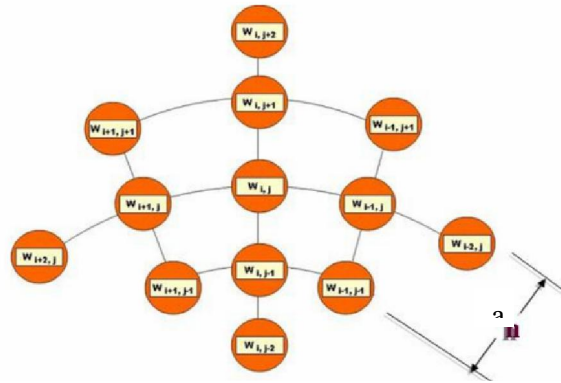


Fig 2.14 Schematic of grid points of disc rotor [42]

In recent years, the finite element method (FEM) play an importance tool for modelling the disc brake systems and providing new insights into the problem of brake squeal. This method makes accuracy of solution through generating finite dimensional approximations for governing equations motion of the components of the brake system, see in left of figure 2.13 [43]. This method was used by authors in reference [3, 43-54]. The most common of this method is to compute matrices M and K of the model, then extract natural frequencies and mode shape of the structure disc brake. Finite element analysis (FEA) was used to incorporate the sliding friction between the rotor and pad surfaces and the dynamic response obtained through this analysis has important implications. The results obtained through this analysis identify the locations of maximum deflection in a contact pressure of brake system, see in right of figure 2.15 [55] .

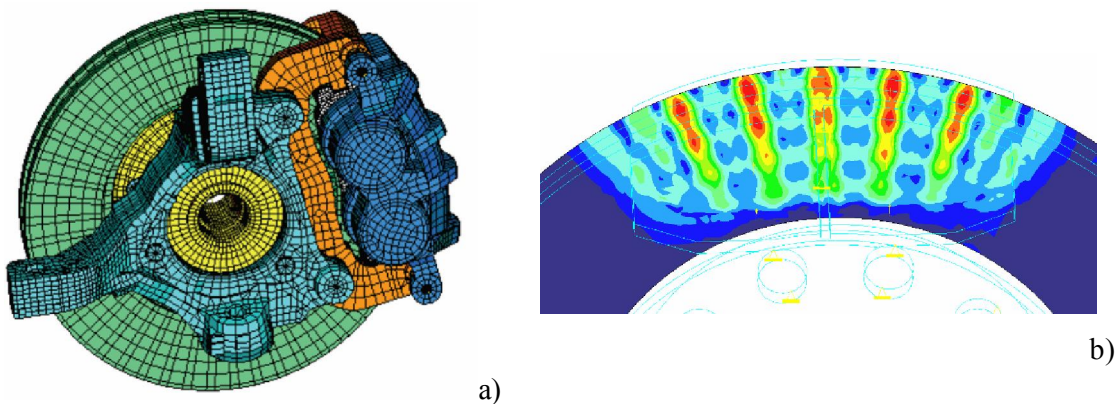


Fig 2.15 Mesh of structure of disc brake system by FEM and contact pressure of disc [56]

Besides, FEA can be contributed in computing complex eigenvalue approach to squeal analysis. The author Cao in [57] performed analysis the unstable frequencies of the disc brake, that are obtained from a linear, complex value and asymmetric eigenvalue formulation established. In 2007, Liu and coworker presented a new functionality of ABAQUS/Standard, which allows a nonlinear analysis prior to a complex eigenvalue extraction in order to study the stability of the brake systems. It was used to analyze the disc brake squeal, see in reference [49]. An attempt is made to investigate the effects of system parameters, such as the hydraulic pressure, the rotational velocity of the disc, the friction coefficient of the contact interactions between the pads and the disc, the stiffness of the disc, and the stiffness of the back plates of the pads, on the disc squeal. The simulation results show that significant pad bending vibration may be responsible for the disc brake squeal. In 2009 Nouby [47] used a combined approach of complex eigenvalue analysis and design of experiments to study the disc brake squeal. Complex eigenvalue analysis has been widely used to predict unstable frequencies in the brake system models, see more in references [45, 58-60]. The results of finite element model are correlated with experimental modal test.

All most temperature and wear analysis are performed by FEM on software as Nastran, Abaqus, Ansys,.. Abu-Bakar (2008) in [61] presented the effecting of wear and temperature on contact pressure distributions. In order to determine the temperature distribution in a medium, it is necessary to solve the appropriate form of heat transfer equation. To express the heat transfer in the disc brake model, several thermal boundary conditions and initial conditions need to be defined as the interface between the disc and brake pads. Heat is generated due to sliding friction. The same author in [62], he used a numerical analysis of disc brake squeal that considering temperature dependent with friction coefficient. The numerical results indicate that some strong unstable frequencies and noise indices are speed-dependent. Other author also used FEM for simulation the effecting of damping to the stability of the brake system as Guillaume Fritz in reference [63].

2.4. EXPERIMENT BRAKE SQUEAL

Since the researchers beginning to develop brake squeal theories for different models, they try to do experiments correlated with experimental test. These theories were starting in 1930's while instrumentation has been available for brake noise measurement since the 1940s. Until now, It took more than five decades to develop useful and reliable

measurement practices [1]. To discuss experimental studies on brake squeal, it is convenient to first consider work prior to the mid-1970s. The working of Felske [64] in 1978 used Dual Pulsed Holographic Interferometry (DPHI) to examine the modes of squealing brakes. DPHI was continued developing until the other optical technique of Electronic Speckle Pattern Interferometry (ESPI) became popular in the late 1990s [65, 66]. While an increased number of experimental researches about tribology of disc and pad in 1980's [10]. In year recently, development of equipments and tools for vibration measurement and noise field are still strong as well. It is adapted for complex test and obtaining high accuracy. A summary of the conclusions reached in the experimental literature and discussions are included: i) identify characteristic of vibration and noise of disc brake system, ii) Tribology friction and characteristic of couple friction between surfaces of disc and pad.

2.4.1 Identify characteristic of brake squeal

Characteristic of disc squeal is complex and dependent by more factors, however, as discuss previous parts the noise occur when the structure of the system unstable. Therefore, normally experimental test focuses on identifying natural frequencies, mode shape in different conditions. Besides, bench tests and dynamometer test, road test also was performed to compare with the theoretical results, see more in reference [47, 58, 67-72]. Most components of the brake system are used for experiment, including rotor and pad. It is most important to evaluate the components of the braking system. That generates noise and vibration and to determine their spectra, frequencies and level. It is also critical to understand the effect of temperature, pressure, velocity, wear and other factors on vibroacoustic activity of the friction pair in brake units [9].

Fosberry and Holubeck's [73, 74] tested on a simple disc brake system and reported characteristic of friction have a relationship with brake squeal. This work indicated that the rotor is the resonant member, vibrating in transverse modes with diametric nodes while the centers of the pads are located near anti-nodal points of this vibration [10]. Next experiments of Spurr [75], Earles and coworkers [76-78], North [79], where the North's experimental result was correlated with model theory. For some models have the complex geometries as full disc brake system assembly, the researchers as Millner [80], Kung [22], Felske [64] used DPHI and ESPI technique for capture three dimensional displacement

fields of the imaged object, these techniques can measure for individual mechanism components of the brake system.

In the years 1980-1990s, researchers used piezoelectric accelerometers for capture vibration and mode shape of structure of brake system components. Murakami and co-worker [81] examined squealing disc brake using DPHI and combine with piezoelectric accelerometers, which mounted on several components of the braking system. The experimental result showed the amount of squeal increased when the natural frequencies of the pads, caliper, and brake rotor were close to each other. Nishiwaki and co-worker [82] also concluded that the vibration mode and frequency of a squealing disc brake rotor are heavily influenced by the natural frequencies and modes of the stationary rotor. Talbot and Fieldhouse [83] used DPHI for experiment when the rotational speed of the mode around the disc is not uniform. By using a Fourier series, they showed in this case that a mode they observed with n modal diameters could be approximated as the sum of forward and backward travelling waves:

$$w_z(r, \theta, t) = G_1(r) \sin(8\theta - \omega t + \alpha) + G_2(r) \cos(8\theta + \omega t + \beta) \quad (2.22)$$

Here, α and β are constants and $G_1(r)$ and $G_2(r)$ are functions of r . These authors also found a similar representation for the tangential displacement w_θ of the rotor [10].

In recent years, development of technology about measure equipments, a number large of modern equipments are applied as accelerometer and microphone with high sensitivity, preamplifier, frequency analyzer, transducers a Laser Doppler Vibrometer (LDV) was used to measure the modes of a full disc brake assembly. However, LDV also has restrains, so Electronic Pulsed Speckle Interferometry (EPSI) is a modern technique of non-contact measurements of deformations in different objects and loading conditions. 3D EPSI ensures the possibility of studying both transverse and longitudinal deformations of the brake disc. An acoustic holography method based on the measurement of sound pressure within the source near field by way of plane microphones and a multiple channel analyzing system performing the fast Fourier transform of the signals from each microphone [9]. Specifically, system in the laboratory was applied for bench test in figure 2.16.

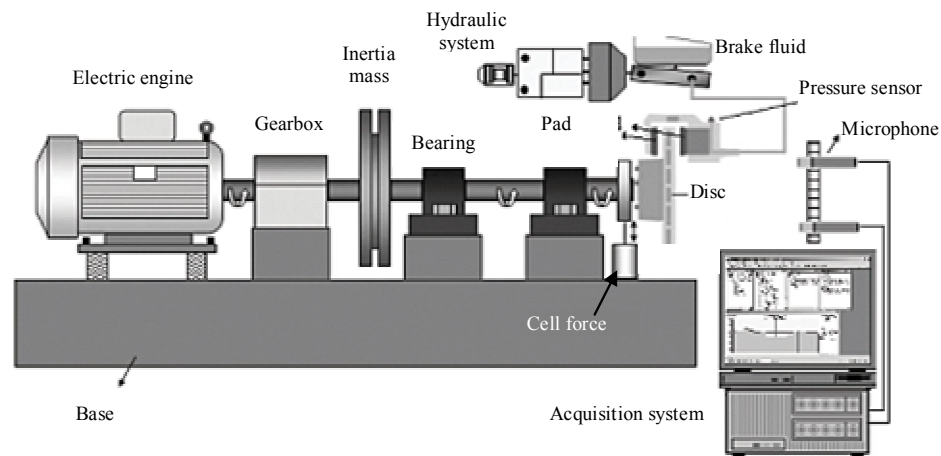


Fig 2.16 Components of an inertial dynamometer [84]

Through this experiment, researchers could capture vibration and noise of the brake system, and can be assessed the friction coefficient. By using the impact hammer and accelerometer and acquisition system, we can obtain natural frequencies and mode shape of the system in various conditions. In figure 2.16 shows a shaft type dynamometer. It is the most common for laboratory brake noise testing and most compact, efficient way of performing such tests in the majority of these cases. However, another approach is to place the vehicle under study in an anechoic chamber with a dynamometer for the vehicle tires to run on such a chassis dynamometer. The dynamometer system shows in figure 2.17.

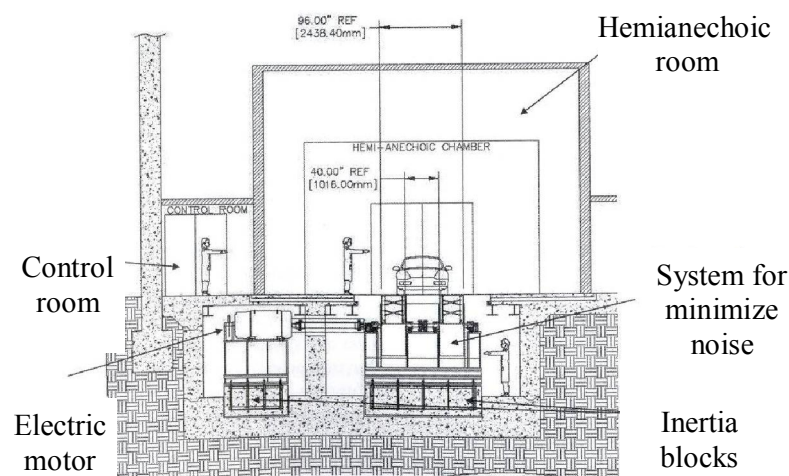


Fig 2.17 Chassis dynamometer drawing [1]

This procedure allows measure noise of the disc brake. It can provide a more realistic test environment in many ways. The required space and cost of such a facility are important considerations. An additional consideration, a full vehicle is needed for testing. In two test systems above, they can experiment effect of temperature and humidity to brake squeal. However, these systems cannot simulate conditions on real road with sufficient accuracy. Therefore, need to test on the road with the real vehicle.

The road test is a procedure in NVH test in real conditions of the vehicle. This is a wide range of on road testing for evaluation the brake noise, including evaluating the potential for the brake squeal, specialized operating condition designed to elicit brake noise. The test was performed in real conditions and factors such as line pressure, temperature and speed of car were considered. Standard for test is including SAE J2625 or city traffic test, the Los Angeles City Traffic (LACT).

2.4.2 Tribology of a couple friction disc and pad

The friction of a couple pads and disc of the disc brake system is a primary cause of brake noise and vibration. Therefore, correct understanding of frictional phenomena and a comprehensive theoretical model are essential to successful simulation and prediction of brake noise. Beside wear plays an importance role in brake squeal, many tribological studies have been performed in this area.

Francesco M. and coworkers in reference [85] presented an experimental tribological analysis. It performed in parallel to the dynamic one and aimed to highlight the role of the contact problems in the squeal phenomena. By using scanning electron microscope (SEM) method for pad and disc surface in conditions without squeal and squeal, the results of the tribological analysis highlight effect of the system vibrations at the contact zone. The high frequency vibrations cause oscillations of the local contact stress as evidenced by the author. Similarly, Chen and his colleague [86] study an effect of surface topography on formation of squeal under reciprocating sliding and the effect of a negative friction–velocity slope on squeal formation is examined with a result: The result demonstrates that not all negative friction–velocity slopes can cause squeal. Eriksson [87] focus on the role played by the tribology of pad and disc interface. He presented friction behavior necessary to increase the understanding of the squeal phenomenon by experiment in various conditions and explain

why the noise occurs. The same author in [88] presented investigation involves a more comprehensive study of the formation, mechanical properties and composition of the tribological surfaces of such pads. By applying high resolution scanning electron microscopy, nanoindentation, energy dispersive X-ray analysis and three-dimensional profilometry with white light optical interferometry, correlation between brake pressure and plateau size for brake pads was assessed. During increasing pressure, the plateaus will be slightly smaller than the equilibrium size at the current pressure.

2.5. CONCLUSION

This chapter presents a number of theories and experimental methods to explain vibration and noise of the brake. After review literatures, it can be seen theories sprag-slip, stick-slip often used for the disc with simple geometries as annular disc. While the brake system has complex geometries as full the brake assembly, it uses the finite element method (FEM). Because it has the capability of generating three dimensional geometry models from a real disc brake system by a solid continuum in software. This method has more advantages, easy add boundaries, loading conditions with high accuracy. Furthermore, the accuracy of a finite element model is typically controlled by the analyst, who may choose to refine the approximation in order to simulate the response of the brake system with a higher degree. Also, can be computed and simulation quickly on the computer, such as strains and stresses, contact pressure, temperature, dynamic of structure. Beside, a number of literatures experiment was presented to identify natural frequencies and mode shape of the disc brake. Effect of factors as wear, temperature, humidity, contact, friction coefficient between pads and disc or material of components to brake squeal are play an important role. Therefore, brake squeal is a complex problem, that is why in this thesis cannot considers full of problems, but the area of study is limited in the conditions of our laboratory at the Technical University of Liberec -TUL. A full of jobs in this thesis were described in figure 2.18. Based on information above, this thesis limits as flowing:

- The frequency range of the squeal is considered from around 1 kHz to 10 kHz.
- The object of study is the disc brake system on the car Renault Traffic, model 2007, Power 66kW of TUL.
- Model of the full disc brake assembly is built by FEM in software Abaqus.

- Simulation of the disc brake to find out natural frequencies and mode shape of the components of the disc brake system.
- A theory of the complex eigenvalue analysis commonly utilized to determine disc brake assembly stability.
- Due to the complex phenomenon of heat transfer and the difficulty of numerical modelling, thermal effects have been ignored in effect to disc brake squeal.
- An analysis of topography, including SEM surfaces and EDS analysis of disc and pad was studied to find the cause of vibration and noise. Beside, predict occurrence noise in different conditions.
- Experimental approaches are essential not only to quantify the nature of squeal and various operating conditions affecting it but also to provide results confirmation simulation results and try to explain occurrence noise. The experiment will be tested in chassis dynamometer type following standard SEA J2521 at the brake laboratory of the Department of Vehicle and Engines, TUL. Besides, another experiment for tribology of couple pads will be conducted at the laboratory of electron microscopy and laboratory of surface technologies Department Material and Science, TUL.

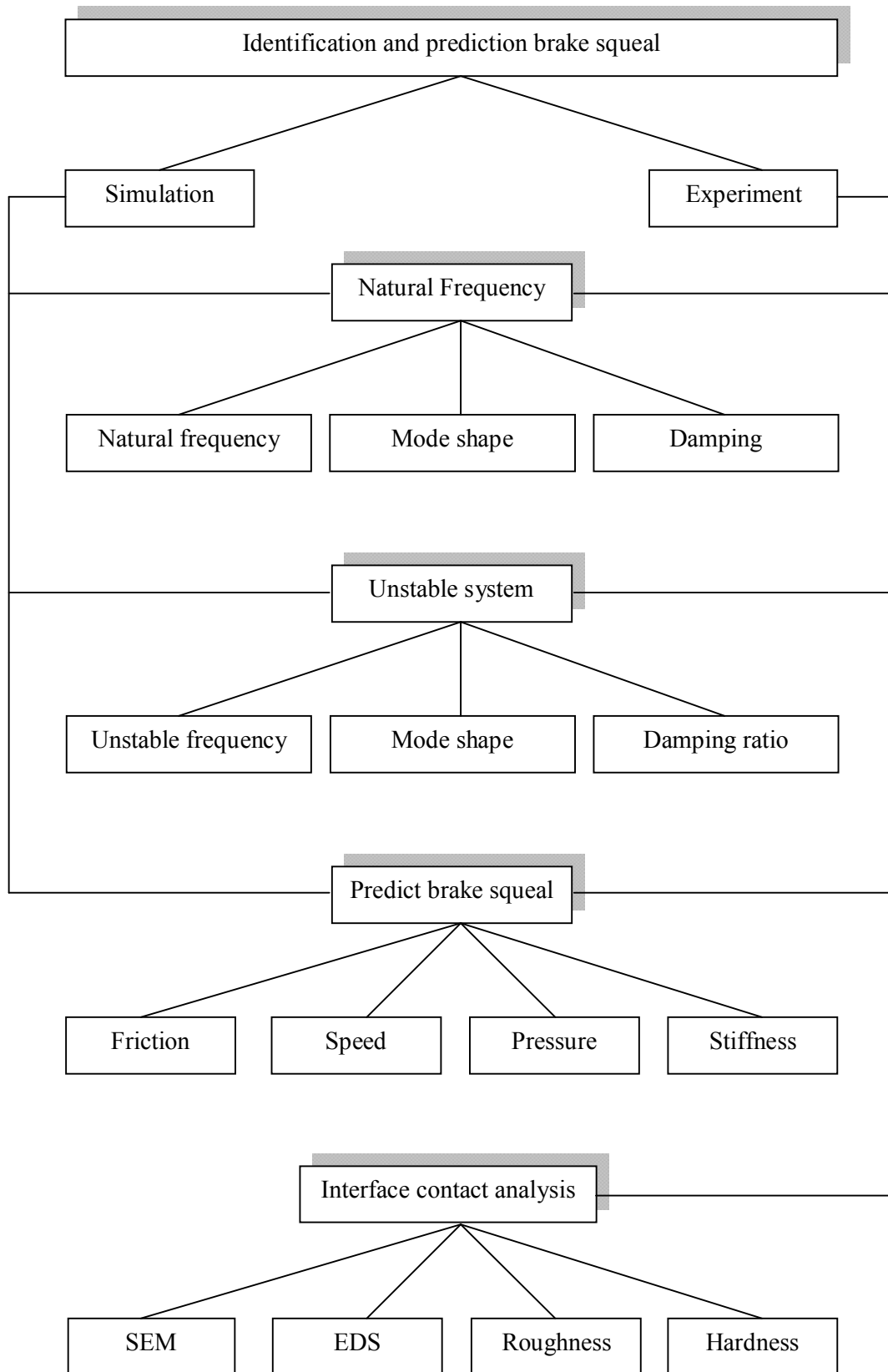


Fig 2.18 Schema for prediction noise of disc brake

FINITE ELEMENT MODEL AND SIMULATION

3.1 INTRODUCTION

Nowadays, with the development of computers and measurement equipment which allow the researcher to simulate brake squeal in different conditions and compare them with experimental results. A number of researchers focus on theory, simulation and experiment to identify natural frequencies and mode shape of the disc brake in different boundary conditions. Bakar (2005) simulates the contact of pair discs and pad with 3-dimensional finite element in relation with the brake squeal [3]. P Liu (2006) considers simulation of the disc brake by software Abaqus. In this study an attempt is made to investigate the effects of system parameters, such as the hydraulic pressure, the rotational velocity of the disc, the friction coefficient of the contact interactions between the pads and the disc, the stiffness of the disc, and the stiffness of the back plates of the pads, on the disc squeal. The simulation results show that significant pad bending vibration may be responsible for the disc brake squeal [49]. Nouby (2009) also used a combined approach of complex eigenvalue analysis and design of experiments to study the disc brake squeal [47]. This study performed simulations of the disc brake squeal by effect of the components of the disc brake and effect of material properties. An experiment to validate for measurement mode shape of the structure disc brake was examined.

This chapter introduces a finite element (FE) model of an actual disc brake, and it will be simulated in Abaqus 6.10 to find out the natural frequencies and the mode shape of components of the disc brake structure. When developing a FE model, it is important to validate it in order for the model to correctly with the geometry and the material properties of the brake structure. Thus, experimental result of the natural frequencies will be compared with the natural frequencies of simulation to adjust material properties. This helps the results to be more accurate.

This work also simulates modal analysis of the disc brake components such as disc, pads, calliper, carrier, piston and bolts in frequencies range from 1000 Hz to 10000 Hz. The

simulation procedure consists of two steps. The first step is to simulate the natural frequencies and the mode shape of the disc brake components in free-free boundary conditions. Next step, a simulation of natural frequencies and mode shape of the full disc brake assembly with full boundary conditions will be performed. The results of two steps are compared with the experiments.

3.2 THEORY OF MODEL

3.2.1 Linear vibration

The concept of damping and natural frequencies can be illustrated by looking at a viscously damped oscillator. The differential equation for a single degree of freedom system is

$$m\ddot{x} + c\dot{x} + kx = 0 \quad (3.1)$$

Where m , c , k and x are the mass, viscous damping, stiffness and displacement respectively. If a harmonic solution is assumed, then,

$$x = Ae^{\lambda t} \quad (3.2)$$

Where A is the amplitude and λ is the eigenvalue. This produces a characteristic equation of the form

$$m\lambda^2 + c\lambda + k = 0 \quad (3.3)$$

For harmonic vibration, the discriminant of the above quadratic equation must be less than zero,

$$c^2 - 4mk < 0 \quad (3.4)$$

When only viscous damping is present, then the roots are complex conjugate pairs.

$$\lambda_{1,2} = \alpha \pm j\omega \quad (3.5)$$

Using Euler's formula for the exponential, the time response can be recovered as

$$x = Ae^{\alpha t} (e^{j\omega t} + e^{-j\omega t}) = 2Ae^{\alpha t} \cos \omega t \quad (3.6)$$

This is the damped harmonic response. From (3.6) the real part of the root is the damping while the imaginary part is the frequency

$$\alpha = \text{Re } \lambda \text{ and } \omega = \text{Im } \lambda \quad (3.7)$$

The roots can be displayed on a root locus diagram that plot frequency versus damping, see in figure 3.1. If a parameter is varied in the system, the movement of the rotor can be displayed versus the parameter.

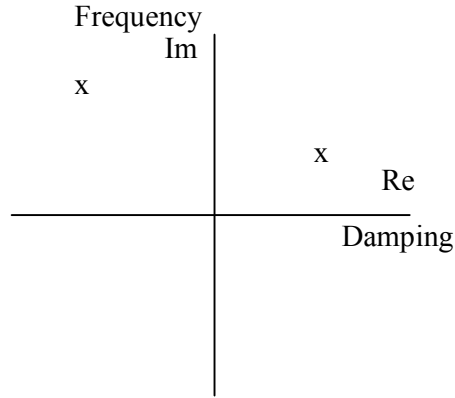


Fig 3.1 Root locus diagram in the complex eigenvalue plane [4]

3.2.2 Equation vibration of multiple degree of freedom

The multi DOF equation is called state equations that are developed with friction that provides work out of the system. Friction is a non conservative force, so the value of K_f is negative. The fully nonlinear state equation for slip-stick vibration is shown in [89, 90].

$$[[M]\{\ddot{q}\} + [C + C_f(\dot{q})]\{\dot{q}\} + [K + K_f(q)]\{q\} = 0 \quad (3.8)$$

Where M , C , K , C_f , K_f , q are corresponding to mass matrix, viscous damping matrix, stiffness matrix, friction damping, friction stiffness and displacement.

Since the vibration velocity is zero at steady sliding, the equilibrium position is found by a nonlinear static solution using a Newton solver.

$$[K + K_f(q_0)]\{q_0\} = \{P\} \quad (3.9)$$

Where P is the brake line pressure and q_0 is the steady sliding position. The complex mode is found as a change of this position,

$$\{q\} = \{\Phi\}e^{\lambda t} \quad (3.10)$$

The characteristic equation that results is

$$[\lambda^2 M + \lambda[C + C_f] + [K + K_f(q_0)]]\{\Phi\} = \{0\} \quad (3.11)$$

3.2.3 Mode shape animation

By animating the unstable modes in a commercial post-processing program, guesses can be made on how to uncouple the modes and hence stabilize the system. Assume there is no friction and viscous damping only then the root occurs in complex conjugate pairs. Suppose the damping is greater than zero (unstable)

$$\Phi_i = a_i \pm jb_i \quad (3.12)$$

$$\lambda_i = \alpha_i \pm j\omega_i \quad (3.13)$$

These are converted into the time domain as

$$y(t) = 2e^{\alpha_i t} |\Phi_i| \cos(\omega_i t + \theta_i) \quad (3.14)$$

$$\tan \theta_i = \frac{b_i}{a_i} \quad (3.15)$$

$$|\Phi_i| = \sqrt{a_i^2 + b_i^2} \quad (3.16)$$

Where λ_i is the eigenvalue; a_i and b_i is amplitude; α_i is real part of the root. Detail of the theory of this model was presented above by Frank. It shows in reference [4].

3.3 PROCEDURES EXPERIMENT

3.3.1 Equipment

To identify natural frequencies and mode shape of the structural components of a brake system, a number of equipment including accelerometer, impact hammer, acquits system, analyzer, computer is used for tests. The arrangement of equipment is shown in Fig. 3.2. There are many types of equipment for testing natural frequencies and mode shape of some companies, but in this research, some equipment available in the department Vehicle and Motor (KVM) was used. Some main specifications of equipment are:

Piezoelectric accelerometer type 4393 of Brüel & Kjær with reference sensitivity at 159,2 Hz ($\omega = 1000 \text{ s}^{-1}$), 100 ms^{-2} and 23°C . Charge sensitivity $0,321 \text{ pC/ms}^{-2}$, Voltage sensitivity $0,409 \text{ mV/ms}^{-2}$. This was used to measure the vibration acceleration of the disc brake and send the signal in the form of voltage to the analyzer system.

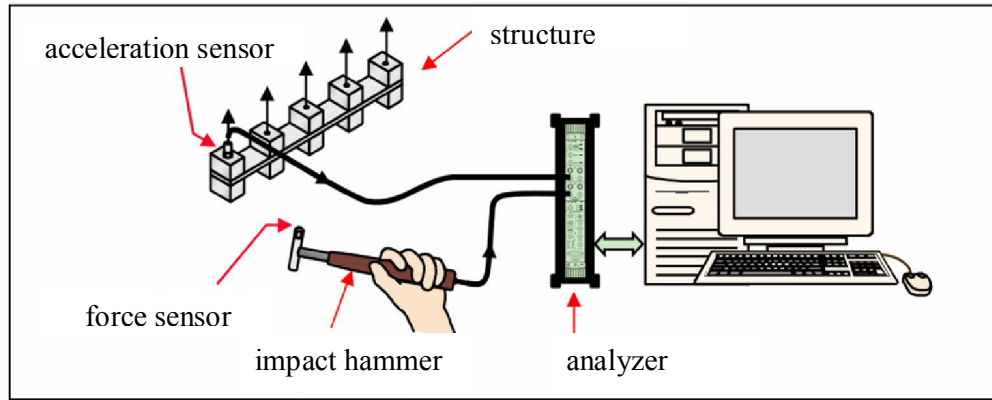


Fig 3.2 Scheme arranges equipments for experiment [91].

Calibration chart for impact hammer type 8202 of Brüel & Kjær with sensitivity at output of the hammer is 1,02 pC/N and including 3 tips with parameters as table 3.1.

Table 3.1. Technical parameters of Hammer 8202 Tips

	Force range (N)	Duration range (ms)	Approx. frequency range (-10dB) (Hz)
Rubber tip	100-700	5-1,5	0-500
Plastic tip	300-1000	1-0,5	0-2000
Steel tip	500-5000	0,25-0,2	0-7000

Calibration chart for force transducer type 8200 was mounted on hammer with reference sensitivity at 159,2Hz ($\omega=1000s^{-1}$) at room temperature is 3,95 pC/N (without static load). Static sensitivity from 0-1000N tension is 3,98 pC/N and from 0-5000N compress is 4,15 pC/N [92].

Impact hammer is a device that produces an excitation force pulse to the test structure. It consists of a hammer tip, a force transducer, a balancing mass and a handle [93]. Depend on material properties, sensitivity and range frequency or structure, different kinds of tip can be used. In this experiment, a steel tip was used.

Dual Channel Real-time Frequency Analyzer Type 2144 B-K was used to obtain and analyze input signals and connect to the computer. Dual channel real time analyzer type

2144 is a portable, digital filter, frequency analyzer for acoustics, electroacoustics and vibration measurements. The 2144 features real time single channel operation, the real time frequency range is halved. The analyzer 2144 is operated by means of user interactive menus. Access to a system of on screen help pages provides you with information about the use of the menus at any time and in any place. A large internal non volatile memory together with back-up disk storage facilities makes type 2144 a powerful data gathering device. The IEEE 488 and RS-232-C interfaces allow-processing and external control [92]. All equipments show in figure 3.3.

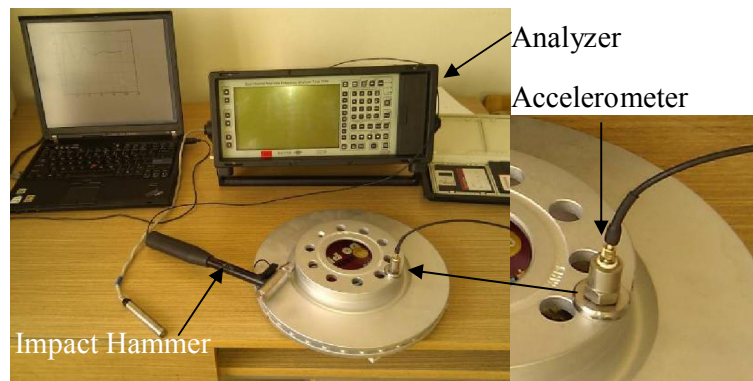


Fig 3.3 Equipment for modal testing

3.3.2 Procedures

Modal testing is an experimental technique used to derive the modal model of a linear time invariant vibratory system. The theoretical basis of the technique is secured upon establishing the relationship between the vibration response at one location and exciting at the same or another location as a function of excitation frequency. This relationship, which is often a complex mathematical function, is known as frequency response function [94].

Experimental modal analysis obtains the modal model from measured frequency response function (FRF) data or measured free vibration response data. This investigation is very important as brake squeal often involves modal coupling between various modes associated to natural frequency [93]. The experimental approach to modelling the dynamic behavior of structures through impact hammer test modal testing consists of four steps as follows:

- Setting up the modal test,
- Taking the measurements,

- Analyzing the measured test data,
- Presenting results and compare them with modelling data.

This experiment focuses on identifying natural frequencies and mode shape of the disc brake system. A natural frequency may be detected experimentally by exciting the structure with a harmonic force and varying the frequency until resonance is achieved. At resonance, the mode associated with the natural frequency may be observed. Figure 3.4 describes a principle for modal test.

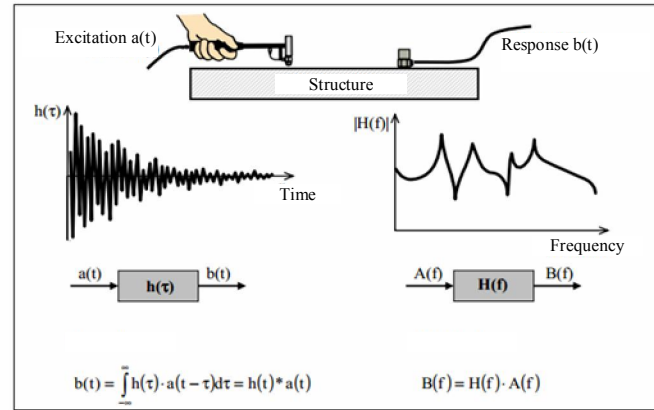


Fig 3.4 Principle for modal test [91]

In figure 3.4, it can be seen $a(t)$ is the input signal of impact hammer and $b(t)$ is the response signal of the accelerometer. Impulse response and frequency response function $h(\tau)$ or can be expressed in form

$$H(f) = \frac{B(f)}{A(f)} \quad (3.17)$$

The frequency response function obtains in time domain, this time data provide very useful information and they can be transformed to frequency domain by Fast Fourier Transform (FFT). The $H(f)$ is invariant because it is a ratio of the response to the input.

To increase accuracy, it is highly recommend that the analyzer should be configured to show both the FRF and the coherence between the input and response channels on the analyzer screen. This allows one to see that adequate resonances have been excited and that there is signal quality where the resonances occur.

An accelerometer is attached on the surface of the circular disc. Repeatedly, the impact hammer is striking on the same position for obtaining signal of two channels and gives to the analyzer for calculating the natural frequencies and mode shape.

3.4 BUILDING MODEL IN ABAQUS

3.4.1 Finite Element Model of the disc brake

3.4.1.1 Disc brake structure

To simulate the brake squeal, the first need to understand about structure of the disc brake components, including disc, pads, calliper, carrier and piston. This will provide information to build a Computer Aided Design (CAD) models in software and setup boundary conditions exactly.

The disc brake is a component of the brake system. Its function is to stop the vehicle completely or control the vehicle speed. Friction causes the disc and attached wheel to slow or stop the vehicle. A disc brake system includes main components such as rotor (disc), a pair pads, calliper, piston and carrier. Figure 3.5 illustrates a real disc brake system from a Renault Traffic car, model 2007, at the department KVM in Technical University of Liberec.

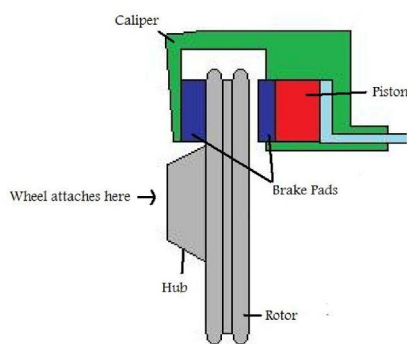


Fig 3.5 Model of disc brake system of car

When braking, the brake pads convert the kinetic energy of the car to thermal energy by friction. Two brake pads are contained in the brake caliper with their friction surfaces facing the rotor. When the brakes are hydraulically applied, the caliper clamps or squeezes the two pads together into the spinning rotor to slow or stop the vehicle [95].

Caliper is the assembly of disc brake system which houses the brake pad and pistons. This is a floating caliper moves with respect to the disc and along a line parallel to the axis of rotation of disc. It also including a carrier helps keep and fixed with knuckle of wheel.

The piston of the brake system is designed for using hydraulic, with calliper has two pots. Therefore, there are two pistons and cylinders are used.

All components of the disc brake system and its assembly were created by SolidWorks software, see in figure 3.6. These models will be used to simulate the natural frequency and mode shape in Abaqus.

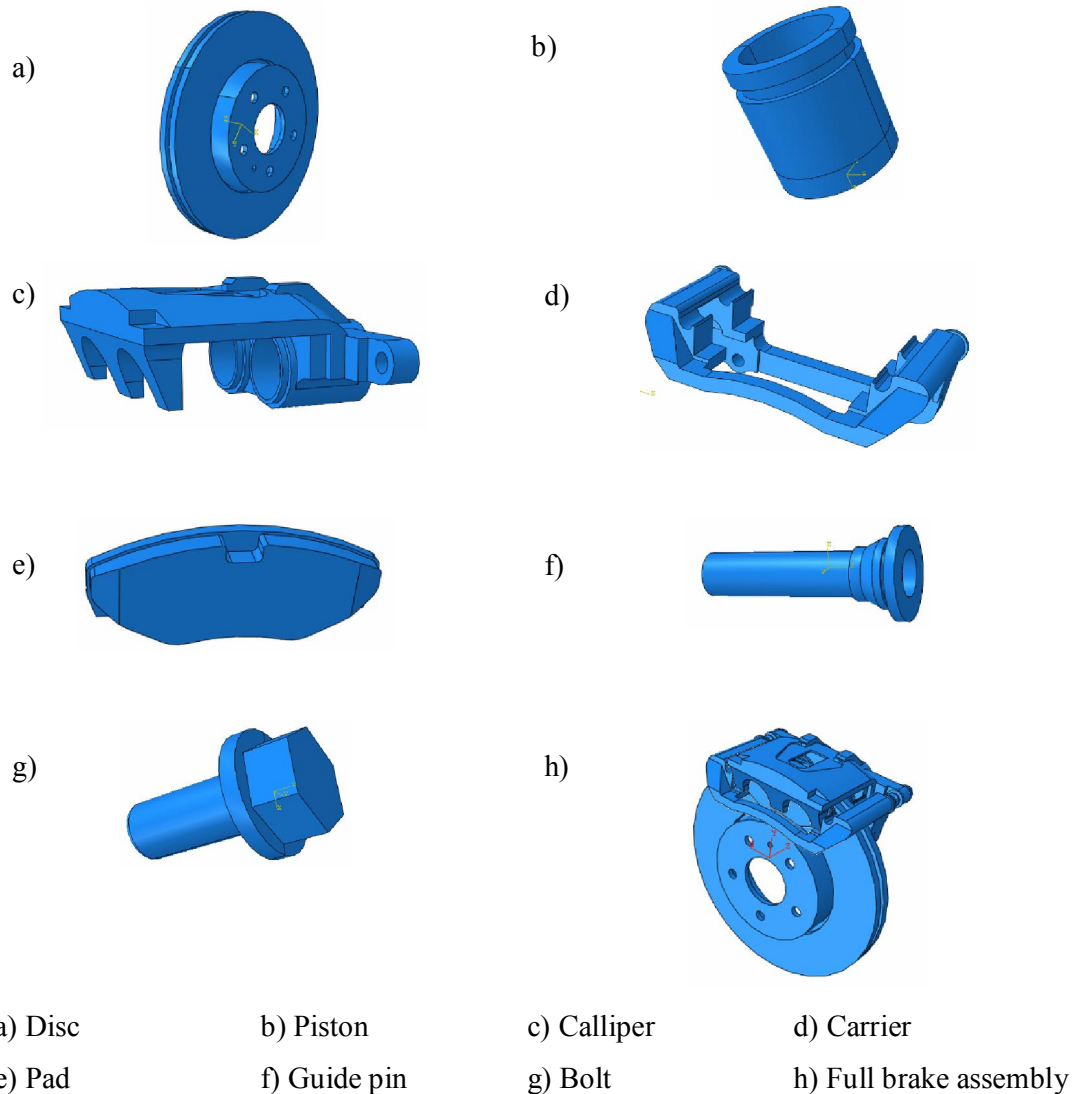


Fig 3.6 Disc brake components

3.4.1.2 Finite element model in Abaqus software

The objective of building a FE model in software Abaqus is to get the simulation results more exactly. Abaqus software is the software affords a straight forward brake squeal analysis methodology. The simulation process consists of the steps: (1) preloading the brake, (2) rotating the disc, (3) extracting natural frequencies, and (4) computing complex eigenvalues. A FE model of the real disc brake must be made for the process. All components of the disc brake system will be developed and validated in 3D FE model. This section will describe analysis natural frequencies and mode shape of disc brake components in free-free boundary conditions. A full disc brake assembly also simulates in full boundary conditions, including pressure load and rotation.

At the free-free boundary condition, firstly, the components of the disc brake system are created in SolidWorks software and then imported to Abaqus software. Secondly, material properties are attached to all components. Next, all components are meshed by elements. There are two types of elements used in this work: element C3D4 – 4 node linear tetrahedron using for the disc brake assembly and element C3D8 – 8 node linear brick using for pad and piston parts. Next step is to setup a job for static analysis and frequency analysis for extracting natural frequencies and mode shape. Lastly, compare simulation results with the experiment results. . If there is any error between the simulation and the experiments results, a procedure to adjust material properties has to be performed until the results between them can be acceptable level.

With the full boundary conditions, a full disc brake system is created in SolidWorks and then imported to Abaqus. Steps for analysis are nearly the same as in the procedure above but they have more settings about contact between surfaces and settings for boundary conditions. The contact between the disc and the pad play an important role, thus a contact type surface to surface is applied. This type helps for calculation the contact stress between two surfaces. Some rest interactions are used type of node to surface as pair caliper with pad, piston with pad, guide pin with carrier and piston with caliper. Procedure pressure load allows the contact pressure distribution on the lining surface of disc and pads. It is using for static analysis, when the disc is not rotating. Next, establish a steady state rotational motion for disc. In this static step, a rotational velocity is imposed on the disc as a predefined field variable. This provides for the modeling of steady state frictional sliding between two bodies

that are moving with different velocities. In this simulation setting $\Omega = 9.7$ rad/s and $\mu = 0.35$ are used. For pressure load, a typical value of 3 bar is directly applied to the cylinder and back piston regions.

In this step, a setting boundary condition for disc brake components is also performed. The disc is completely fixed at the five counter bolt holes and the ears of the pads are constrained to allow only axial directional z movements.

To analysis this job in Abaqus software, a requirement of procedures contains the following four steps [22, 49] and in figure 3.7 is a scheme of steps analysis:

- Nonlinear static analysis for the application of brake pressure.
- Nonlinear static analysis to impose a rotational velocity of the disc
- Normal mode analysis to extract the natural frequency to of undamped system.
- Complex eigenvalue analysis to incorporate the effect of friction coupling.

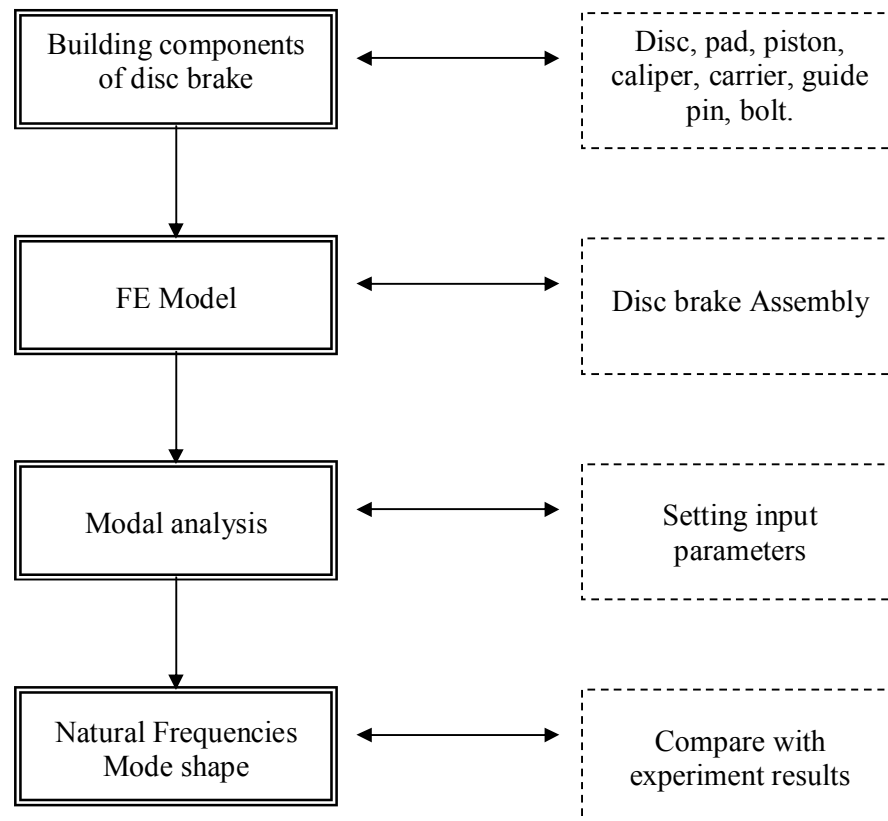


Fig 3.7 Scheme of procedure for modal analysis

3.4.2 Material properties

To obtaining analysis results exactly, the first information on material properties has to be taken from the companies where the products were manufactured. Then it will be adjust density and Young's Modulus suitable with an actual structure model. This is reason why most of the FE models do not exactly resemble the actual geometry of the brake components. Thus, in order to have almost identical dynamic properties between the FE model and real brake components, those parameters need to be tuned. Procedure for adjust material properties including: putting standard density of the material, Young's Modulus, Poisson's coefficient of the materials in to the software. The calculation of weight of the model can be identified by checking its volume obtained from Abaqus. The next step is to compare the weight of the model in software with the weight of the real components in the disc brake system. If there is difference between the two masses, density needs to be tuned until mass of the finite element model is similar to the real brake component. Modal tuning is also used to get an optimum correlation between finite element method and experimental method based on Young's modulus. Frequency response based modal tuning is applied to achieve best correlation. The relationship between natural frequency and Young modulus with a free-free boundary condition is shown as equation 3.17 [96].

$$f_n = \frac{\lambda^2}{2\pi a^2} \sqrt{\frac{Eh^3}{12\gamma(12-\nu^3)}} \quad (3.17)$$

Where	a - Length of the part	λ – Constant
	E - Young modulus	h – Thickness
	γ - Mass per unit	ν - Poisson's ratio.

The purpose of the part validation is to make the value of natural frequency of simulation, f_s closer to the experimental natural frequency f_E . The procedures to simplify $f_E = f_s$ is shown as below. The relationship between experimental equation and simulation equation was computed by divided f_E with f_s .

$$\frac{f_E}{f_S} = \frac{\left[\frac{\lambda^2}{2\pi a^2} \sqrt{\frac{Eh^3}{12\gamma(12-\nu^3)}} \right]_E}{\left[\frac{\lambda^2}{2\pi a^2} \sqrt{\frac{Eh^3}{12\gamma(12-\nu^3)}} \right]_S} \quad (3.18)$$

Equation 3.18 can be simplified into equation 3.19 as the value of λ , a , h , γ and ν are constant in experiment and simulation.

$$\frac{f_E}{f_S} = \frac{\sqrt{E_E}}{\sqrt{E_S}} \quad (3.19)$$

$$\left(\frac{f_E}{f_S} \right)^2 = \frac{E_E}{E_S} \quad (3.20)$$

When $f_E = f_S$ thus $E_E = E_S$. The difference between simulated and measured data can be obtained as follows:

$$Error = \frac{f_S - f_E}{f_E} \times 100\% \quad (3.21)$$

From the procedures above, table 3.2 is summaries material properties of disc brake system.

Table 3.2 Material properties of disc brake components

	Density (kg/m³)	Young's Modulus (MPa)	Poisson's Ratio
Disc	7050	95.3	0.29
Pad Friction	2700	13	0.4
Pad Plate	7850	210	0.3
Pist	8020	200	0.3
Calliper	7130	200	0.3
Carrier	7010	170	0.3
Bolt	9530	53	0.3
Guide Pin	7800	650	0.3

3.5 MODAL ANALYSIS

Modal analysis is the process of determining the inherent dynamic characteristics of a system in forms of natural frequencies, damping factors and mode shapes, and using them to formulate a mathematical model for its dynamic behavior. The formulated mathematical model is referred to as the modal model of the system and the information for the characteristics are known as its modal data [94]. Modal data is an extremely useful piece of information that can assist in the design of almost any structure. The understanding and visualization of mode shapes is invaluable in the design process. It helps to identify areas of weakness in the design or areas where improvement is needed [97]. The development of a modal model is useful for simulation and design studies as structural dynamic modification and vibration a noise of brake disc.

Modes are inherent properties of a structure, and are determined by the material properties including mass, damping, and stiffness, and boundary conditions of the structure. Each mode is defined by a natural frequency, modal damping, and a mode shape so-called modal parameters. If either the material properties or the boundary conditions of a structure change, its modes will change. For instance, if mass is added to a structure, it will vibrate differently.

After creating model and setting loads, boundary conditions as described above, a modal analysis of the disc brake components and the disc brake assembly are performed. These analyses are performed in free-free boundary conditions type (there are no constraints imposed on the components). Next step, the disc brake was simulated with the full boundary conditions for modal analysis. This case is only performed in Abaqus software and it isn't to compare with the experiment due to the limitation of equipment and tools available in the laboratory of TUL.

3.5.1 Disc

The 3D CAD model of the full structure disc in the disc brake system of Car Renault Traffic is presented in figure 3.8. Totally, disc was meshed with 8660 elements C3D4. The results of a modal analysis which extract natural frequencies and mode shape at free-free boundary conditions are shown in figure 3.9.

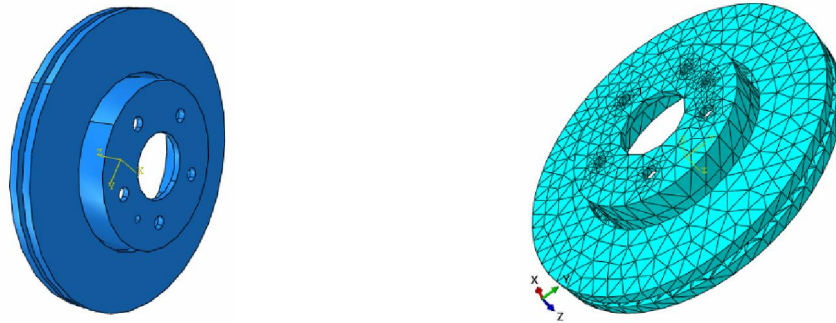


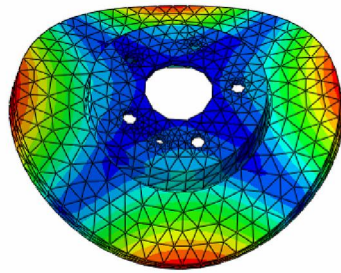
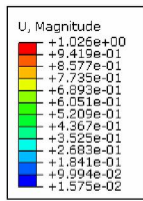
Fig 3.8 FE Model (left) and meshed (right) of disc brake

There are a lot of modes at different frequencies of the disc, but here only considers frequencies that have nodal diameter (ND) because they are dominant in the squeal phenomenon as in table 3.3 below. The number of nodal diameters based on a number of nodes and anti node appearing on the rubbing surface of the disc. Coupling between modes increases the natural frequency of the rotor disc, which generates the squeal phenomenon.

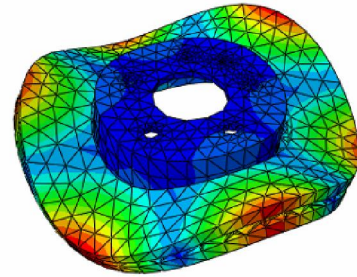
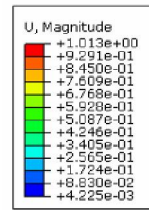
Table 3.3 Comparing natural frequencies of disc in simulation and experiment results.

	2 ND	3 ND	4 ND	5 ND	6 ND	7 ND
Simulation (Hz)	1197	2678	4447.6	6415.3	8580	10764
Experiment (Hz)	1200	2576	4432	6384	8512	10544
Error %	0.25	-3.96	-0.35	-0.49	-0.80	-2.09

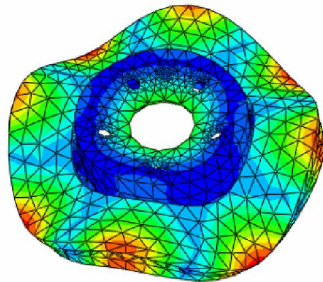
From the observation of the mode shape of the disc brake, it can be seen that the vibration mode pattern of the disc brake consists of anti-node and nodes as illustrated in figure 3.9. Frequencies range from 1 kHz to 10 kHz have maximum of mode with seven nodal diameters.



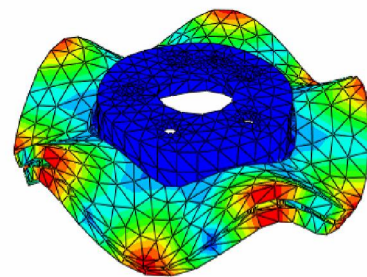
a) 2ND at 1197 Hz



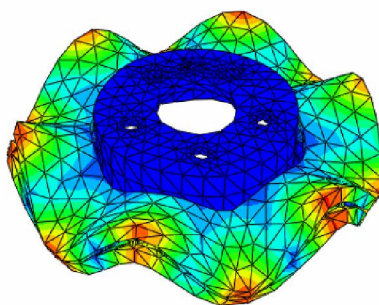
b) 3ND at 2678 Hz



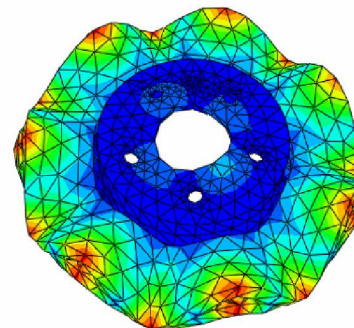
c) 4ND at 4447.6 Hz



d) 5ND at 6415.3 Hz



e) 6ND at 8580 Hz



f) 7ND at 9812 Hz

Fig 3.9 Modes shape of disc at different frequencies

For experiment modal of the disc, we mark site of points on the surface of brake. These points will be created again in software of modal analysis. In this research, the disc was divided into 27 points as in figure 3.12. After creating the reference points, an accelerometer and an impact hammer can be used for measurement. Corresponding with the points, the analyzer obtains the signal of accelerometer and impact hammer. It was calculated for each of points by the formula 3.22 then the results are shown on screen of analyzer or saved. Details of mode shape experiment of the disc are illustrated in figure 3.10.

$$H_{ji}(\omega) = \frac{B_j(\omega)}{A_i(\omega)} \quad (3.22)$$

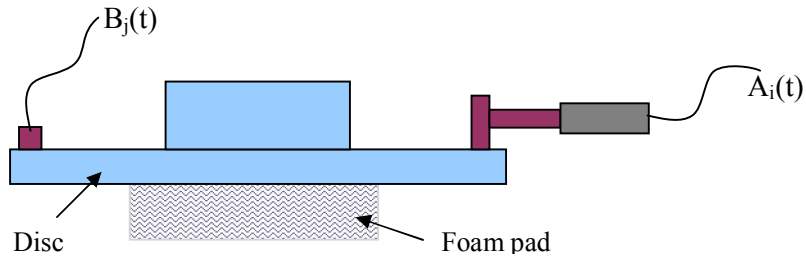


Fig 3.10 Scheme experiment modal shape for disc

At free-free boundary condition, the disc was tested by supporting the structure with soft materials is foam pad. Natural frequencies of the disc brake were obtained from the peaks of FRF. It shows in figure 3.11. An experiment of modal analysis was showed on MDSView software, see in figure 3.12

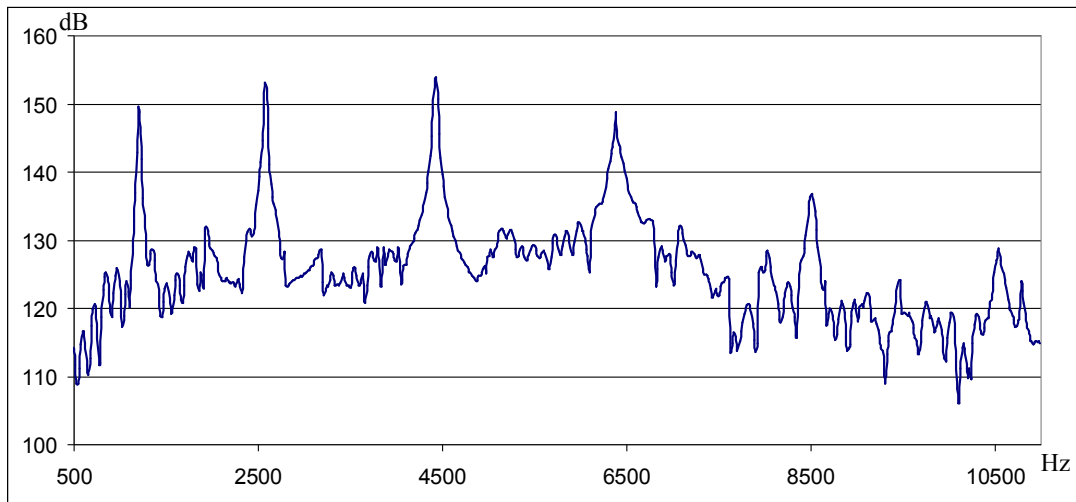
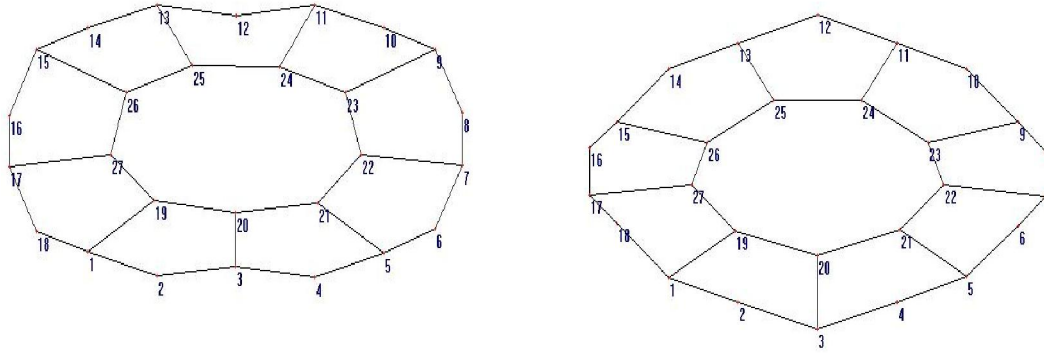
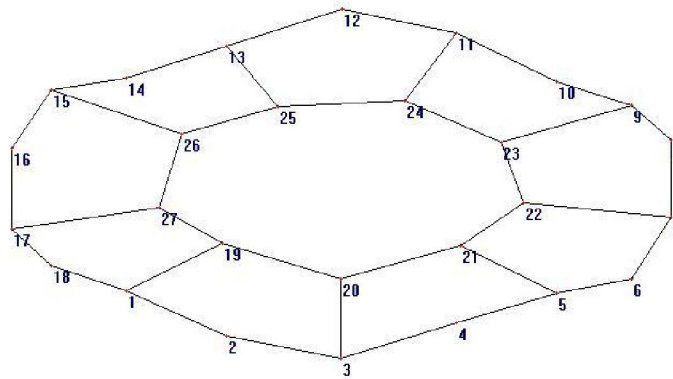


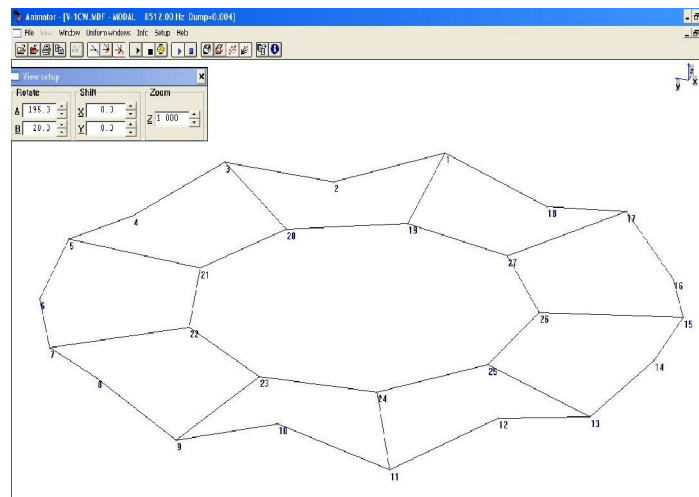
Fig 3.11 Frequencies response function of disc



a) Experiment mode shape at 1200 Hz



b) Experiment mode shape at 4432 Hz



c) Experiment mode shape at 8512 Hz

Fig 3.12 Experiment modal analysis of disc

3.5.2 Pad structure

Brake pad is a component of the disc brake. Brake pads consist of steel backing plate structure and a solid of friction material bound to the surface of the disk brake rotor. A couple pads play an important role in contributing noise and vibration to the brake disc system. The FE model of a pad brake and the meshed model are presented in figure 3.13. The pads were meshed with 1261 elements C3D8.

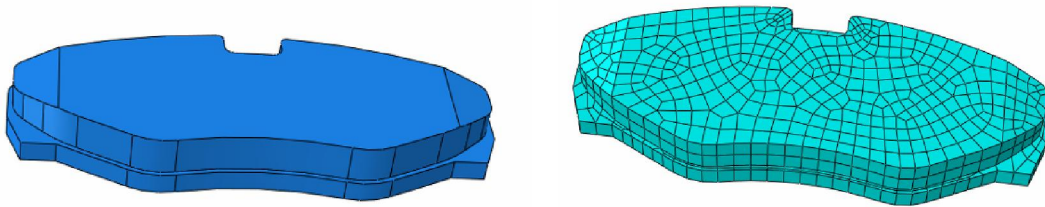


Fig 3.13 Pad brake in FE model (left) and meshed (right)

The same way of simulation for the disc, a modal analysis is simulated in frequencies range from 1 kHz to 10 kHz. The result shows four main modes shape, see in figure 3.14.

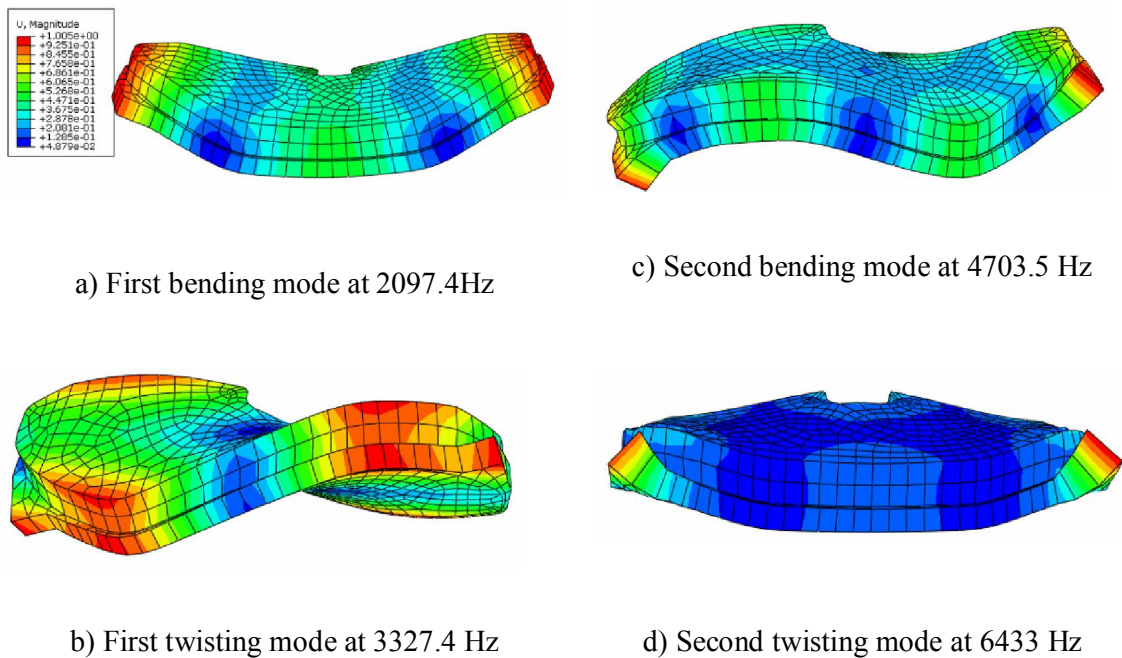


Fig 3.14 Mode shape of pad

From the results of simulation, it can be seen the first mode occurs at the frequency of 2079.4 Hz while the second bending mode occurs at 4703.5 Hz. The first twisting mode is at the frequency of 3327.4 Hz and the second twisting mode is at 6433 Hz. The results of simulation and experiment are shown in table 3.4.

Table 3.4 Comparing natural frequencies of pad in simulation and experiment results.

	1 ND	2 ND	3 ND	4 ND
Simulation (Hz)	2097.4	3327.4	4703.5	6433
Experiment (Hz)	2080	3312	-	-
Error %	-0.84	-0.46	-	-

An experiment was performed to compare with the simulation results in table 3.4. But the test only captured two modes of pad. The cause can be due to damping of pad and the tip of impact hammer is large and heavy. The frequency response function of the pad with natural frequencies is peaks of FRF shows in figure 3.15

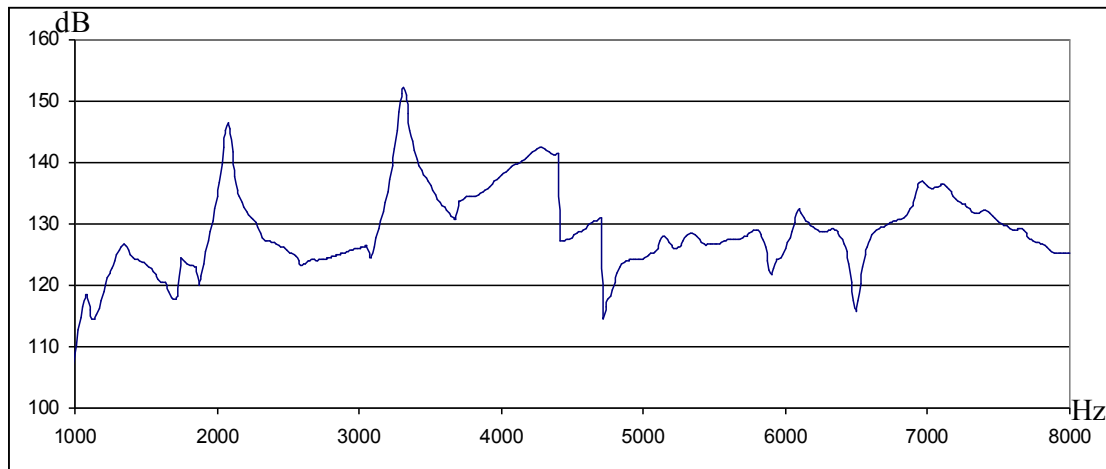


Fig 3.15 The frequencies response function of pad

3.5.3 Calliper

The brake caliper is the assembly which houses the brake pads and pistons. In figure 3.16 is a FE model of caliper (left) and the meshed on right with 6820 elements C3D4.

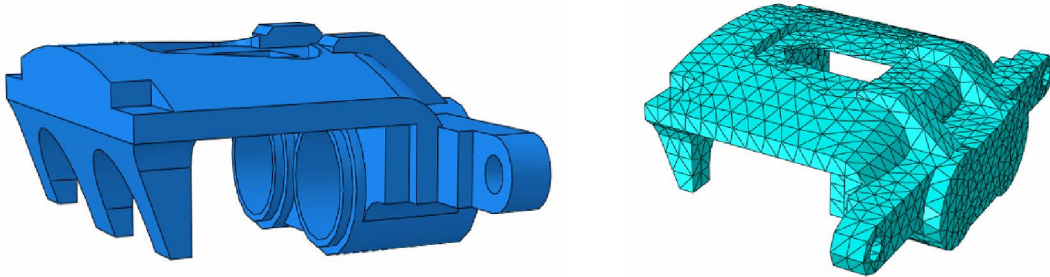
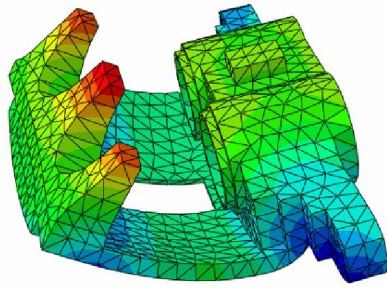


Fig 3.16 FE model of caliper (left) and meshed (right)

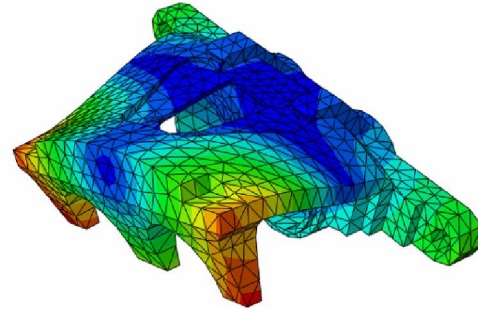
At free-free boundary condition, a modal analysis of caliper was performed in range 10 kHz. Totally, there are 17 modes found as in table 3.5 which the value at mode 7, mode 10 and mode 15 are close to the natural frequencies of disc. The first bending occurs at 1455 Hz and the first twist occurs at 1558 Hz. The simulation results shows in figure 3.17.

Table 3.5 Natural frequencies of caliper

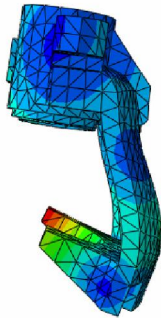
Mode	Natural Frequencies	Mode	Natural Frequencies
1	1455.8	10	8640.2
2	1558	11	9246.4
3	4168	12	10125
4	5070.4	13	10264
5	5721.3	14	10540
6	5811.2	15	10936
7	6567.3	16	11322
8	7476.2	17	11864
9	8482.4		



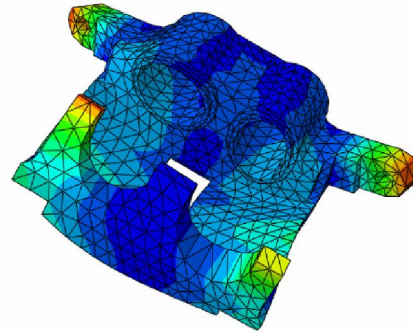
a) First bending 1455 Hz



b) First twist 1558 Hz



c) Mode shape at 4168 Hz



d) Mode shape at 8640 Hz

Fig 3.17 Mode shapes of caliper

3.5.4 Carrier

Similar simulation of other components, at free-free boundary condition, the carrier was modeled as in figure 3.18 (left) and meshed with 6354 elements C3D4 as in figure 3.18 (right). Modal analyses performed and extract natural frequencies and mode shape. It was presented in figure 3.19.

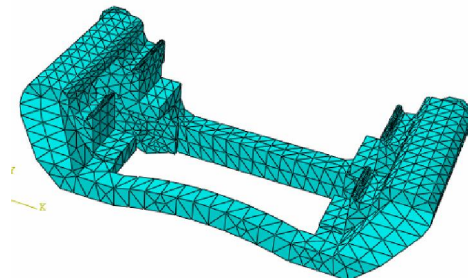
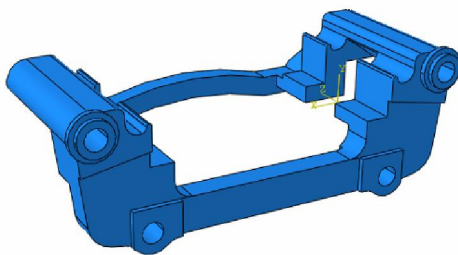
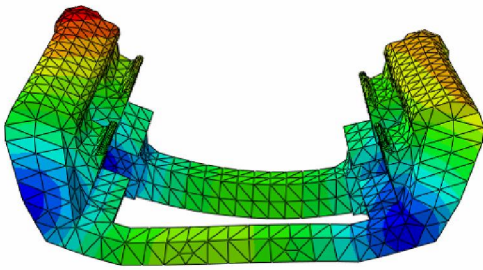
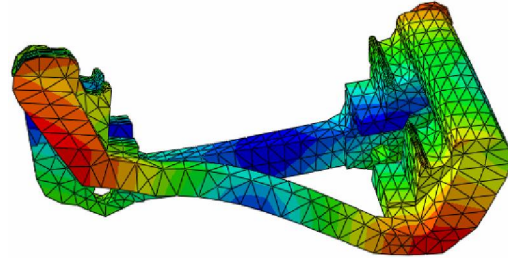


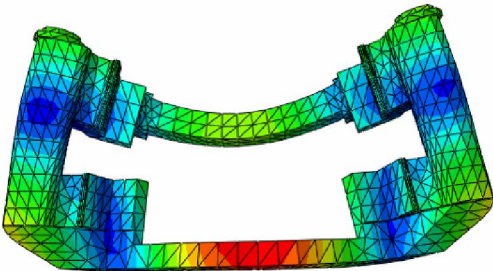
Fig 3.18 FE model (left) and meshed (right) of carrier



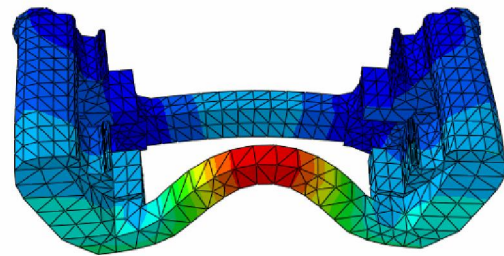
a) Mode shape at 893.38 Hz



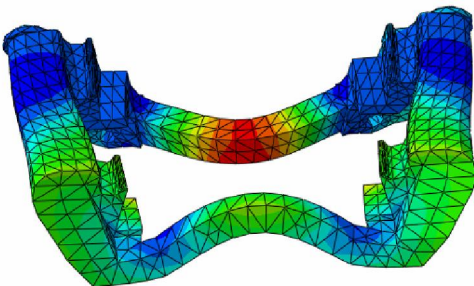
b) Mode shape at 941.22 Hz



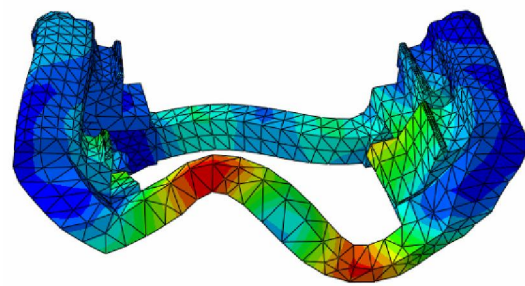
c) Mode shape at 2448 Hz



d) Mode shape at 3560.9 Hz



e) Mode shape at 4827.4 Hz



f) Mode shape at 8620.9 Hz

Fig. 3.19 Mode shapes of carrier

In the investigated range of frequency, there are nineteen modes found as shown in table 3.6. The value at mode 6, 11, 13 and mode 17 are closed to natural frequencies of disc.

Table 3.6 Natural frequencies of carrier

Mode	Natural Frequencies	Mode	Natural Frequencies
1	893.38	11	4827.4
2	941.22	12	6343.4
3	1379.3	13	6360.6
4	1495.3	14	6641.8
5	2448	15	6728.9
6	2551.8	16	7411.4
7	2767.9	17	8620.9
8	3560.9	18	8796.2
9	4111.3	19	9027.2
10	4366.8		

3.5.5 Piston

Piston is a part of the disc brake system thus it is also considered for simulation. The FE model of piston and the meshed model with 1244 elements C3D8 are shown in figure 3.20. Similarly, a modal analysis was performed to find out natural frequencies and mode shape of the piston.

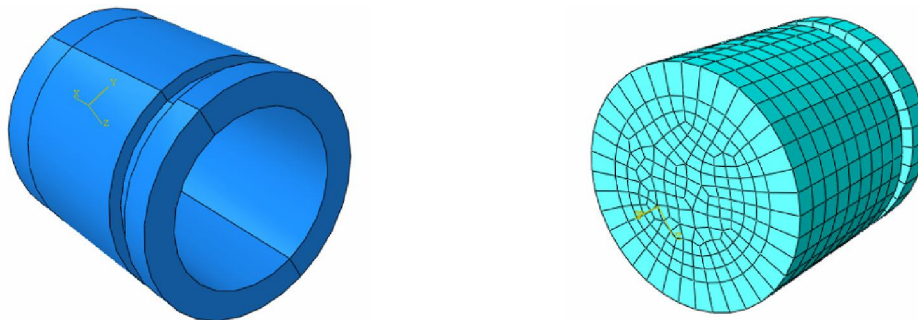


Fig 3.20 FE model (left) and meshed (right) of piston

In the investigated range of frequencies, the piston has only one mode shape at 9020 Hz as in figure 3.21.

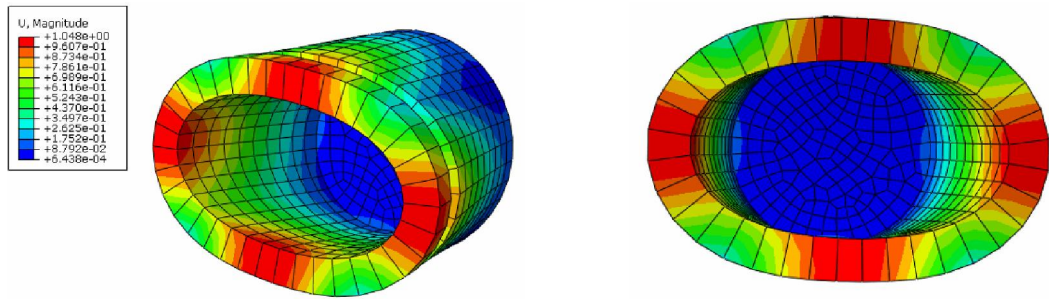


Fig 3.21 Mode shape of piston at 9020 Hz

3.5.6 Bolt and Guide Pin

Bolt and guide pin was also simulated, the FE model and the meshed model with 682 elements C3D4 are shown in figure 3.22. From the simulation results, it can be seen that there are two mode shapes with first bending at 6586.1 Hz and first twist at 10890 Hz as in figure 3.23.

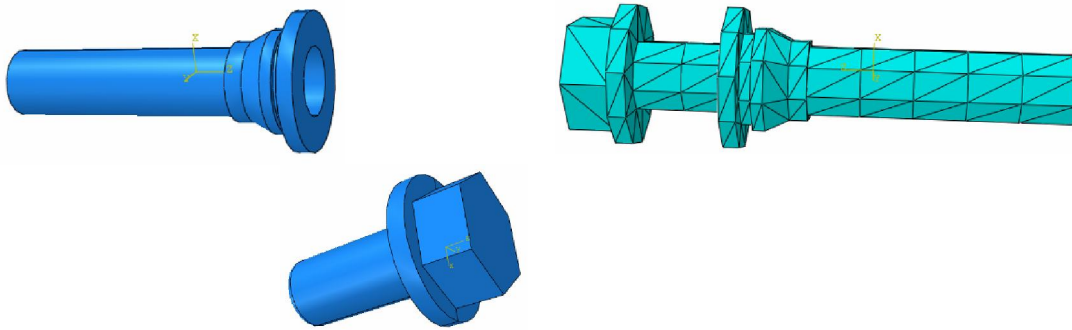


Fig 3.22 FE model (left) and meshed (right)

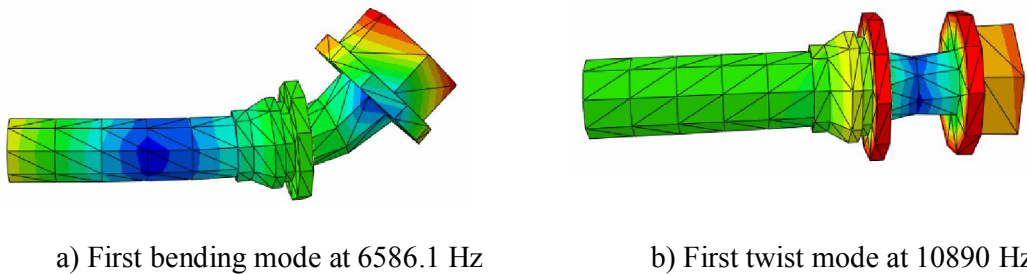


Fig 3.23 Mode shape of bolt and guide pin

3.5.7 Disc brake assembly

The disc assembly is meshed by 39293 elements C3D4 type, see in figure 3.24. The results of simulation natural frequencies of the full disc brake assembly with boundary conditions are shown in table 3.7. The results of mode shape are shown in figure 3.25.

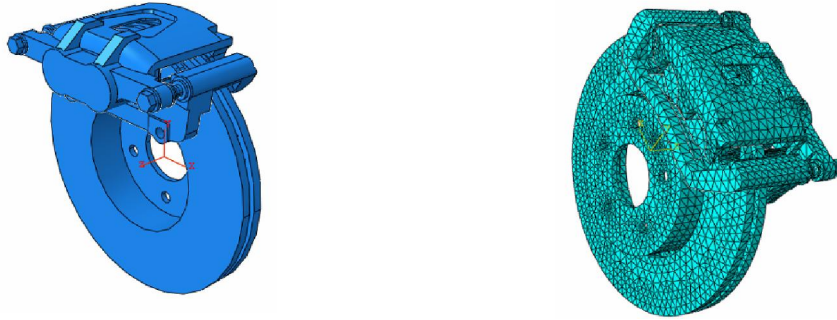
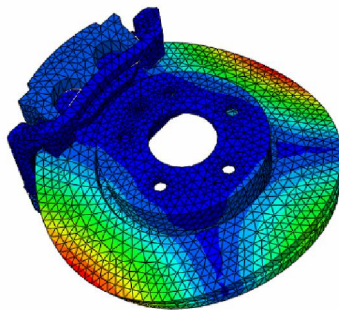


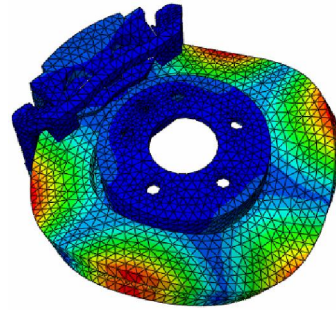
Fig 3.24 FE model (left) and meshed (right) of disc brake assembly

Table 3.7 Natural frequencies of disc brake assembly.

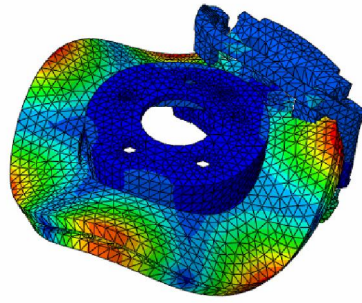
	2 ND	3 ND	4 ND	5 ND	6 ND	7 ND
Simulation (Hz)	1110.1	2277.1	3565.8	5752.3	7033.7	9013.2
		2980		5782.5	7514.1	



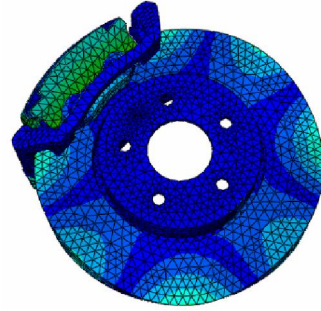
a) 2 ND at 1110.1 Hz



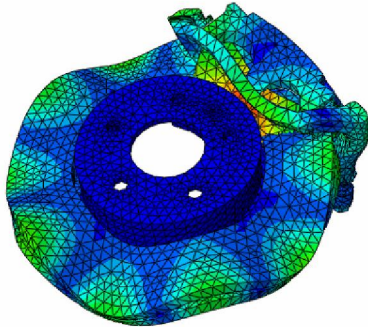
b) 3 ND at 2277.1 Hz



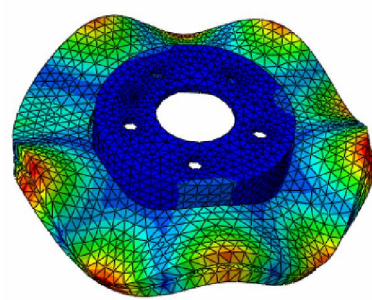
c) 3 ND at 2980 Hz



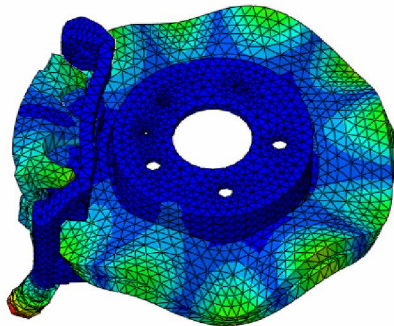
d) 4 ND at 3565.8 Hz



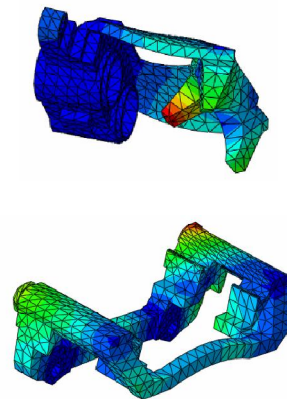
e) 5 ND at 5752.3 Hz



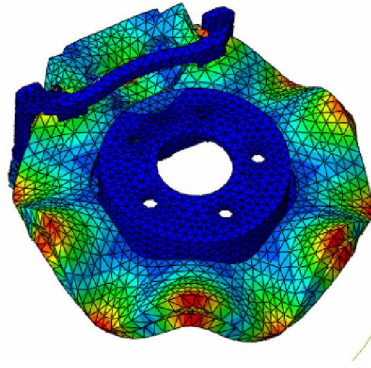
f) 5 ND at 5782.5 Hz



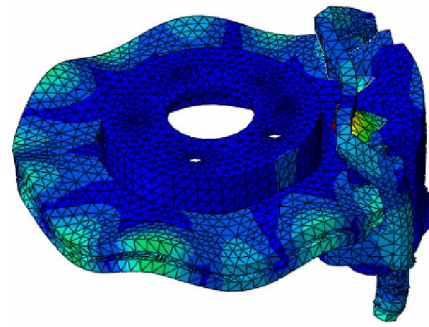
g) 6 ND at 7033.7 Hz



h) Caliper and carrier at 7514.1 Hz



i) 6 ND at 7514.1 Hz



k) 7 ND at 9013.2 Hz

Fig 3.25 Mode shapes of disc brake assembly

3.6 CONCLUSION

The objective of this chapter is to develop a FE model from the actual disc brake of car Renault Trafic. Through this model, a modal analysis was performed to found the natural frequencies and the mode shape of the disc brake components at free-free boundary conditions and the disc brake assembly at full boundary conditions. From the main objective, some of results can be drawn as follows:

Modal analysis is a method that can be used to describe a structure in terms of its natural characteristics which are the frequency, damping and mode shapes. It is useful for designer to observe the structure characteristics and having adjusted or optimization structure. Meanwhile experimental modal analysis is used to explain a dynamics problem, vibration or acoustic and confirm with simulation results. The theoretical finite element analysis and experiment modal analysis have been very separate engineering activities aimed at solving common problem. This is the first stage for prediction vibration and noise of disc brake, which will be mentioned in the next chapter.

The resonance is often the cause of vibration and noise that occur in structure when operating machinery or at least contribute factor to vibration and noise in general structure system. Thus, modal analysis has become a widespread means of finding the modes and natural frequencies of the disc brake components. In this analysis, there are some values of

modes of caliper, carrier, pad and disc closes to natural frequencies together. Thus, maybe it makes resonance and occurs vibration and noise.

An experimental investigation of researchers shows that the vibration frequencies of a disc squeal are influenced by natural frequencies. While vibration mode and frequency of a squealing disc brake's rotor are influenced by natural frequencies and modes of stationary rotor. Thus, brake squeal occurs in the vicinity of natural frequencies of disc brake components. In this chapter, a finite element model was analyzed then compares it with experiment method. Hence, the results are more confident.

Mode shape and natural frequencies depend on the material properties and geometries of structure. In this simulation, the natural frequency at mode shape that has bending was presented because it effects directly to vibration in perpendicular plane with rotor plane. In this analysis, finding out natural frequencies of components of disc brake will provide information to predict vibration and noise of the disc brake and its contribution on brake squeal.

PREDICTION STABILITY OF DISC BRAKE SYTEM

4.1. INTRODUCTION

In the previous chapter, a modal analysis was performed to find out natural frequencies and mode shape of the disc brake components. This is the first step for identifying and predicting vibration and noise of brake system. But, this problem is complicated and influenced by many factors, this lead to difficult to predict the brake squeal. Most theories explain the cause of vibration and noise of disc brake due to loss stability of the structure. The main causes due to a couple friction between rotor and pads. For this reason a number of studies were performed to explain the cause and investigated the effect of factors such as geometries of components, material properties and frictional coefficient. Some of these researches are summarized as follows.

AbuBakar and Ouyang [58] in 2006 presented a complex eigenvalue analysis and dynamic transient analysis in predicting disc instable of disc brake. In this research, three different contact regimes are examined in order to assess the best correlation between the two methodologies. The results compare the prediction between two schemes: finite sliding and small sliding. AbuBakar et al [98] analyzed the stability of disc brake squeal considering temperature effect. They performed a complex eigenvalue analysis to predict disc brake squeal with and without temperature effect. The result shows that temperature is important in the prediction procedure but it is difficult to compare experiment results with FE simulation results.

In 2007, Liu [49] performed an nonlinear analysis and complex eigenvalue for extraction instability modes of disc brake. An attempt was made to investigate the effects of factors such as the hydraulic pressure, coefficient friction between pads and disc surface, rotational velocity of the disc, the stiffness of the disc, pads. Through this analysis the author shows the solution to reduce vibration and noise of the disc brake as decreasing the friction coefficient, increasing the stiffness of disc, using damping material for back plates of pads

or modifying the shape of the brake pads. This is a good solution to design the disc brake system.

A theoretical model is presented in the research Joe and co-worker [99]. They performed an analysis of disc brake instability due to friction and the effect of parameter model. The model consists of lumped and distributed parameter components. The disc is modelled as a simple beam with infinite degrees of freedom. From the governing equation of an Euler-Bernoulli beam in sliding contact with an elastic medium, the eigenvalue problem can be calculated in order to solve for the eigenvalues and eigenvectors. The theoretical results were compared with the experiment results. The effect of some factors to the brake squeal was also simulated by EF analysis.

In 2009, some authors also used the complex eigenvalue for modal analysis and extract to find instability mode of frequencies. Nouby [47] studied a combined approach of complex eigenvalue analysis and design of experiments to study disc brake squeal. A simulation of influence of friction coefficient on brake noise was performed by Meng [100]. Sinou presented transient non-linear dynamic analysis of automotive disc brake squeal [101]. In 2012, Nouby continued to study a FE analysis and modal testing of a commercial disc brake assembly [102]. The purpose of this study is to predict at the early stage of design disc brake and explain the cause of vibration and noise of couple modes.

Based on some recent researches that mentioned above, the main methods are used for predicting brake squeal including: complex eigenvalue analysis and dynamic transient analysis. Although authors try to use these methods for studying natural frequencies, mode shape, instability of frequencies modes and effect of some parameters as friction, velocity of disc, material properties,.. But they have some limitations such as:

- (1) The simulation results used for explaining why the noise and vibration occur do not focus on predict noise and vibration for real disc brake in variable operation cases.
- (2) Theoretical model used for calculation is not exact to the real disc brake car because the geometries of the disc brake are complex.
- (3) The limitation of experimental results shows only at the modal experiments, identify natural frequencies and mode shape of disc brake.

Due to these limitations, a deep study is presented in this chapter. First, a theory of complex eigenvalue analysis is presented for applying to the simulation. Second, a simulation from

previous chapter is continued to extract the real part and imagine part of eigenvalue for prediction unstable modes in frequency range from 1 kHz to 10 kHz. A full real disc brake with full boundary condition will be simulated in different operational cases. It plays an important role in prediction of vibration and noise in the real operating conditions. Next, a sensitivity analysis of factors as coefficient friction, pressure line, rotation of the rotor and material properties of a disc effect on brake squeal is studied

4.2. COMPLEX EIGENVALUE ANALYSIS

There are many important areas of structural analysis in which it is essential to be able to extract the eigenvalues of the system and, hence, obtain its natural frequencies of vibration or investigate possible bifurcations that may be associated with kinematic instabilities [103]. The mathematical eigenvalue problem is a classical field of study, and much work has been devoted to providing eigenvalue extraction methods. For many important cases, the matrices are symmetric. The algorithm for eigenvalue problem is introduced by Abaqus [104] as below. The eigenvalue problem for finding natural frequencies and modes shape with small vibration of a finite element model is

$$M\ddot{x} + C\dot{x} + Kx = 0 \quad (4.1)$$

The governing equation can be rewritten in classical matrix notation as

$$(\lambda^2[M] + \lambda[C] + [K])\{\phi\} = 0 \quad (4.2)$$

Where $[M]$ is the mass matrix, $[C]$ is the damping matrix, which includes friction induced contributions, and $[K]$ is the stiffness matrix, which is asymmetric due to friction which λ is eigenvalue, Φ is eigenvector – the mode of vibration. The eigensystem in (4.1) in general will have complex eigenvalues and eigenvectors. This system can be symmetrized by assuming that is symmetric by neglecting $[C]$ during eigenvalue extraction. The symmetrized system has real squared eigenvalues, λ^2 , and real eigenvectors only.

Typically, for symmetric eigenproblems we will also assume that $[K]$ is positive semidefinite. In this case λ becomes an imaginary eigenvalue, $\lambda = i\omega$, where ω is the circular frequency, and the eigenvalue problem can be written as

$$(-\omega^2[M] + [K])\{\phi\} = 0 \quad (4.3)$$

To solve eigenvalue extraction for symmetric systems Eq.(4.3) Abaqus provides eigenvalue extraction procedures for complex eigenproblems that the subspace projection method is used. In the subspace projection method the original eigensystem Eq.(4.2) is projected onto a subspace spanned by the eigenvectors of the undamped, symmetric system Eq.(4.3). Thus, the symmetrized eigenproblems must be solved prior to the complex eigenvalue extraction procedure to create the subspace onto which the original system will be projected. Next, the original mass, damping, and stiffness matrices are projected onto the subspace of N eigenvectors:

$$\begin{aligned} [M^*] &= [\phi_1, \dots, \phi_N]^T [M] [\phi_1, \dots, \phi_N] \\ [C^*] &= [\phi_1, \dots, \phi_N]^T [C] [\phi_1, \dots, \phi_N] \\ [K^*] &= [\phi_1, \dots, \phi_N]^T [K] [\phi_1, \dots, \phi_N] \end{aligned} \quad (4.4)$$

Then, the projected complex eigenproblem become

$$(\lambda^2 [M^*] + \lambda [C^*] + [K^*]) \{\phi^*\} = 0 \quad (4.5)$$

Finally the complex eigenvectors of original system can be obtained by

$$\{\phi\} = [\phi_1, \dots, \phi_N]^T \phi^* \quad (4.6)$$

The eigenvalues and the eigenvectors of Eq. (4.2) may be complex, consisting of both a real and imaginary part. For under damped systems the eigenvalues always occur in complex conjugate pairs. For a particular mode the eigenvalue pair is:

$$\lambda_{i,1,2} = \alpha_i \pm i\omega_i \quad (4.7)$$

The damping ratio is defined by formula:

$$\zeta_i = -2 \frac{\alpha_i}{|\omega_i|} \quad (4.8)$$

Where α_i is real part that indicating the stability of the system and ω_i is imaginary part expressed the mode frequency. The value of $Re(\lambda)$ is positive, it causes the amplitude of oscillations to increase with time. Therefore the system is unstable when the damping coefficient is positive. Related to damping ratio, if the damping ratio is negative the system becomes unstable that is same with case of the value of real part is positive.

4.3. STABILITY PREDICTION OF DISC BRAKE

4.3.1 Procedure for prediction

As mentioned above, the main cause of vibration and noise of the disc brake is friction between two surfaces of disc and pads when pressure applied. When the structure of rotor and pad becomes loose stability and raising vibration cause of occurrence noise. To predict vibration and noise of disc brake, a complex eigenvalue analysis can be performed. This analysis was completed by helping of software Abaqus. The simulation was performed in operational cases of the brake system.

According to chapter 3, a simulation with extraction of the instable complex eigenvalue theory was performed. The natural frequency and mode shape of the components of the disc brake are found. It is continued analysis with complex eigenvalue analysis to incorporate the effect of friction coupling. First, the model is set up in software Abaqus as modal analysis but there is a little change in the friction coefficient ($\mu = 0.35$), the angular velocity ($\Omega = 9.7$ rad/s) and the pressure ($P = 3$ bar). Next, a simulation was investigated for a changing coefficient friction between rotor and pads from 0.1 to 0.6 for observation unstable modes of structure in frequency range from 1 kHz to 10 kHz.

4.3.1.1. Coefficient friction

Coefficient friction between disc and pads plays an important role for vibration and noise of the disc brake. Thus, identifying exactly coefficient friction will help to have good results of prediction. However, in actual operating regime it is difficult to identify exactly this coefficient friction. Because coefficient friction between the two surfaces of rotor and pads depend on many factors as temperature, pressure, velocity, humidity, material, and shape of surface contact. For some reasons above, TRW Company made an experiment of coefficient friction in fifteen operational regimes of disc brake as shown in figure 4.1. The experiment was performed with a couple commercial pads against the rotor when pressure was applied. Value of coefficient friction $\mu = 0.35$ is the average value of fifteen tests. Base on this experiment, a simulation with coefficient of friction $\mu = 0.35$ is used for complex eigenvalue analysis.

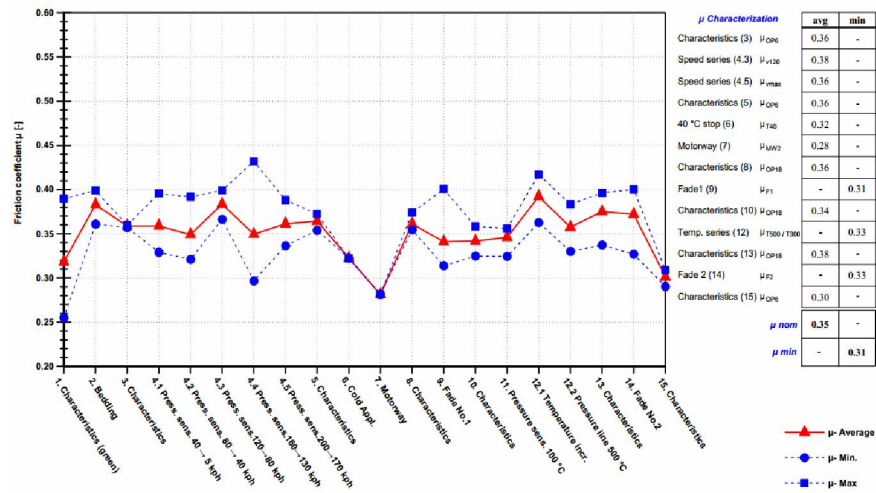


Fig 4.1 Experiment coefficient friction

4.3.1.2. Friction Model

For simulation in Abaqus, when surfaces are in contact, they usually transmit shear as well as normal forces across their interface. Generally, there is a relationship between these two force components. The relationship, known as the friction between the contacting bodies, is usually expressed in terms of the stresses at the interface of the bodies. The friction model is used in this analysis including type of classical isotropic Coulomb friction model. It allows the friction coefficient to be defined in terms of small slip rate and contact pressure and it is used as a constant in static and a kinetic friction coefficient with a smooth transition zone defined by an exponential curve (see [104] for details). Table 4.1 presents a setting for interaction between couple surfaces as pads and disc, caliper and pad, carrier and pad, or carrier and guide pin. Where, normal behavior with hard contact model refers that the surfaces transmit no contact pressure unless the nodes of the slave surface contact the master surface; no penetration is allowed at each constraint location; there is no limit to the magnitude of contact pressure that can be transmitted when the surfaces are in contact.

Table 4.1 Type of interaction between surfaces in software Abaqus

Pair friction	Type	Behaviour
Pad and Disc	Surface to surface	Normal = Hard contact Tangential = friction
Caliper and Pad	Surface to surface	Normal = Hard contact
Carrier and Pad	Surface to surface	Normal = Hard contact
Piston and Caliper	Surface to surface	Normal = Hard contact Tangential = friction
Piston and Pad	Surface to surface	Normal = Hard contact Tangential = friction
Carrier and Guide pin	Surface to surface	Normal = Hard contact

Rest of pair friction contact that have tangential behaviour, a coefficient of friction $\mu = 0.05$ were setting. With tangential behaviour properties, the penalty method approximates hard pressure over closure behavior. In this method, the contact force is proportional to the penetration distance, so some degree of penetration will occur. Some advantages of the penalty method include: (1) numerical softening associated with the penalty method can mitigate over constraint issues and reduce the number of iterations required in an analysis; (2) the penalty method can be implemented such that no Lagrange multipliers are used, which allows for improved solver efficiency more. Details of the method can be seen in [104].

4.3.2 Predict vibration and noise of brake squeal

4.3.2.1. Extraction of the unstable complex eigenvalue

Disc brakes operate by pressing a set of brake pads against a rotating disc. The friction between the pads and the disc causes deceleration, but it may also induce a dynamic instability of the system, known as brake squeal. One possible explanation for the brake squeal phenomenon is the coupling of two neighboring modes. Two modes, which are close to each other in the frequency range and have similar characteristics, may merge as the friction contribution increases. When these modes merge at the same frequency (become coupled), one of them becomes unstable. The unstable mode can be identified during complex eigenvalue extraction because the real part of the eigenvalue corresponding to an unstable mode is positive. The brake system design can be stabilized by changing the

geometry or material properties of the brake components to decouple the modes [104]. The purpose of this analysis is to identify the unstable modes cause of vibration and noise in a particular disc brake system.

The first, consider case of normal operational regime of disc brake system at $\mu = 0.35$, $P = 3$ bar, $\Omega = 9.7$ rad/s. A simulation for this case was performed and the results of unstable modes in range 1 kHz to 10 kHz shown as table 4.2.

Table 4.2 Unstable modes at $\mu = 0.35$

Mode	Frequencies (Hz)	Real part	Effective damping ratio
Mode 33	4521.4	14.430	-0.00102
Mode 36	4837	19.248	-0.00127
Mode 72	9437.1	275.53	-0.00929

Totally in range 10 kHz, there are 76 modes but only have three unstable modes, first at 4521.4 Hz, second mode at 4837 Hz and the last mode at 9437.1 Hz. In this analysis performs eigenvalue extraction to calculate the complex eigenvalues and the corresponding complex mode shapes of a system. It can be seen through the positive of real part value and negative of effective damping ratio value, as shown in figure 4.2.

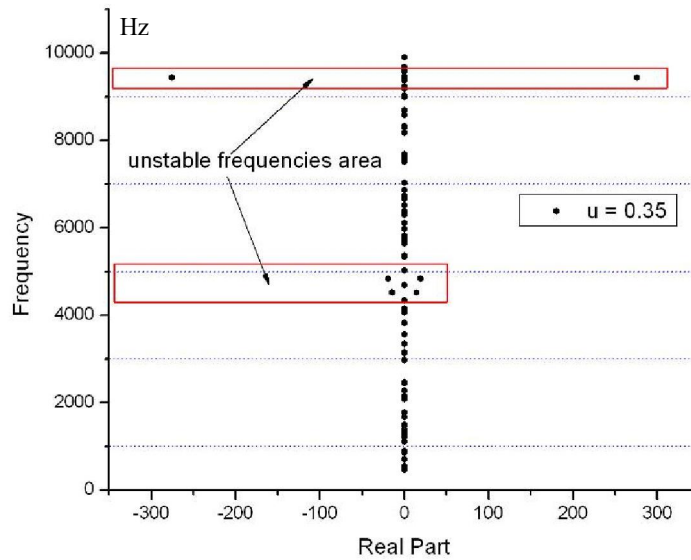


Fig 4.2 Real part of all modes

Figure 4.2 shows the real part at unstable frequencies of disc brake and figure 4.3 shows the effective damping ratio value. By following the theory as mentioned above, when the damping ratio is negative that means the system will be unstable. Mode shape of frequencies is shown in figure 4.4.

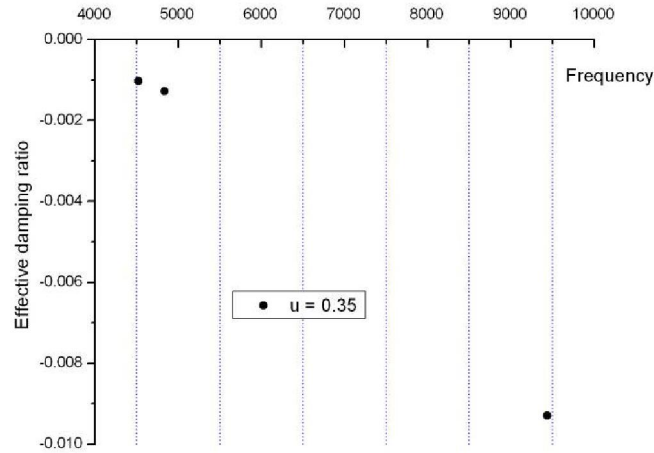


Fig 4.3 Effective damping ratio value of unstable modes

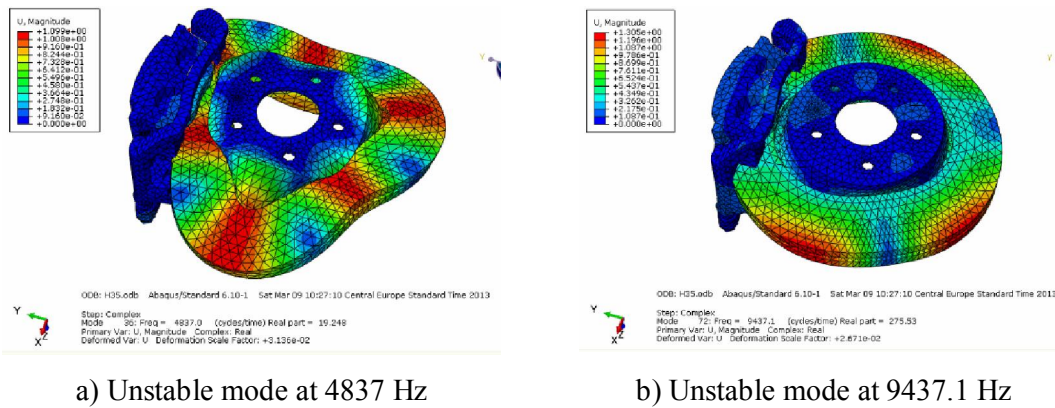


Fig 4.4 Mode shape of unstable frequencies

To observant the effect of the unstable frequency by friction coefficient, a sensitivity analysis with friction coefficient from 0.1 to 0.6 was performed. The results of simulation unstable modes are shown in table 4.3.

Table 4.3 Unstable frequencies at various friction coefficients

Friction coefficient	Mode	Eigenvalue	
		Real part	Frequencies (Hz)
0.1	36	2.0397	4836.5
	36	10.459	4836.5
0.2	44	6.1645	5755.3
	72	64.178	9423.2
	36	17.641	4836.9
0.3	44	35.179	5757.5
	72	130.8	9430.5
	32	10.721	4519.9
0.4	36	21.107	4837
	72	366.3	9441.5
	36	21.771	4836.3
0.5	58	5.9007	7655.6
	72	402.97	9446.2
	36	24.126	4836.3
0.6	58	15.659	7655.8
	69	219.4	9332.1
	72	596.5	9461.8
	76	131.08	9877.4
	36	24.126	4836.3

Through this sensitivity analysis, it can be seen that corresponding with small friction coefficient there exist a little bit number of unstable frequencies while with higher friction coefficient that exist more number of unstable frequencies. In table 4.3, there is only one unstable frequency at the $\mu = 0.1$ and there are three unstable modes at $\mu = 0.2$ to 0.5 but when $\mu = 0.6$ there are five unstable modes. Range of unstable frequencies occurs at 4500 Hz to 10000 Hz, for frequencies less than 4500 Hz there are more stability thus in this analysis there is no unstable frequency occurrence. Especially, mode 36 always occur in the investigated range, and mode 72 occurs at fiction coefficient $\mu = 0.2 \div 0.6$. Figure 4.5 presents the real part value at unstable frequencies.

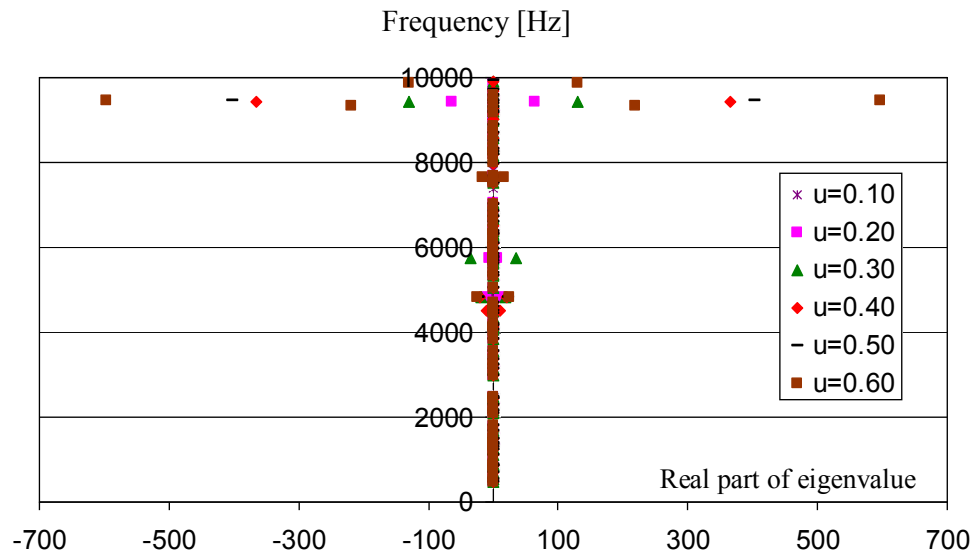


Fig 4.5 the real part value at various friction coefficient

In this figure, it can be seen that with various friction coefficient, the unstable frequencies occur at 4.8 kHz, 5.7 kHz, 7.6 kHz and 9.4 kHz. All of modes have real part negative value except modes which created stable-unstable pair. That means the eigenvalue occurs in conjugate pairs and symmetric about the frequency axis. At mode shapes which have the same frequency, they have the same mode shape form as shown in figure 4.6.

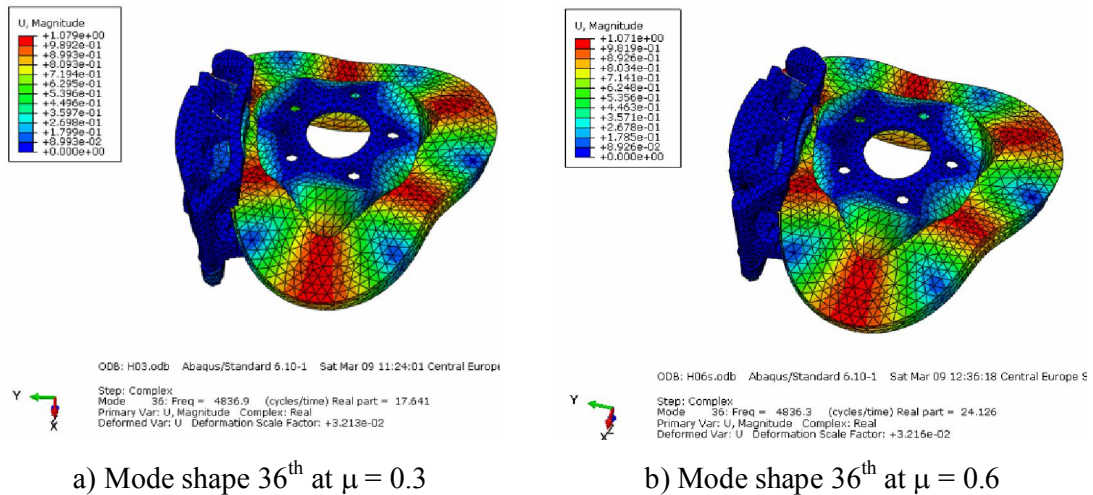


Fig 4.6 Mode shape 36th at various friction coefficients

4.3.2.2. Analysis a couple unstable modes

To demonstrate the squeal occurrence of the disc brake, it is necessary explanation about couple mode. The FE analyses by complex modes indicate that when two modes close to each other in the frequency range coalesce under the influence of friction and then become coupled, thus the system becomes unstable [102]. This analysis shown that there are seven couple adjacent modal at unstable frequencies as pair of mode 31 & 32, mode 35 & 36, mode 43 & 44, mode 57 & 58, mode 68 & 69, mode 71 & 72 and mode 75 & 76. Below, presents some of modal modes with comments on this analysis. Consider mode 35 and 36 in figure 4.7 and figure 4.8 as below.

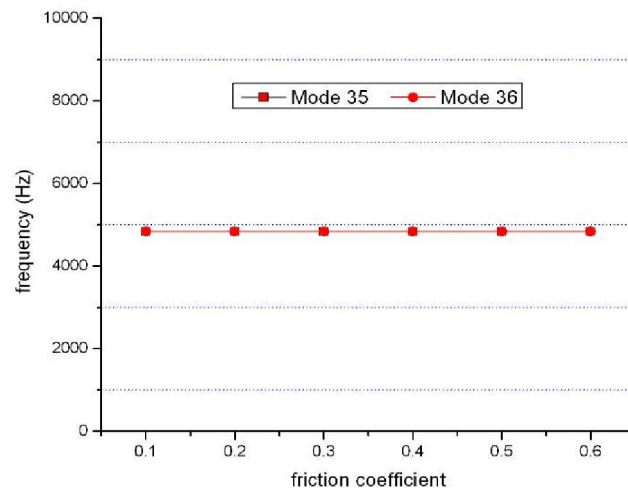


Fig 4.7 Unstable frequencies at various friction coefficients

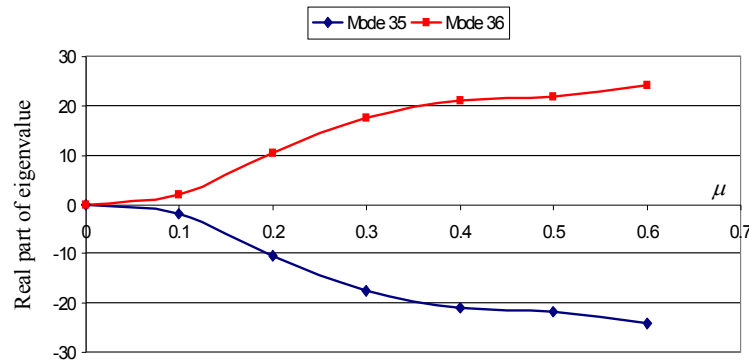


Fig 4.8 Value of real part at different friction coefficients

It can be seen, mode 35 & 36 has the same value of eigenvalue including real part and frequency, thus this pair always unstable when the friction coefficient have changing.

Next, a modal couple of mode 57 & 58 was considered as in figure 4.9 and 4.10. The figure 4.9 shown coupling of two neighboring modes. At the initial stage, the two modes have different frequencies in friction coefficient range from 0.1 to 0.4 but at $\mu = 0.4$ they merge together at the same frequency and become coupled. One of them is unstable (mode 58). When the two modes merge to one, it illustrates that the friction coefficient value at this point becomes critical friction coefficient and the system will be unstable. It is clearer to see in figure 4.10. At $\mu = 0.4$, the real part is zero and its value starts to increase, at friction coefficient 0.5 and 0.6 it becomes positive and is divided into two lines that opposite together.

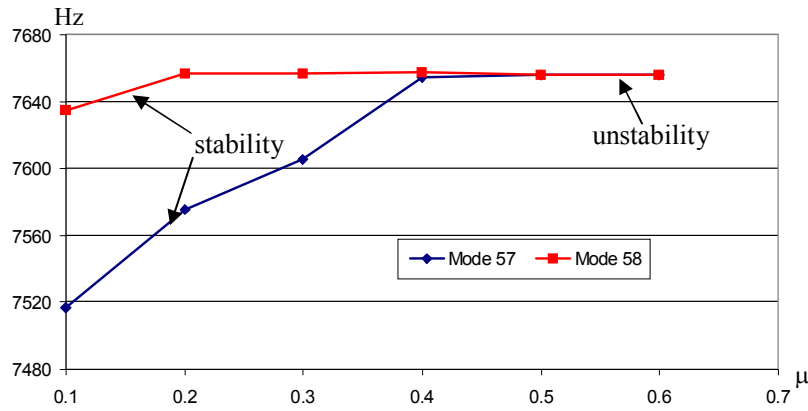


Fig 4.9 Unstable frequencies at various friction coefficients

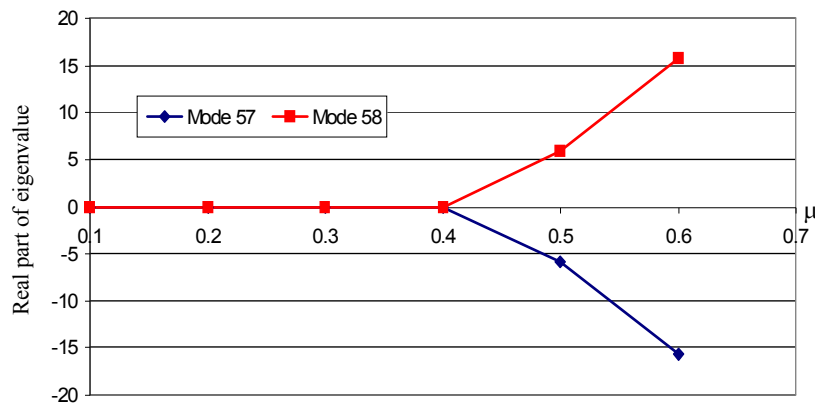


Fig 4.10 Real part value at different friction coefficients

An effect damping ratio of modes 57 & 58 are shown in figure 4.11. By the same ways, the rest of modal modes are presented as in figures below. Figure 4.12 shows two modes 68 & 69 with high stable at $\mu = 0.6$. The two modes are merged and become unstable complex mode.

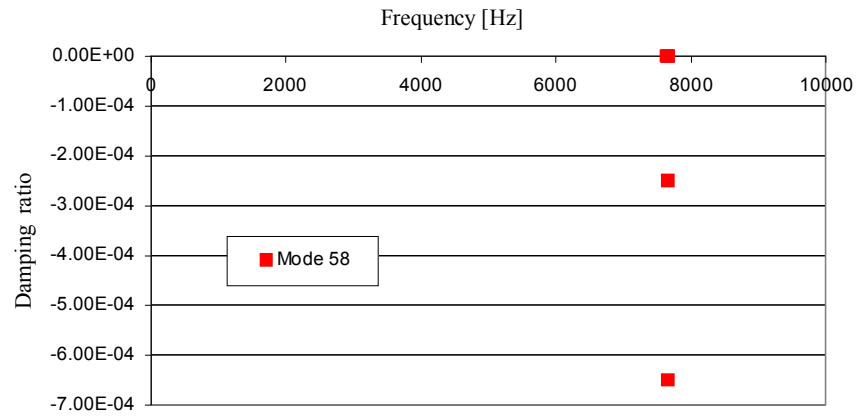


Fig 4.11 Effect damping ratio of mode 58

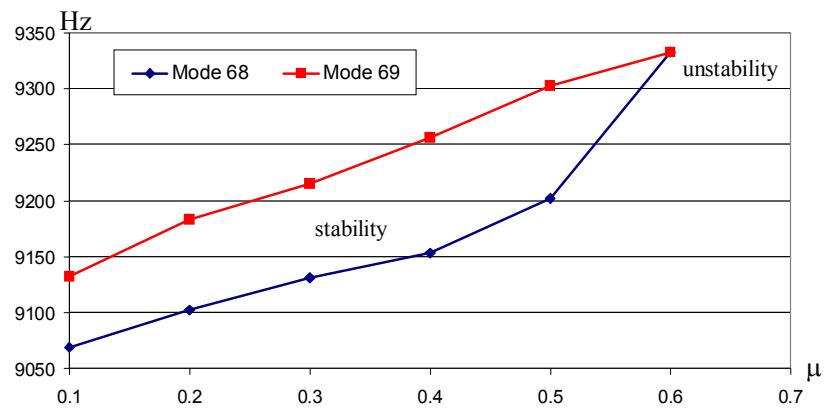


Fig 4.12 Unstable frequencies at various friction coefficients of pair mode 68 & 69

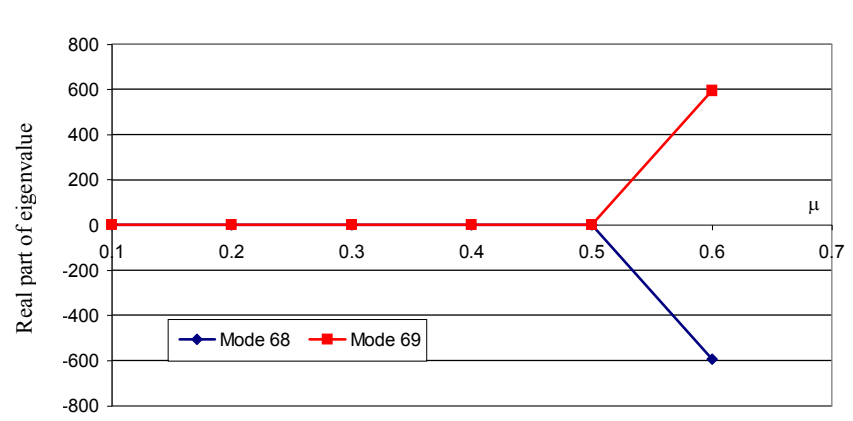


Fig 4.13 Real part value at different friction coefficients of pair mode 68 & 69

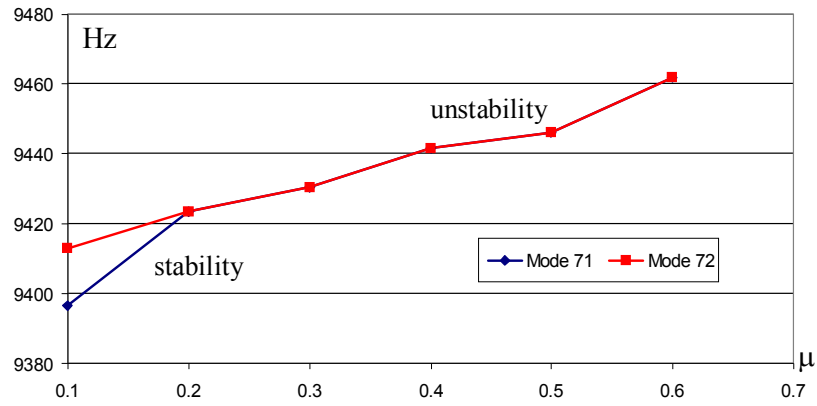


Fig 4.14 Unstable frequencies at various friction coefficients of pair mode 71 & 72

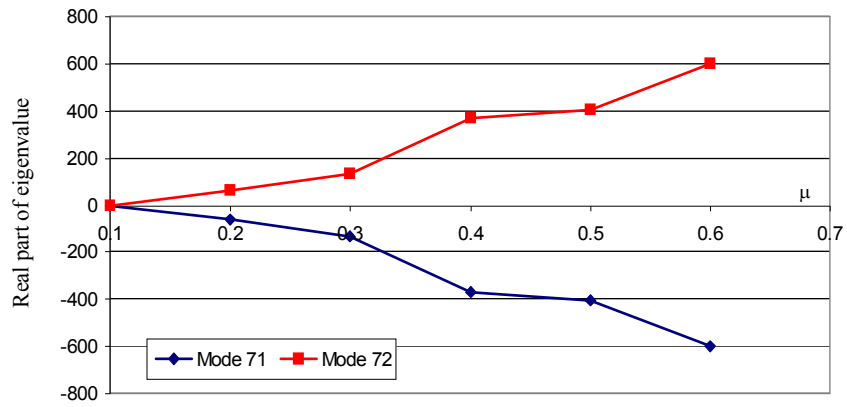


Fig 4.15 Real part value at different friction coefficients of pair mode 71 & 72

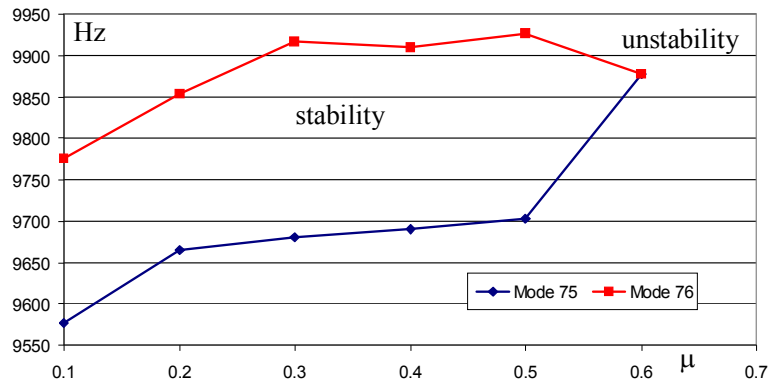


Fig 4.16 Unstable frequencies at various friction coefficients of pair mode 75 & 76

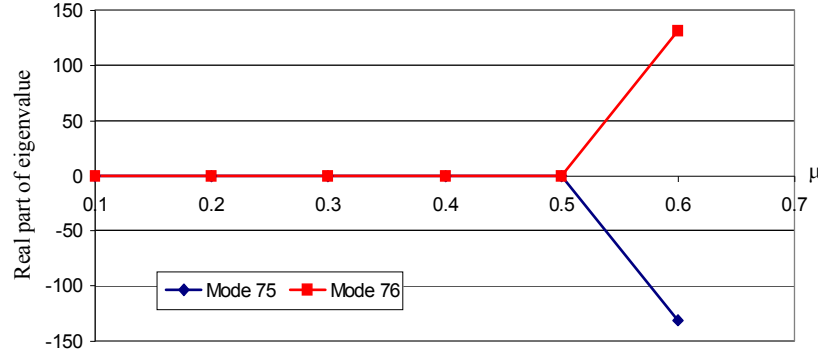


Fig 4.17 Real part value at different friction coefficients of pair mode 75 & 76

For modal couple of modes 71 & 72 as shown in figure 4.14, it can be seen that the stable mode only at friction coefficient $\mu = 0.1$ and loss stability since $\mu = 0.2$. At that friction coefficient, mode 71 and mode 72 which are close to each and overlap together with the same frequency and they have similar characteristic. At $\mu = 0.2$ is the limitation of friction coefficient that keep the system is stable. Therefore, it is called critical friction coefficient. For modal couple of modes 75 & 76 as showed in figure 4.16 and 4.17. These modes are opposite with the modes 71 & 72. The stability of structure is stable in friction coefficient range from 0.1 to 0.5. At $\mu = 0.6$ these mode merge to one and loss stability as in figure 4.16 and in figure 4.17 shows value of real part that is positive at this friction coefficient.

Figure 4.18 presents some of modal modes at unstable frequencies that occur noise. With modal modes have the nodal diameter in bending form as mode 44 and mode 69. It is easy to occur noise because they have vibration in Z direction or XY plan. The modes 44 have five nodal diameters. From this simulation, noise occurrence at unstable frequencies of system can be predicted because they have relationship with the natural frequencies of disc brake components. For example, mode 32 with frequency of 4519 Hz (figure 4.18a) is near natural frequency (4447.6 Hz) at mode 18th of the rotor. Mode 36 at frequency of 4836.3 Hz in figure 4.6 has frequency near natural frequency of the pad (4703.5 Hz). Similarity, mode 58 of 7655.6 Hz in figure 14.8c has frequency close to natural frequency of mode 14th at 7624.1 Hz of the pad, frequency of mode 72 in figure 4.18e is close to that of mode 21 of the pad (9423.2 Hz with 9435.7 Hz). Frequency of mode 76 in figure 4.18f is close to that of mode 25 of the pad (9877.4 Hz with 9849.7 Hz). Thus, squeal frequencies are often close to

nature frequencies of one or more of the disc brake components or near some natural frequencies of the statically coupled system.

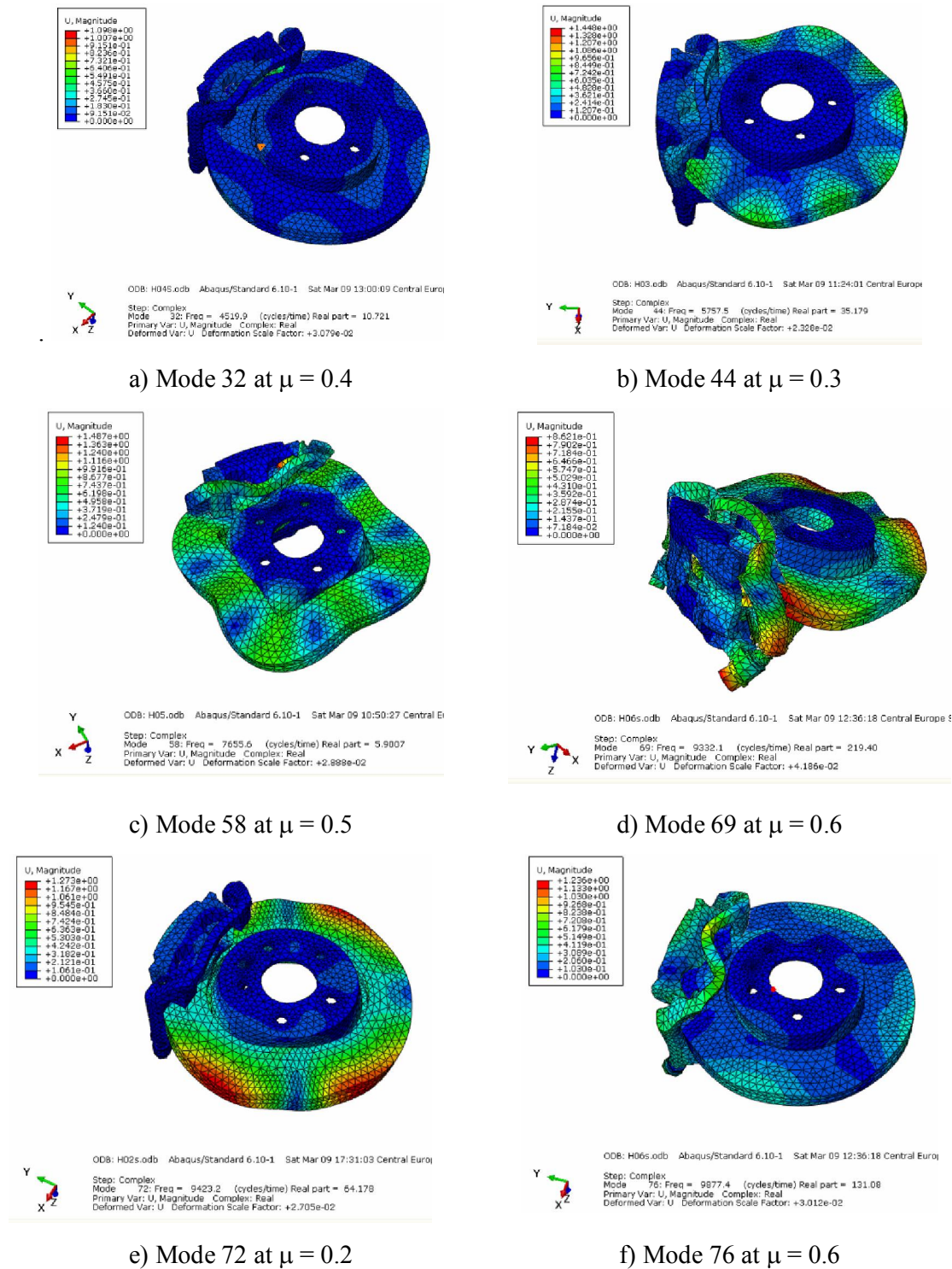


Fig 4.18 Mode shape of unstable mode in range frequency 1 kHz to 10 kHz

4.4. EFFECT OF FACTORS TO STABILITY

According to the analysis above, friction coefficient has more effect to stability of system. It is the main cause of vibration and relate directly to noise of disc brake. Besides, there are more factors that affect significantly to the brake squeal such as material properties, damping, velocity or pressure. Thus, to observe more clear, a study on the effect of Young's modulus of disc and pad, and angular velocity, and pressure will be presented.

4.4.1 Effect of Young's modulus

Young's modulus, also known as the tensile modulus or elastic modulus, is a measure of the stiffness of an elastic material and is a quantity used to characterize materials. The elastic constants of friction or lining material can have significant influence on the stability of a brake system as shown by complex mode analysis. In this section, a sensitivity analysis with various Young's modulus of disc and pad will be performed. First, for the rotor component, initial parameter for analysis will be take from previous part with parameters as pressure $P = 3$ bar, angular velocity $\Omega = 9.7$ rad/s, friction coefficient of 0.35 and initial Young's modulus of disc $E_d = 95.3$ GPa. Then, change Young's modulus as $0.7E_d$, $0.8E_d$, $0.9E_d$, $1.1E_d$ and $1.2E_d$ for sensitivity analysis. The result is shown in figure 4.19. It can be seen that the real part value of disc trend increase of amplitude and frequency when the Young's modulus increases. The unstable modes occur at higher frequency, for example mode at $1E_d$ value compare with mode at $1.1E_d$ value compare with mode at $1.1E_d$.

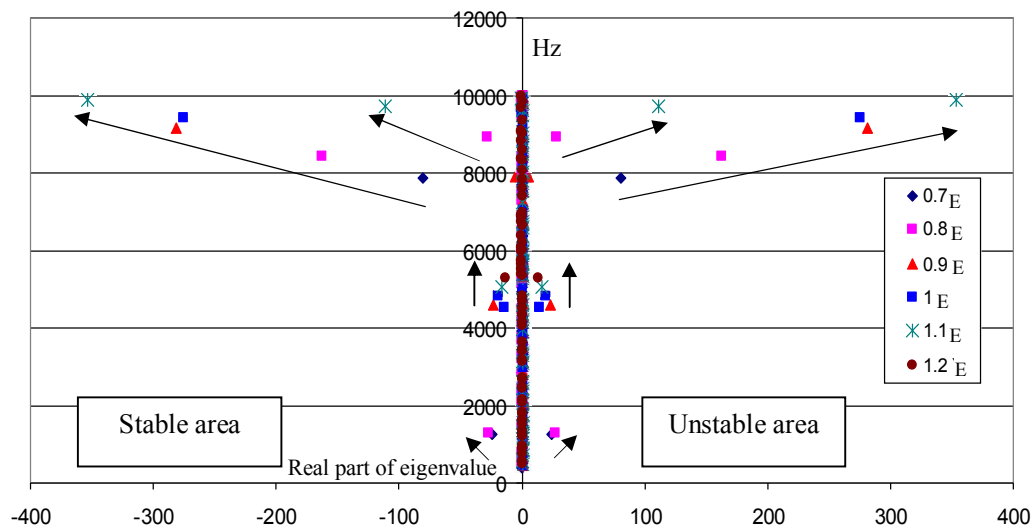


Fig 4.19 Effect of Young's modulus to unstable frequency of disc

Besides, in frequency range 10 kHz, with the value of small E_d , it can be seen that there are more modes than the value of high E_d as in figure 4.20. Complex modes of mode 35 and 36 are presented in figure 4.21 and the real part value is shown in figure 4.22.

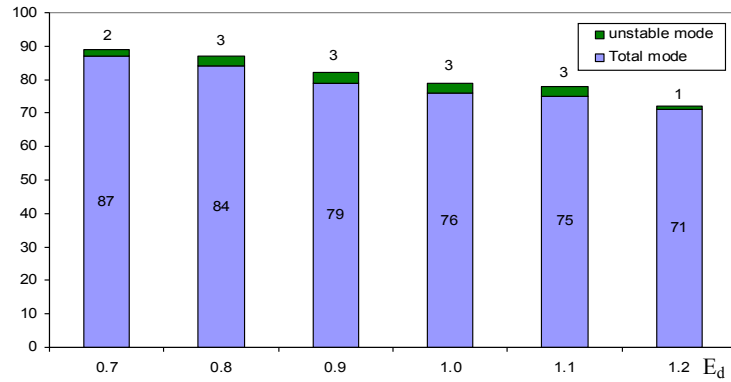


Fig 4.20 Number of modes in various Young's modulus

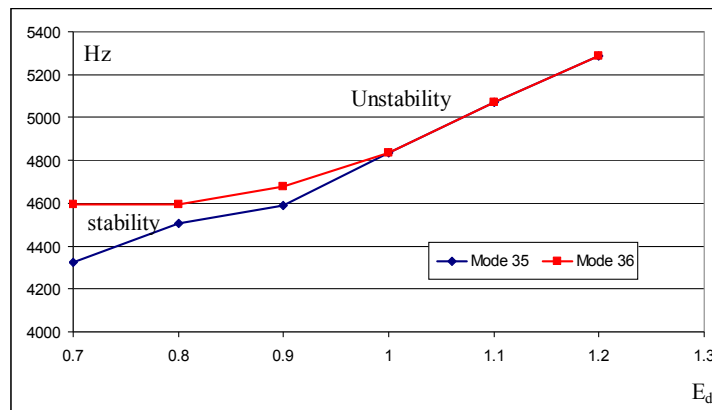


Fig 4.21 Complex modes in various Young's modulus

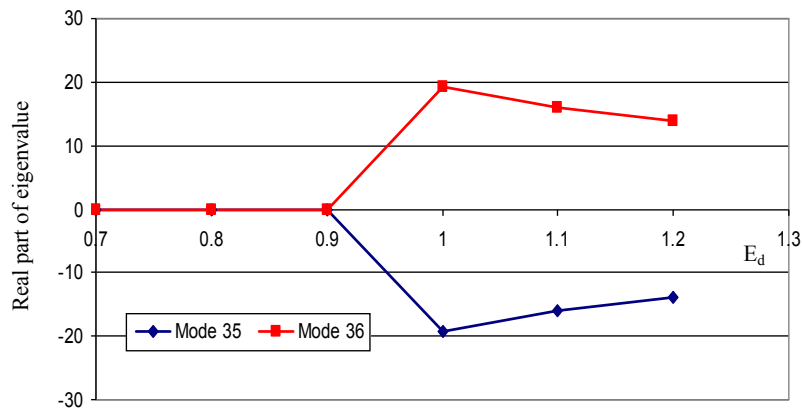


Fig 4.22 Real part value of complex mode 35 and 36 in various Young's modulus

Next, considers the changing of unstable frequency of disc brake when changing Young's modulus of pads. By change Young's modulus of pads from $E_p = 13$ MPa, to $0.7E_p$, $0.8E_p$, $0.9E_p$, $1.1E_p$, $1.2E_p$ while keeping the initial parameters. The results of unstable frequencies are shown in figure 4.23.

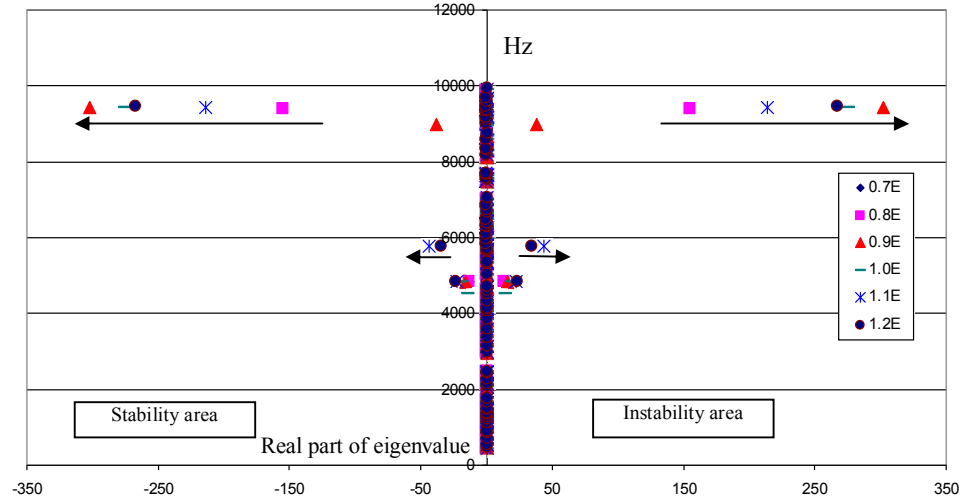


Fig 4.23 Real part value at unstable frequencies

This figure shows that when change the value Young's modulus of pad, the unstable frequencies are not significantly changed. At mode 36 with frequency of 4837 Hz, There is a little change from 4835.7 Hz at $0.7E_p$ to 4838.4 Hz at $1.2E_p$. Most of unstable frequencies are the same with case of $1.0E_p$ except case of $0.9E_p$, $1.1E_p$ and $1.2E_p$ occur unstable modes at 8997.7 Hz, 5766.7 Hz and 5776.3 Hz.

4.4.2 Effect of angular velocity

Simulation condition real operation of brake system with initial parameters as friction coefficient $\mu = 0.35$, pressure $P = 3$ bar, angular velocity $\Omega = 9.7$ rad/s. Next, changes angular velocity of the wheel with values as 1 rad/s, 4.9 rad/s, 9.7 rad/s, 48.7 rad/s and 97.5 rad/s. This analysis is conducted to predict the unstable frequencies of the system in range 10 kHz. The results showed that there are three unstable modes at around 4521 Hz, 4837 Hz and 9441 Hz as in figure 4.24. When changing Ω , there is a little bit change of unstable frequencies and they maintain at three unstable modes except at $\Omega = 97.5$ rad/s where the unstable frequencies is different as in figure 4.24.

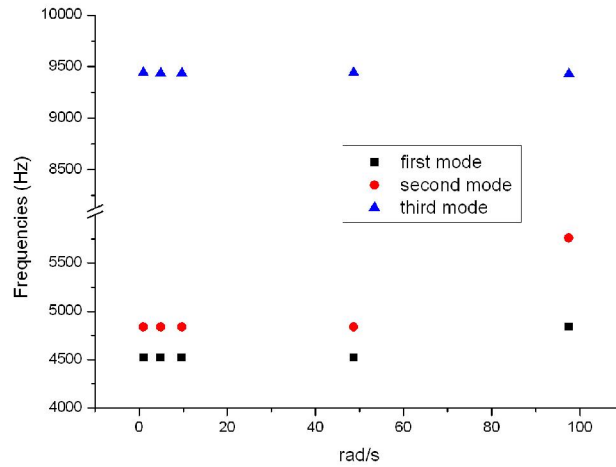


Fig 4.24 Predict unstable frequency by effect of angular velocity

4.4.3 Effect of pressure

The effect of braking pressure is introduced into the FE model through the variation of the contact stiffness between the rotor and pads and it can be describe as function of braking pressure [48]. Thus, this is related directly to the friction coefficient of the pad and the disc and presented in section 4.3.2.1. To simulate the influence of the pressure on brake squeal, a sensitivity analysis of pressure parameter was performed with values of pressure as 1 bar, 3 bar, 5 bar, 10 bar, 20 bar and 30 bar. The results of simulation are shown in figures 4.25 and figure 4.26.

In figure 4.25, it can be seen that with high braking pressure the noise tends to be louder. Specifically, in pressure range from 1 bar to 10 bar three are unstable modes around 4521 Hz, at 4837 Hz and 9437 Hz but when pressure is at 20 bar and 30 bar, there are five unstable modes are observed. The highest frequencies of fifth mode is 9886.6 Hz. Figure 4.26 shows the real part of eigenvalue in various pressure values. It can be seen that the real part is large when high pressure is applied, at the pressure value of 30 bar the highest value of the real part is 523. If the real part is high that means the system is easier to occur loss stability. A consequence of this analysis is the conclusion that when applying high pressure brake more unstable frequencies will occur and noise may higher.

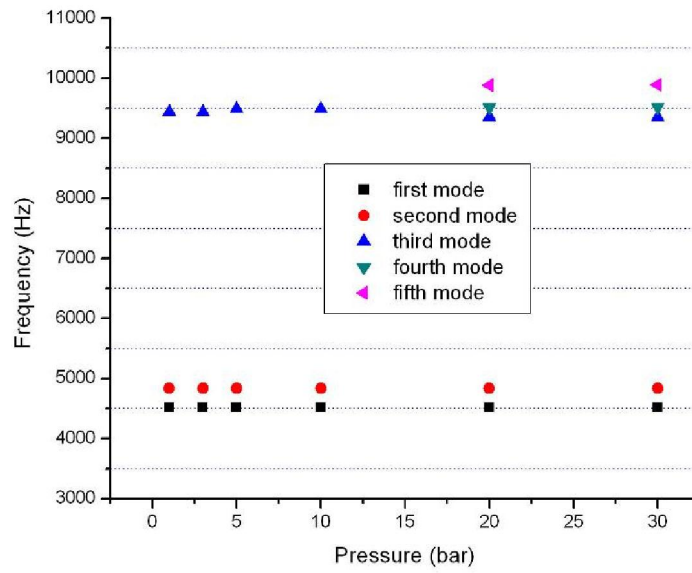


Fig 4.25 Predict unstable frequencies by braking pressure

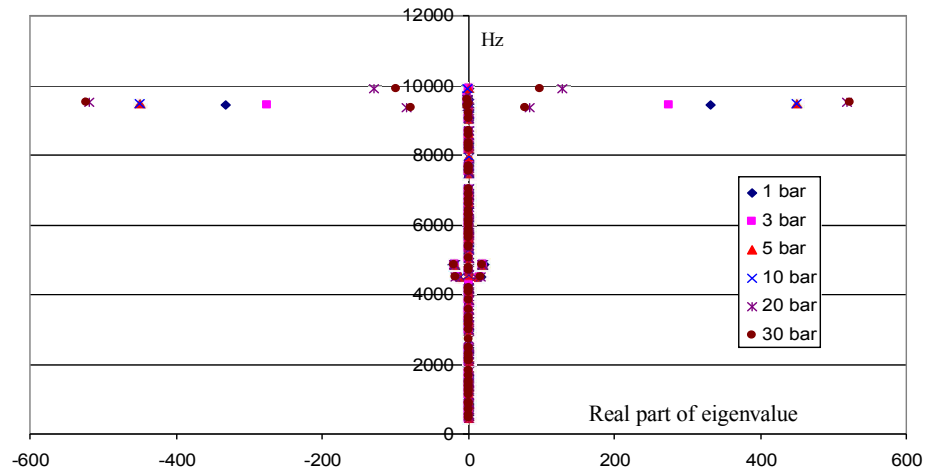


Fig 4.26 Real part at unstable frequencies

4.5. CONCLUSION

The purpose of this chapter is to investigate the disc brake mechanisms that may generate brake squeal. The simulation focuses on prediction unstable frequencies of disc brake system may lead to vibration and noise by complex eigenvalue analysis method. If the real part of complex eigenvalue is positive or damping ratio is negative that mean the system will be unstable and it may lead to vibration and noise. This is a comfortable method for

simulation a real disc brake assembly of car by software. It was modeled with full boundary conditions and simulated at real operation conditions of the disc brake. The results of simulation are shown that.

The eigenvalue of coupling modal is a pair of conjugate complex root. It is extracted to find mode shape and unstable frequencies. According to the result of simulation, at unstable mode shape, they have relationship with natural frequencies of disc brake components. If unstable frequencies of system are nearly or close to natural frequencies of components, the system may resonance and appears noise.

Friction plays the most important role in contribution on brake squeal. When friction coefficient increases, there are more unstable modes appear that leads to more neighboring modes to become coupled and one of them becomes unstable. Especially, if two neighboring modes close to each other in the frequency range and have similar characteristics, which may lead to occurrence noise. Through this analysis the critical friction coefficient at each unstable mode can be also identified.

A sensitivity analysis of parameters Young's modulus of the disc and the pad shown that there are more unstable modes appear when the Young's modulus of the disc increase while they have a little bit change for the pad case. It is clear that, when the Young's modulus of the disc increases there are more unstable frequencies appear and that may lead to noise. For the prediction on the unstable frequencies of angular velocity, there are only three unstable modes to be found and the unstable frequencies do not change significantly. But in the case of the prediction on the pressure, with an increase of high pressure that will lead to increase the unstable state of the system. The real part value in this case is big, so it can easily make losing unstable of structure and lead to occur noise. Lastly, this chapter provides a tool for identify and prediction brake squeal, they will be compared with the prediction results by experiment method in the next chapter.

EXPERIMENT PREDICTION DISC BRAKE SQUEAL

5.1 INTRODUCTION

The brake squeal is one of the complex problems in NVH field, it is influenced by many factors. Researchers have tried to solve this problem by theoretical models but these theories do not fully describe the brake squeal. Therefore, it must be monitored in experiment. Because, vibration and noise prediction of disc brake is complex, generally high standard laboratories which have good equipment and test facilities are needed. Thus, it is often performed at companies or institutes. The following are some experiments which can be used as background knowledge about experiments on brake squeal.

In 2001, Cunefare identified brake squeal by using dither and control squeal. The experimental investigation into the application of dither control for the active control and suppression of automobile disc brake squeal by vibro-acoustic analysis of a disc brake system during squeal determined the acoustic squeal signature to be emanating from the brake rotor. This squeal was eliminated, and could even be prevented from occurring, through the application of a harmonic force with a frequency higher than the squeal frequency. Through this experiment, a conclusion demonstrates that the dither control system could prevent squeal from occurring. So long as the dither amplitude was high enough, brake squeal was not generated, but as soon as the control was deactivated, squeal appeared [70].

Next in 2002, Oliviero and his coworker [105] presented a model for brake squeal prediction in the laboratory as shown in figure 5.1. The disc brake, which was designed and manufactured to evaluate the behavior of a real automotive brake, consists of a rotating disk and a caliper that can be appropriately adjusted to simulate different operating conditions. The model describes the modal interaction of the disk and the caliper coupled together through friction.

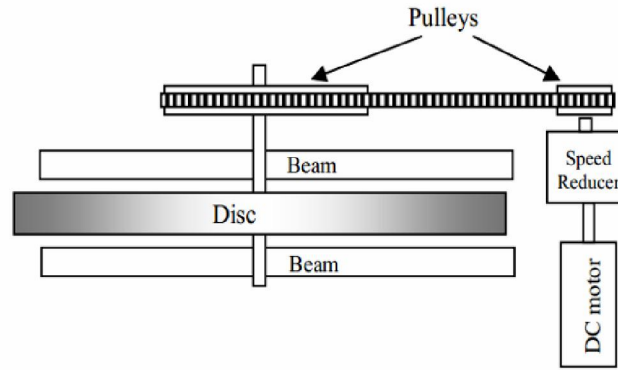


Fig 5.1 Experiment system of disc brake [105]

The result shows that the squeal frequency increases linearly when the normal load increases within each cluster of squeal. Squeals at a beam eigenfrequency can have two clusters which close to each other due to small differences between the two beams that lead them to have different eigenfrequencies. As a result, each of the beams becomes unstable at their respective eigenfrequency, producing two different squeal responses. The results also show a tendency for the higher frequency squeals to develop at higher normal loads [105]. The model can predict good brake squeal and the effect of factors to brake squeal in the laboratory. Oliviero and his coworker also did another study of brake squeal noise on a laboratory brake setup [68]. In this study the authors performed measurements on disc brake system to characterize its dynamical behavior and its squeal behavior. The tests are aimed at identifying the key parameters controlling the squeal phenomenon that are necessary to build a model of the setup. The result of experimental analysis identifies several characteristics that lead to instabilities and correlates them with the operating parameters as well as with the dynamic behavior of the system [68].

In 2003, Ouyang [59] performed an experiment for noise prediction of disc brake in range 7 kHz for two models, one for the non-moving-load approach (model A) and the other for the moving load approach (model B). The theory for prediction is also complex eigenvalue analysis and the results show that model A has more unstable frequencies than that of model B while the results of experiment show that model B covers all the experimental squeal frequencies up to 7kHz, and model A is correlated well with major squeal frequencies bands.

Amr et al [106] presented an experiment to identify noise of disc brake system on dynamometer caused by friction force. The study also considers influence of factors as velocity and pressure in various operating condition. The result of this study shows that the peak value of the pad vibration amplitude is highest. The vibration amplitude is dependent on the brake pressure and the vehicle speed. The vibration level decreases with the increase of sliding speed. Furthermore, it shown that the vibration level decreases with the increases of applied pressure [106].

From above researches, it can be seen that most of them performed with the operational conditions of disc brake system as changing the pressure and speed and measure noise of brake system. Thus, in this study, an experiment was performed to compare with the simulation results that presented in the previous chapter. The experiment is including measurement vibration and noise of disc brake according standard J2521. Through this experiment, noise frequencies and identify percentage occurrence noise of disc brake in different operation can be identified. A series of two pads with various worn were used for measure on the real car.

This experiment was performed on a chassis dynamometer system at the brake laboratory of TUL. This system has become a sophisticated test platform for identifying the propensity of a brake to generate squeal and diagnosing squeal noise problems. The result of experiment can be used to identify and predict vibration and noise of disc brake to develop a good model of disc brake that has high quality.

5.2 MEASUREMENT EQUIPMENTS

5.2.1. Transducers and data acquisition

For vibration and noise measurement of disc brake, the important key is the transducers, the data acquisition and data analysis system. As shown in figure 3.3, the collection of vibration and noise signals in brake squeal testing is done with a microphone for sound and an accelerometer for acceleration. Each of these transducers is very effective and highly specialized. Both the accelerometer and the microphone need a preamplifier or a signal conditioner to bring their output signals to sufficient voltage to be used as input to a frequency analyzer. These transducers convert sound or vibration into electrical signals that can be acquired and processed in a manner to understand the physical phenomena. The data acquisition and analysis systems convert the raw time based signals from the transducers to calibrated time or frequency domain data for storage and further analysis [1]. In figure 5.2 are equipment for vibration and noise measurement of disc brake. Some transducers as accelerometer and data acquisition have been introduced in section 3.3 of chapter 3.

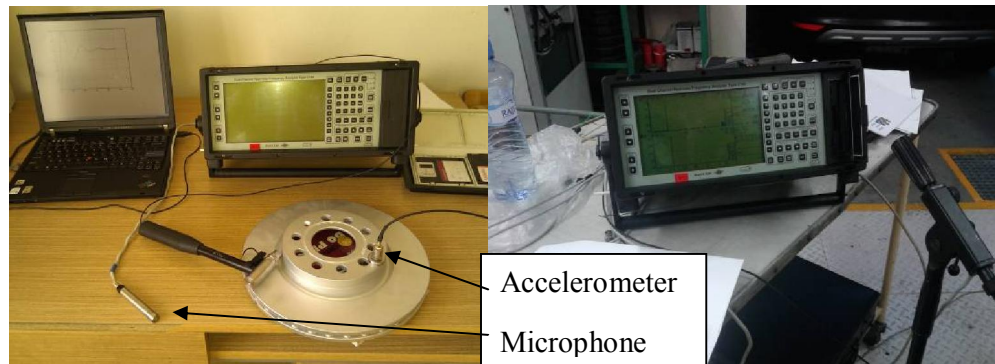


Fig 5.2 Transducers and data acquisition for experiment

Here, the transducer is microphone type 4165 of B&K Company. Some technical parameters of this transducer are: sensitivity $S_0 = -25.9$ dB re 1V/Pa (50.7 mV/Pa), cartridge capacitance 19.8 pF and frequency range to 20 kHz. The microphone is typically designed for optimum performance in a particular sound field. Therefore, it is important to select a microphone to improve accuracy.

5.2.2. The chassis dynamometer

The dynamometer consists of two steel rollers, each has a diameter of 1219 mm, see figure 5.3, which are flanged directly onto a direct current machine. The DC machine is regulated via a thyristor set. Mass inertia simulation is controlled both electrically and mechanically, whereby the basic inertia is 1361 kg. In addition, a flywheel may also be actuated electromagnetically [107]. The capacity test of the system is:

- maximum speed: 200 km/h,
- maximum tractive force: 3000 N,
- max motor output 100 kW,
- vehicle inertia range 907-2711 kg.

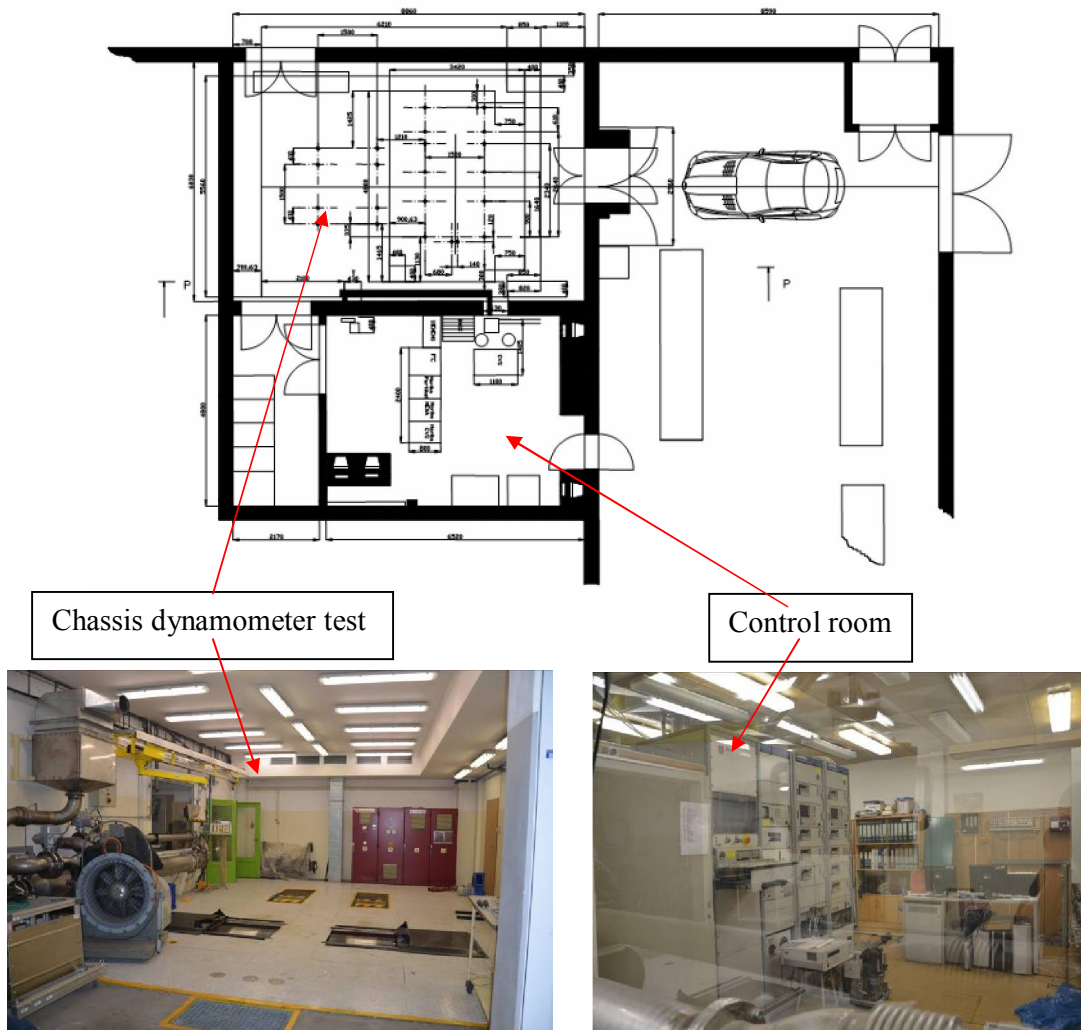


Fig 5.3 Chassis dynamometer test plan

The system can be used to simulate vehicle with the basic mechanical inertia of the rotating test rig components of 1358 kg. Vehicle mass in the range between 907 kg and 2722 kg is electrically simulated with the direct current machine. Figure 5.4 provides the diagram of the layout chassis dynamometer test system for experiment.

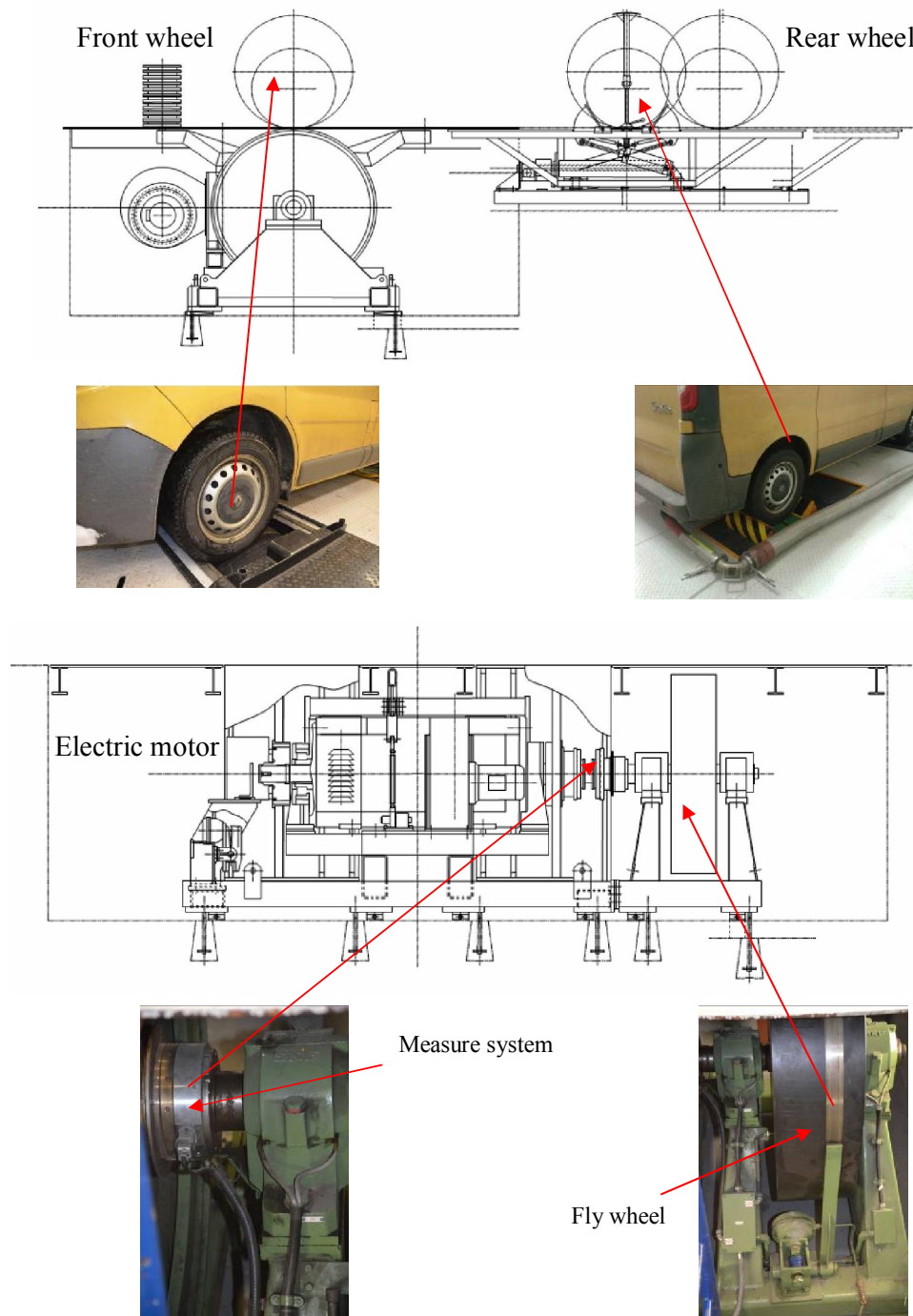


Fig 5.4 Full chassis dynamometer test system

This system can provide information of parameters for measurement as brake force, pressure, inertia, time, speed, etc by types of sensor system. Therefore, it is suitable for experiments on brake squeal. Especially, the system is useful to simulate operational cases of disc brake in real conditions.

5.3 EXPERIMENTAL PROCEDURES

5.3.1. Prepare experiment

The objective of a brake noise dynamometer test is to reproduce the conditions and the noise experienced by a brake on an operating vehicle. This experiment uses a chassis dynamometer, in which a driven road wheel is used to drive a tire that drives the brake assembly.

The experiment need to be prepared carefully to obtain good results. The experiment is arranged as in figure 5.5. First, the car is fixed in position on the test bench to secure the vehicle and the retention device. In order to ensure the correct vehicle position during the test procedure, the non-driven wheels are anchored with retention flaps (see in figure 5.5). Also, the exhaust gases of the car are conducted out of the room for safety and hence there is no sound effect.



Fig 5.5 the car on chassis dynamometer test system

The retention flaps are integrated into the adjustment facility and are pneumatically retracted and extended. In order to combat the transverse and longitudinal forces which occur in the case of front-wheel drive vehicles, a vehicle retention device is additionally secured to the vehicle's front towing eye. This is securely attached to the test rig frame.

The environment in which the brake is operating is also important to reproduce the noise found on the vehicle. This includes the acoustical environment and the environmental conditions. The experiment was performed at the brake laboratory and measurement noise of the room before experiment was approached 65 dB.

Normally, an accelerometer is located on the brake assembly for measurements of vibration in addition to noise. The data from the accelerometer may be compared to the microphone data to ascertain whether the suspected squeals are from the brake and not extraneous noise from the dynamometer.

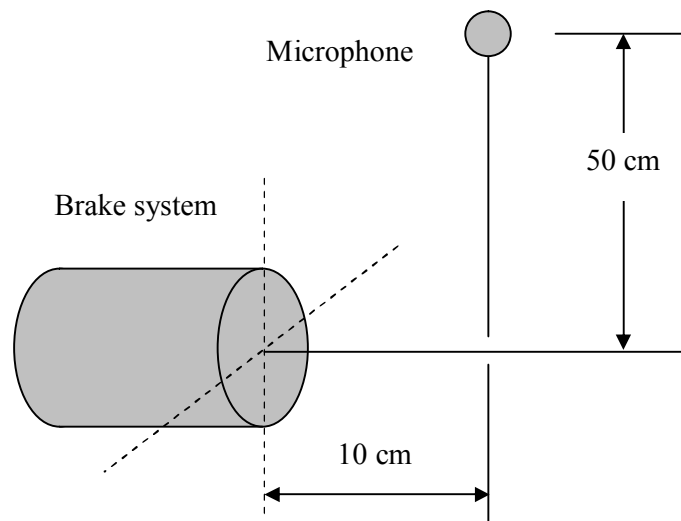


Fig 5.6 Setup a microphone for measurement

The noise transducer is located as a distance of 10 cm from the wheel hub face and 50 cm above the centerline of the axle as in figure 5.6. According to standard J2521, a frequency range threshold of ≥ 70 dB will be used for recording sound but in this experiment a sound level ≥ 75 dB will be considered noise because they are subjected influence of more various noise source as tire, engine and fan.

5.3.2. Experimental Procedure

To do the experiment on brake squeal, standard J2521 “Disc and drum brake dynamometer squeal noise matrix” is used. It is very important to be aware that this standard is for a brake noise screening procedure. Because this standard is too large for experiment at the laboratory of TUL, especially the total number of brake stops is too large, about 2321 applications. Therefore, in this experiment, a limited number of brake stops is used (as in table 5.1) for two pads with various worn to save cost and time. First, the experiment is used to predict occurrence noise frequencies of real car in frequency range 1 kHz to 10 kHz. Then noise of two pads in various worn is compared. The procedure of the experiment for noise prediction of disc brake is shown at table 5.1.

Table 5.1 Experiment testing noise and vibration in different operation conditions.

Test number	# Of application	Initial speed	Final speed	Pressure (bar)	Initial temperature
01	30	10 km/h	0 km/h	3.2	Measure
02	30	50 km/h	0 km/h	7.8	Measure
03	30	100 km/h	0 km/h	9.7	Measure

The experiment was performed with pad (10% worn) at the front and left wheel of car. Firstly, run the car at 10 km/h and then apply the brake with pressure by brake operation as in figure 5.7 until the wheel stops. The stop operation is shown in figure 5.8.

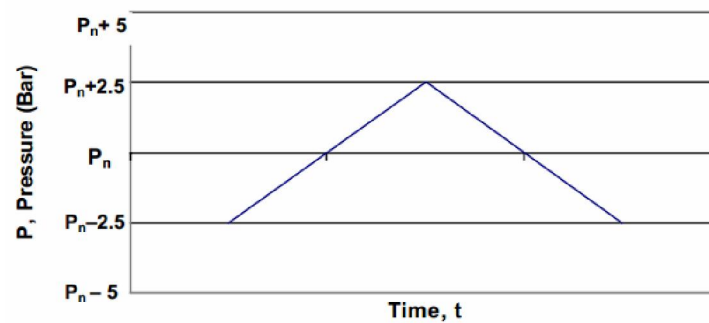


Fig 5.7 Braking pressure ramp [108]

In order to stop operations are required according to the braking pressure ramp shown in figure 5.7. Brake applications are performed from the highest speed required in the test matrix to a speed lower than 0.5 km/h as is shown in figure 5.8.

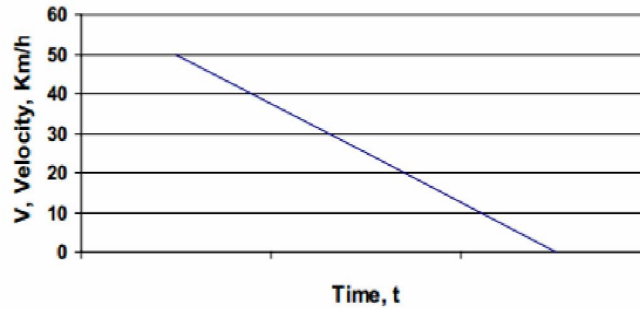


Fig 5.8 Stop operation [108]

During braking process, the measurement system starts to record data, microphone captures the signal of noise and sends to the acquisition system. An accelerometer mounted on the caliper in parallel direction with the rotation axis of wheel, it will capture the signal of vibration in-plan direction of disc brake (see figure 5.9). A setting cross spectrum analysis for measurement of data acquisition is used with two channels; channel A for noise signal and channel B for vibration signal. Frequency was measured in range maximum 10 kHz with bandwidth 1/3 octave.

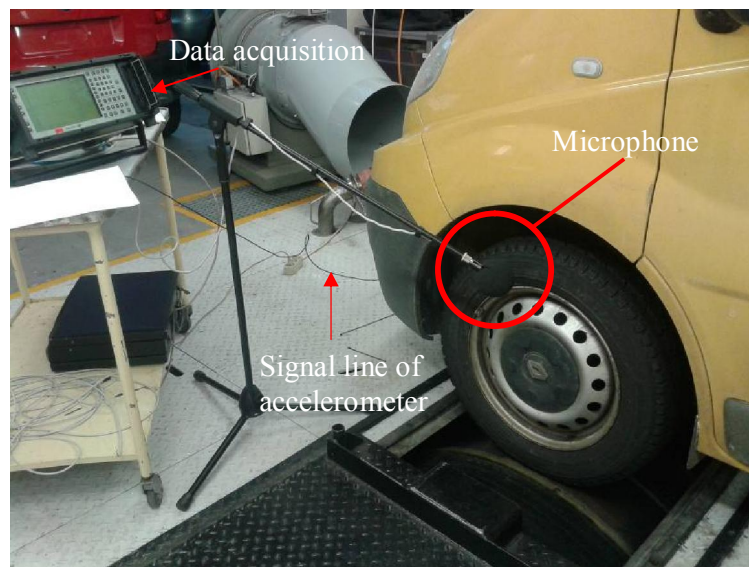


Fig 5.9 Setup equipments for measurement noise and vibration

In the same way, continue to measure for old pad with 70% worn on the right wheel of car and save data to compare with the percentage occurrence noise of two pads. The temperature of disc surface will be measured by infrared thermometer CEM DT-8819.

5.3.3. The inertia simulation

To perform realistic simulation of stopping a vehicle, the effect of the inertia of the vehicle must be considered. Without the proper inertia, brake would not provide the right amount of energy dissipation and the noise performance may be different since noise is highly dependent on speed, temperature, deceleration and clamping force [1]. One of the important benefits of the inertia simulation system is that it negates any effects of bearing friction or windage in the rotating discs when it is applied. For dynamometer with electric simulation, there is a simple formula to predict the amount of inertia that may be simulated on a particular machine.

$$I_a = \frac{9549.3 * P_m * R_r}{a * g * \Omega} \quad (5.1)$$

Where

a- Vehicle acceleration (m/s²)

g- Gravitational acceleration = 9.8 (m/s²)

I_a – Available inertia (kg.m.s²)

P_m – Maximum motor power, typically 150% of rated power (kW)

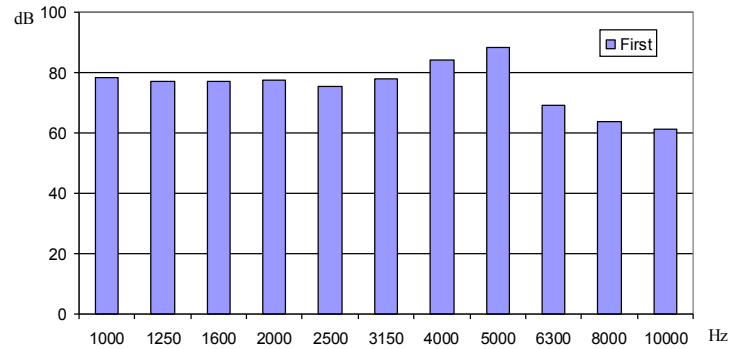
R_r – Tire rolling radius, m

Ω - Rotational speed of interest, rpm

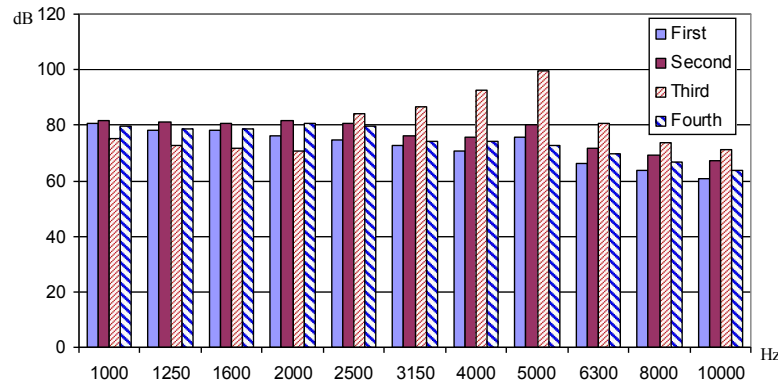
Whatever method is used, the goal is to match the inertia of the vehicle as closely as possible to assure the accurate recreation of the stop. In this experiment, a chassis dynamometer with an electronic motor is used to simulate the inertia of the vehicle as shown in figure 5.4. The value of inertia is automatically calculated by the control system. The input value of mass inertia is 1700 kg. A close matches ensure that the brake under test sees the same torques, stop time, temperatures and number of revolutions as seen on the vehicle.

5.4 RESULTS

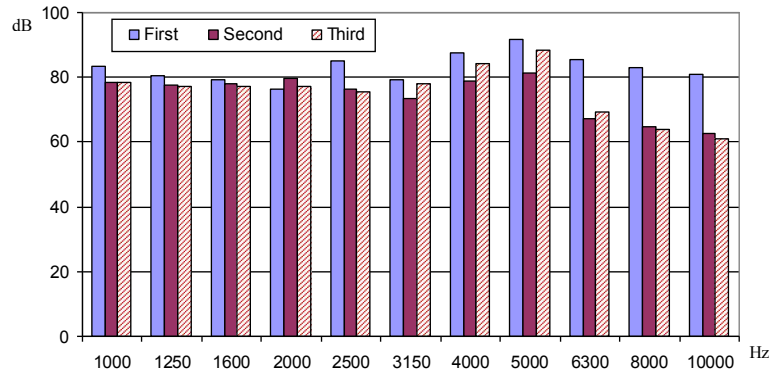
A summary of experimental results for test 01, test 02 and test 03 is displayed in figure 5.10. From this figure, it can be seen that the major frequencies that occur noise at 5000 Hz with the maximum of squeal value at 99 dB. For test 01, the result of squeal only occurs one time in 30 test stop applications. The maximum of noise is 88 dB occur at frequency of 5000 Hz. While with test 02 there are four times occur noise and for test 03 with three times occur noise in total 30 test stop applications. Generally, the noise occurs during test range of frequency but the major frequency is at 5000 Hz, other noise occurs at around 2000 Hz. But for high frequency the noise level is decreased. For example, at frequency of 8 kHz and 10 kHz only two squeals of test 02 and test 03 are found. However, for test 01 it is invisible occurrence noise at these frequencies.



a) The noise level of test 01



b) The noise level of test 02



c) The noise level of test 03

Fig 5.10 Measure noise of disc brake system in various operation conditions

To compare results between simulation in previous chapter and experiment result, a summary at frequencies may occur noise is shown in figure 5.11.

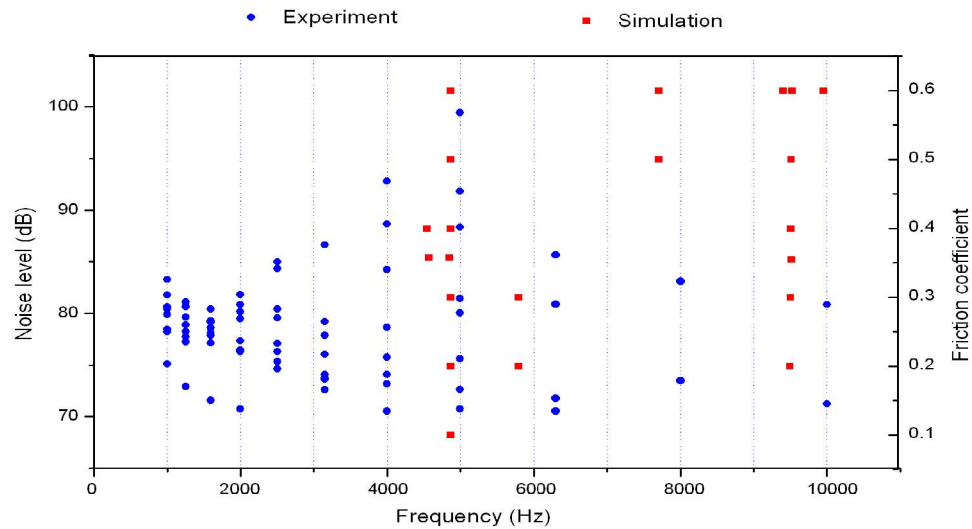


Fig 5.11 Frequencies occurrence noise by experiment and predict by simulation

This graph expresses the data that was collected by experimental tests in various operation conditions of disc brake and the results of prediction of unstable frequencies corresponding to different coefficients of friction. At the friction coefficient of $\mu = 0.35$, the simulation result shows that there are three mode unstable frequencies which may occur noise include mode 33 at 4521.4 Hz, mode 36 at 4837 Hz and mode 72 at 9437.1 Hz. Compare with the experiment result, it can be seen that the main frequency of prediction noise is mode 36 that is closed to experimental result at frequency of 5000 Hz. While mode 33 occurs between

two frequencies of 4000 Hz and 5000 Hz. Mode 72 occurs near frequency of 10 kHz. According to the simulation result, an increase of friction coefficient leads to increase instable modes that mean occur more unstable frequencies, that may make noise. For example, in figure 5.11, there is one unstable mode at $\mu = 0.1$ but there are five unstable modes when $\mu = 0.6$, thus the possibility of noise occurring at $\mu = 0.6$ is high. Comparing this case with the experiment, it can be seen that the unstable frequencies of the simulation results are closed to those of the experiment. Where mode 36 at 4836 Hz is compared with 5000 Hz and mode 76 at 9461.8 Hz is compared with 10000 Hz of experimental results. By comparing the simulation results and the experimental result, it can be seen that frequencies below 4 kHz are missed in the simulation result.

Relate to experiment noise of disc brake, an accelerometer is used to measure vibration frequencies signal then this results will be compared with the noise frequencies. A detail of vibration frequencies are shown in figure 5.12.

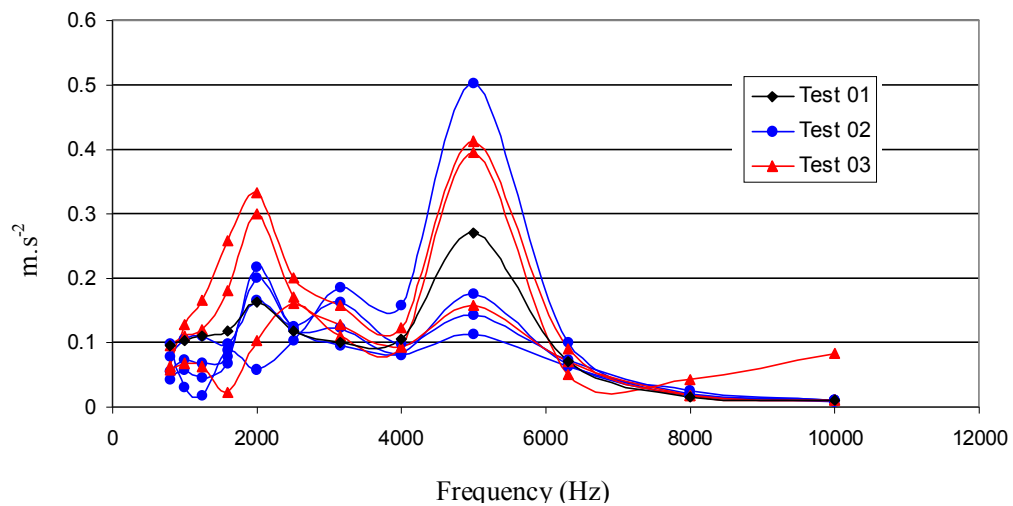


Fig 5.12 Vibration frequencies of disc brake in three test cases

The graph shows that the amplitude of vibration approaches peaks at two main frequencies of around 2000 Hz and 5000 Hz. In three tests all most vibration frequencies appeared at the same in frequency but different in amplitude. Because they were measured in the same type and operation conditions therefore they can be compared with the results of noise frequencies in figure 5.10. It can be seen that the peak of the highest value in test 2 is corresponding to the highest vibration amplitude of test 2. Based on the analysis above, it

can recommend that there is a relationship between the vibration in plan of disc with the creation of noise of the disc brake.

The temperature and humidity in this experiment were not deep study but the author had tried to measure two these parameters to provide more information about the test. The humidity of the test room is approximate 50% at the time of experiment. The temperature for test 1 is in range from 48 °C to 80 °C, from 90 °C to 125 °C for test 2. The temperature for test 03 is higher, from 100 °C to 175 °C.

When observing the results, it can be commented that the noise level or sound pressure level often occur in range of 75 dB to 80 dB, as shown in figure 5.13. A little bit occur at the high noise level more than 90 dB. This is the result of tests 01, 02 and 03 in different operation conditions of disc brake.

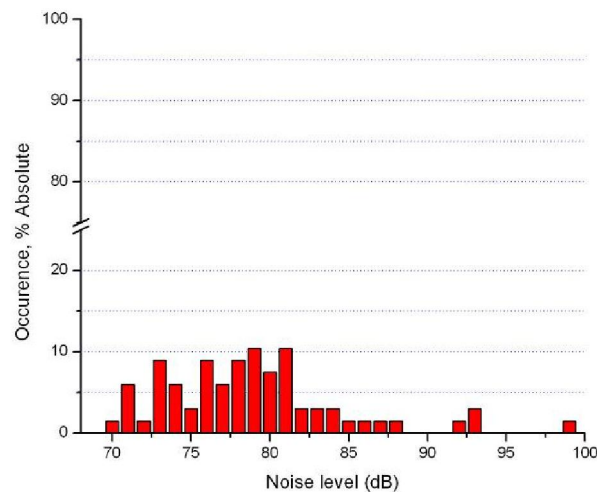


Fig 5.13 Percentage noise level occur in various operation

An experiment result of different pressures (given in table 5.1) is presented in figure 5.14. It can be seen that at the speed of 50 km/h with pressure of 7.8 bar, the percentage of noise occurring is highest with 13.5%. The next is the case of pressure 9.7 bar and the last is for the case of pressure 3.2 bar. It can be compared with the simulation effect of pressure on brake squeal that is presented in the previous chapter. That mean with higher the pressure may occur more unstable frequency and may much noise is created.

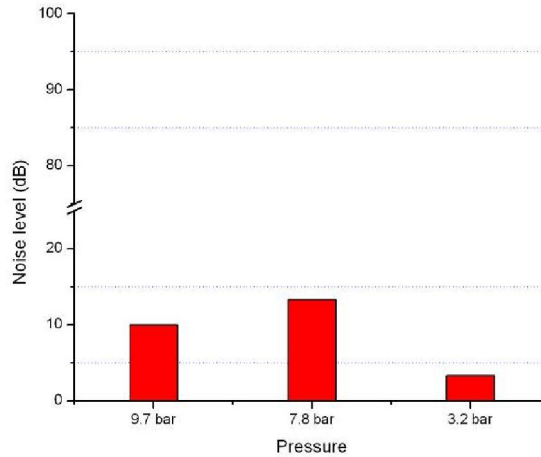


Fig 5.14 Effect of pressure on percentage occur noise

The experiment for pad with 70% worn (pad B) is done in the same way and the result is shown in figure 5.15. Figure 5.16 presents a comparison on percentage of noise occurrence of the 2 pads. The percentage of noise occurrence is calculated by dividing the number of brake applications with a threshold ≥ 70 dB by the total number of brake applications in the test.



Fig 5.15 Two pads with 10% worn (left) and 70% worn (right)

The result of figure 5.16 shows that two models of pad effect to brake squeal during braking. According to the test result, it can be seen that percentage of noise occurrence of pad B is higher than that of pad A. Totally, with 90 stop applications the percentage of noise occurrence of pad B is 11.1% and pad A is 8.8%. Particularly, in the case of 10 km/h, pad B occurs noise 10% higher than pad A (3.3%) while at the speed of 50 km/h the percentage of noise occurrence of pad A is higher than pad B (13.3 % compared with 10%) and contrary to the case of 100 km/h. The results are suitable with the actual operation conditions, with the disc have high worn may occur noise than new pad. A deep study about that will be continued to explain in chapter 6, include effect by damping, material and surface characteristic.

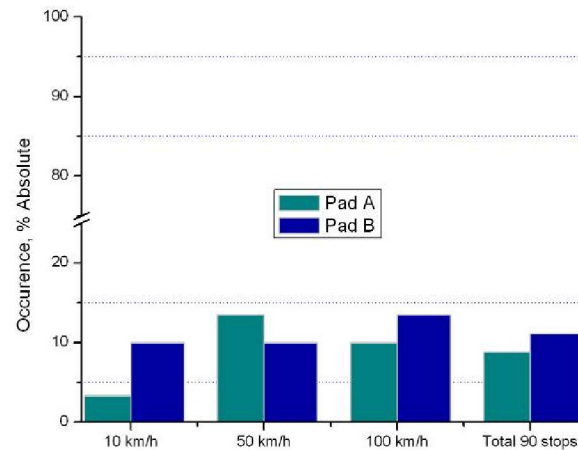


Fig 5.16 Percent occurrence noise of two pads with various worn

5.5 CONCLUSION

This chapter presents an experiment method to identify and predict vibration and noise of disc brake. Based on the standard J2521 the experimental procedure for measure noise and vibration signals in different operation conditions is set up. Through this experiment, some conclusions can be given as follows:

Measurement on chassis dynamometer is an advantage. This provides the most accurate simulation of actual vehicle operation and dynamics. Combine with the real disc brake system will permit to identify and predict brake squeal in various operations. Changing several operational and physical parameters during experiment such as the rotational speed of wheel, pressure apply to disc, etc will occur different squeal at many frequencies. Data obtained from measurements have been used to develop a modal model that can predict the occurrence of squeal and its frequency.

A comparison between two pads with various worn lead to a recommendation that the noise occur at the pad which has high worn more than the pad which has low worn. Here, we can see that pad B (70% worn) appears more than 2.3% of noise compare with pad A (10% worn). All most noise or pressure noise level occurs in range from 76 dB to 82 dB.

The experiment also shows the influence of pressure to the brake squeal. Applying a high pressure may lead to a high percentage of noise occurrence. Compare with the simulation result, it shows that tending occurs noise by influence of pressure is the same case of experiment case, except at pressure 9.7 bar small than pressure value at 7.8 bar of experimental result.

The experimental results are closed to the simulation results at the main frequency 5 kHz but the amount of noise in experimental results occur at frequencies below 4 kHz while simulation results not found unstable frequencies below 4 kHz. For high frequencies, the noise of experimental results decreases while the simulation results show that the noise may occur at the high frequencies from 9-10 kHz. Using accelerometer to check and compare with results that was measured by microphone. It shows that the main vibration frequencies appear at 2 kHz and 5 kHz. Therefore, there is no difference in the results which is measured by the microphone.

From some of results analyzed above, it can be say that identifying and predicting noise and vibration area complex problems in theory and experiment. It is subjected influence of factors. The study described in the next chapter will consider some factors that effect to brake squeal, especially the surface contact between the disc and the pads.

EFFECT CHARACTERISTIC OF PAD SURFACES ON BRAKE SQUEAL

6.1 INTRODUCTION

As introduced in the previous chapters, brake squeal is a complex problem, to identify and predict percentage of noise occurrence approach high accuracy that needs to combine between theory, simulation and experiment. In the previous chapters, a simulation and an experiment are performed to identify and predict noise of disc brake. Some good results have been obtained but there are noise frequencies that missed in the prediction range. This leads to find reasons to explain the causes that make noise in this case. As known that the main cause of brake noise is friction between two surfaces of disc and couple pad, thus it is necessary to study them. Because the characteristic of pad surface is more effect to brake squeal, therefore, this chapter will present a study about characteristic of pad surface and its effect on brake squeal.

Along with the development of measurement technology, especially technology of analysis surface as Scanning Electron Microscopy (SEM) or Energy Dispersive X-ray Spectroscopy (EDS, EDX, or XEDS). These technology permits to evaluate the characteristics of pad surfaces and disc to explain why the noise can be generated and to predict the causes of noise occurrence. Mikael [109] and Filip [110] studied the surface characterization of brake bad and the influence of disc topography on the generation of brake squeal. Mikeal chose two pads which are of metal-fibre reinforced organic type for experiment. An operation case of disc brake by high speed and pressure were performed then an analysis of characterization of the brake pad surface was performed after interrupting the testing under both silent and squealing condition. The results show that the brake squeal has a relationship with the number of contact plateaus, their size and the total contact area. While, Filip used two brake discs that were shot-blasted to produce small pits in the disc surface. This study was performed with a increase of friction coefficient from 0.3 to 0.45 and a recommendation is given that there is no brake squeal generated below critical friction coefficient 0.4.

Besides, a SEM analysis was performed before and after the test to assess the surface characteristic of the disc which effect to brake squeal by changing the microscopic friction conditions. The same authors in papers [88] and [111] presented their study about mechanical properties and composition of tribological surfaces as pads by using high resolution SEM, nanoindentation, energy dispersive X-ray analysis. This analysis shows the relationship between surface characteristics and friction phenomena. Some of contact situation of organic binder brake friction materials against cast iron disc were studied.

In recent years, there is a number of researches about wear of pad and disc surfaces, especially contact surface topography which effect to brake squeal. Bettge [112] used interference microscopy for analysis surfaces of disc. Francesco and co-worker [113] presented a study on the surface topography of pad after test. The task shows that the surface topography of the disc is not affected by the squeal. The main squeal is caused by the surface topography of pads. The result of the tribological analysis highlights a further problem that can arise from squeal vibration as the reduction of the pad life-time. In fact, the fragmentation of some components is properly added to reduce the wear rate and the fatigue damages of the superficial pad material can seriously affect the wear rate of the material. Hetzler [114] studied the influence of contact tribology on brake squeal. Author studied a series of disc brake pads in various wear and they were experimented to obtain data for setting a contact model. A simulation by numerical method was performed to analysis stability and the influence of the contact models on the steady state stability [114].

This chapter presents the study about the effect of the characteristic of pad surface to brake squeal. A series of four pads with different worn will be studied to assess surface wear. This result will be compared with the experimental results. The experiment of noise measurement was performed on the chassis dynamometer system while the experiment of surface analysis was performed at the laboratory of electron microscopy and surface technology of the Material Science Department - TUL. The effect of damping and hardness on brake squeal will also be presented. Therefore, this chapter will contribute a part in prediction vibration and noise of the disc brake system by analysis surface of pad.

6.2 EXPERIMENT PROCEDURE

6.2.1 Measurement instruments

The experiment was performed on the test system of SEM and EDS while the surface of disc and brake pad specimens were investigated by using a light interferometer and a scanning electron microscope. The SEM is used for qualitative examination of the surface properties by equipment Carl Zeiss axio image M2M as in figure 6.1.



Fig 6.1 Equipment microscopy for measurement

For testing the hardness of disc and pad surfaces, MH180 hardness tester is used as in figure 6.2. For noise measurement, some of equipment which introduced in the previous section will be used.



Fig 6.2 MH180 hardness tester

The Energy-dispersive X-ray spectroscopy (EDS, EDX, or XEDS) is an analytical technique used for the elemental analysis or chemical characterization of a sample. It relies on the investigation of an interaction of some source of X-ray excitation and a sample. Its characterization capabilities are due in large part to the fundamental principle that each element has a unique atomic structure allowing unique set of peaks on its X-ray spectrum system [115]. Figure 6.3 illustrates an EDS system including four primary components of the EDS setup. They are the excitation source, the X-ray detector (in the right of figure 6.3 is the detector X-Max with size 20 mm²), the pulse processor and the analyzer. This experiment has been tested at the Laboratory of electron microscopy, Department of Material Science, TUL.

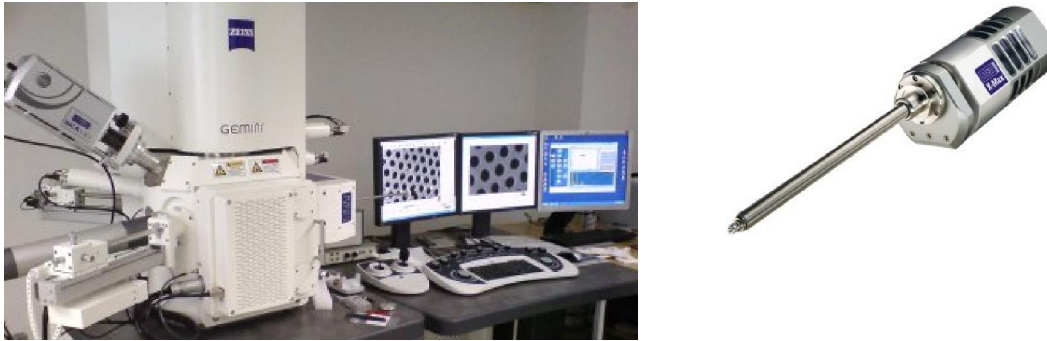


Fig 6.3 Equipments of an EDS system

6.2.2 Experimental procedures

First, the ingredients and chemical properties of a brake pad need to be identified by EDS analysis. This procedure provides information about the material of the brake pad. Then a analysis of surface properties including roughness and SEM method was performed for identifying surface characteristic. This information is very useful to study the surface friction of pads. In this study, a series of four pads with different worn were performed. The wear understood by thickness of friction material of brake pad is decrease. A new pad is brake pad that used for some times while old pad with 10%, 60% and 70% are corresponding with a decrease thickness of brake pad 10%, 60% and 70%. All tests were performed in 30 stop sequences.

6.3 FRICTION AND BRAKE SQUEAL

Brake squeal is known as the cause of friction between disc and pads. That is the main cause that leads to occur vibration and noise. The materials used in the manufacture of brakes are known as friction materials. In contrast to other tribological applications, a relatively high friction coefficient in the range of $\mu = 0.3\text{--}0.7$ is normally desirable when using brake lining materials [116].

From the complex eigenvalue analysis in chapter 4, it can be seen that as friction coefficient increases to a certain level, two modes merge or become a couple form of complex mode which may become unstable. A prediction of unstable frequencies is shown in figure 6.4.

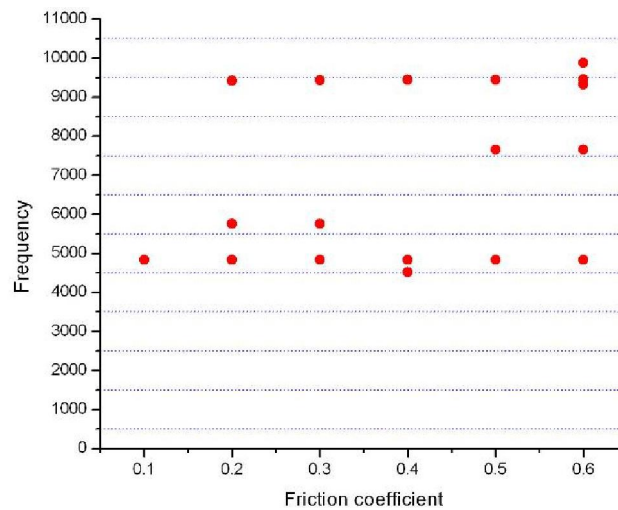


Fig 6.4 Predict unstable frequencies following friction coefficient

Figure 6.4 shows that more unstable frequencies appear when the friction coefficient increase. In this simulation, a threshold value of the friction coefficients appears in each mode. If the friction coefficient is higher than this value, the system will be unstable. The simulation also shows that the prediction results are missed unstable frequencies below 4000 Hz but in the experiment of noise measurement of disc brake system on the chassis dynamometer the noise still occur below 4000 Hz. It is because the conditions of the real test are a little bit different. Actual squeal occurrence at higher or lower coefficient of friction depends on the operation conditions of triggering mechanisms or excitation forces. When temperature is low and humidity is high, it has been found that for some lining materials the variation of friction coefficient is high which may be the cause of cold or

morning squeal [4]. The experiment of noise measurement in chapter 5 for two pads with different worn also shows that the percentage of noise occurrence with old pad is higher than new pad, see figure 6.5. Because for old pad, to approach friction coefficient equivalent with friction coefficient of new pad that is need to increase high pressure line. These results show that higher friction coefficient always tends to make a brake system more unstable.

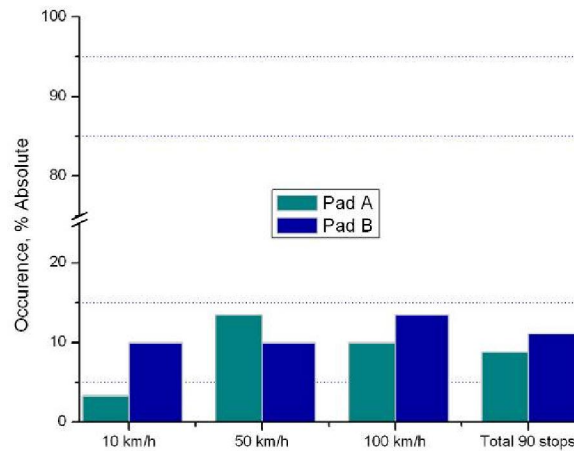


Fig 6.5 Percentage occurrence noise with various worn pads

6.4 ANALYSIS SURFACES OF PAD

6.4.1 Chemical properties of pads

Pad material plays an important role in the brake functions by converting the vehicle's kinetic energy into heat energy. During braking the heat energy is first borne by the two contact surfaces of the brake, namely the brake disc and the brake pad and then transferred to some components of the disc brake. It is the main cause of noise occurrence in the disc brake system. Figure 6.6 shows a commercial pad that used for analysis.



Fig 6.6 Brake pad used for analysis

In general, there are three classifications of materials which can be used to produce pads in industry. First, metallic material includes ingredients as predominantly metallic, such as steel fibers, copper fibers, etc; second is material of semi-metallic including mixture of metallic and organic ingredient; the last is material of non-asbestos organic including predominantly organic, such as mineral fibers, rubber, graphite, etc [117]. Most of them have subcomponents:

- Friction additives which determine the frictional properties of the brake pads and comprise a mixture of abrasives and lubricants.
- Fillers which reduce the cost and improve the manufacturability of brake pads.
- Binder which holds components of a brake pad together
- Reinforcing fibers which provide mechanical strength

Figure 6.8 presents the result of analysis area of the sample pad (figure 6.7) on the EDS system with the magnification at 31X, detector type X-Max.

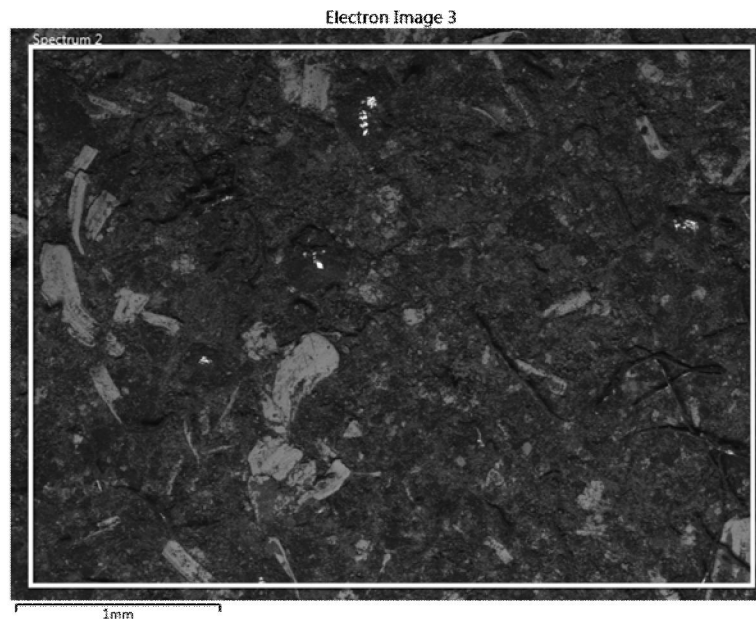


Fig 6.7 A sample of brake pad for experiment

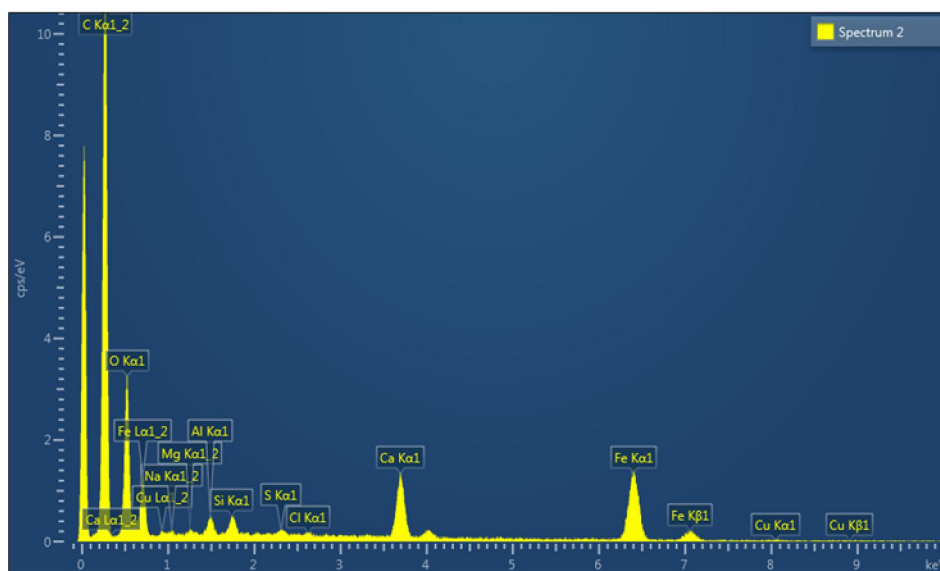


Fig 6.8 Map sum spectrum of chemical components

The analysis results of chemical compounds of the sample pad are presented in table 6.1. It can be seen that the element *Fe* takes a large amount of the total percentage of weight and atomic. Next is element O. The others such as Ca, Al, Si, Cu, S, Na, Mg, Cl keep a small amount.

Table 6.1 Chemical compounds of pad

Element	Weight %	Wt% Sigma	Atomic %	Standard Label
O	26.43	0.48	52.23	SiO ₂
Na	0.56	0.19	0.76	Albite
Mg	0.50	0.13	0.65	MgO
Al	2.00	0.14	2.35	Al ₂ O ₃
Si	1.84	0.13	2.07	SiO ₂
S	0.60	0.11	0.59	FeS ₂
Cl	0.45	0.10	0.40	NaCl
Ca	12.52	0.28	9.88	Wollastonite
Fe	53.28	0.62	30.16	Fe
Cu	1.83	0.51	0.91	Cu
Total:	100.00		100.00	

6.4.2 Surface profiles

The surfaces of pad and disc are the ones that cause on noise of brake, because they effect to the friction coefficient. For two pads with the same parameters but different wear and different plateaus, the pad which has more plateaus will have high friction coefficient than that of the pad which has smooth surface. It is the same of properties of new pad and worn pad. Therefore, an analysis surface of pad is necessary for understanding topography of pad surface.

In this research, four pads with various worn are used for experiment, see figure 6.9. The pad with 70% worn was used on the real car for two years, the pad with 10% worn was replaced one year ago in the brake system of Renault car at TUL. Therefore, they are pads that have natural wear. All of four pads are used for measurement roughness and SEM.



Pad A - with 10% Worn



Pad B - with 70% worn



Pad C is a new pad 0% worn

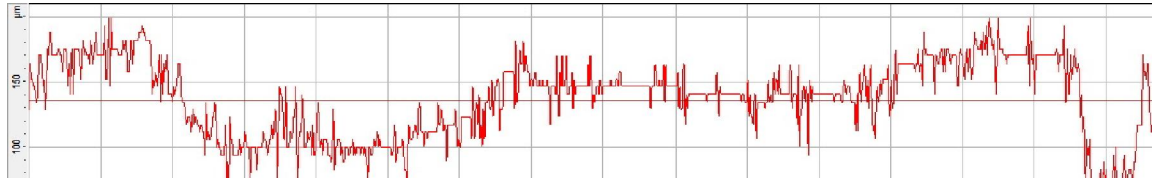


Pad D – with 60% worn

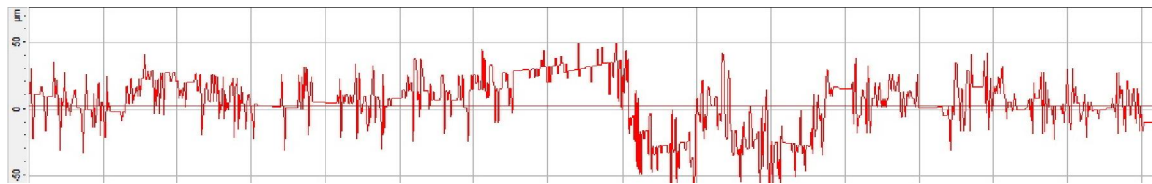
Fig 6.9 Four pads with various worn

The surfaces of the four pads will be analyzed on the SEM system. This equipment detects roughness parameters of the disc and pad surfaces. Originally, the roughness parameters are used for line scans only. Methods are proposed to apply for 3D surfaces. Besides the global roughness, the software Axio Vision with Z-stack and topography module can generate the images of local roughness. For each pixel, the roughness is computed from an area around the pixel. Local roughness is used as information for multi-parameter analysis of the. Some standard roughness parameters such as R_a (arithmetic mean surface roughness), R_q (root

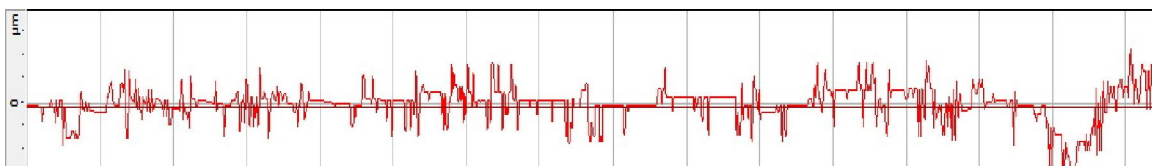
means squeal) and R_t (total height of the roughness profile) are computed from the interference microscopy [118]. The profiles of the surface roughness values of the pads are shown in figure 6.10.



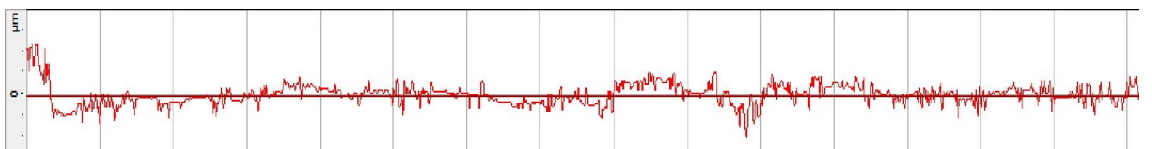
a) New pad 0% worn



b) Pad with 10% worn



c) Pad with 60% worn



d) Pad with 70% worn

Fig 6.10 Roughness of four pads with different wear

The center area of the pad is chosen to measure its surface roughness and the measurement direction is diagonal from the sidling velocity, see figure 6.11. The magnification of the results is 100x. The result shows that the surface roughness of the pad which has small worn is higher than that of the pads which has large worn. It can be seen that the roughness amplitude of the new pad, pads with 10%, 60% and 70% worn are $\pm 80 \mu\text{m}$, $\pm 50 \mu\text{m}$, $\pm 25 \mu\text{m}$ and $\pm 20 \mu\text{m}$ respectively. The values of R_a and R_q are shown in figure 6.12.

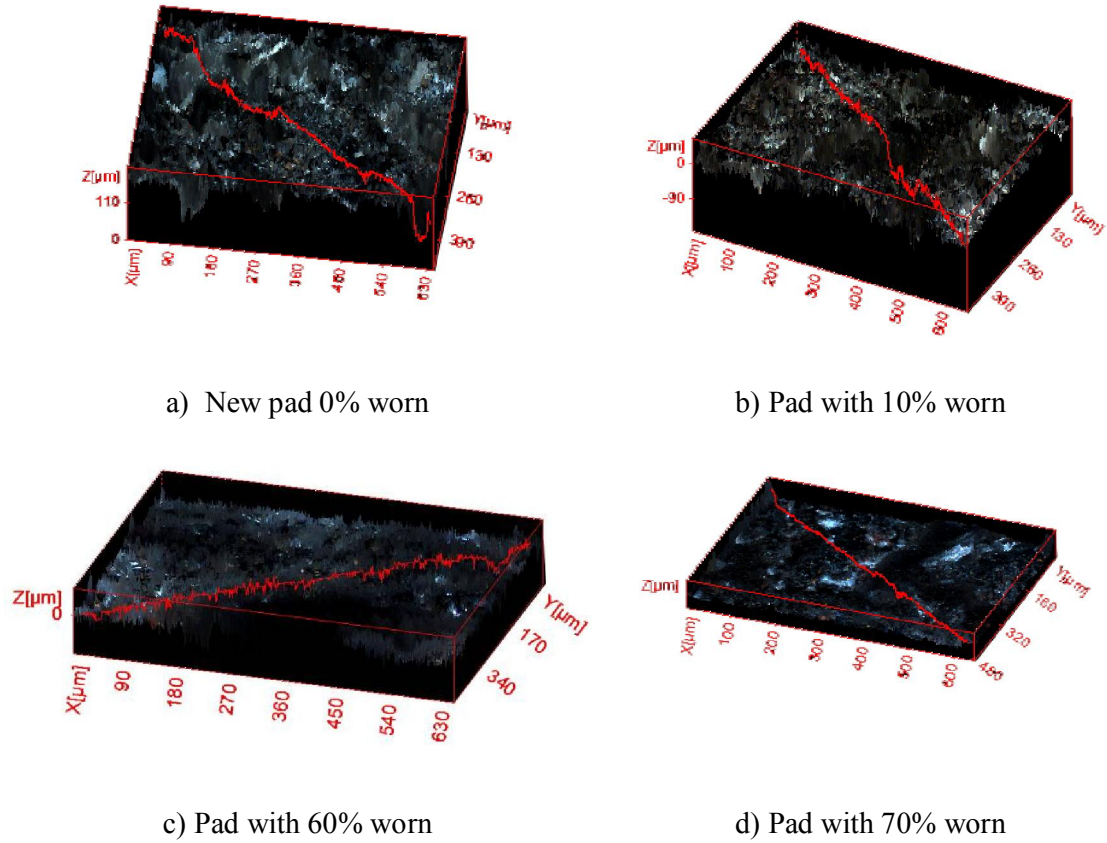


Fig 6.11 Measurement roughness of various worn pads in surface and line

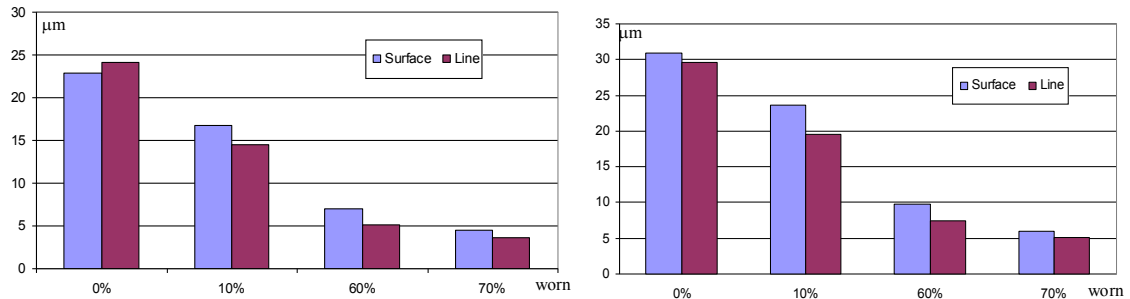


Fig 6.12 The value of roughness R_a (left) and R_q (right) with different worn pads

From figure 6.12, it is recognized that the surface roughness of the high worn pads are smaller than those of the pads which have light worn. The difference of roughness value between the new pad and the old ones is too much. For the new pad, the R_a value is approximate 20 μm -25 μm , but for the old pads (60%-70% worn) the R_a value is approximate 5 μm . The difference in R_a among them is high that lead to effect directly to friction coefficient and damping between disc and pads.

6.4.3 Contact plateaus

The contact surface between pad and disc plays an important role in creating noise of disc brake because it effects directly to quality of contact or effect to friction coefficient. Therefore, considering contact surfaces focus on size and numbers of contact plateaus also distributing material components of the pad that are necessary for understanding about occurrence noise. For example an SEM analysis of a new pad is presented in figure 6.13 and roughness of this pad is shown in figure 6.14.

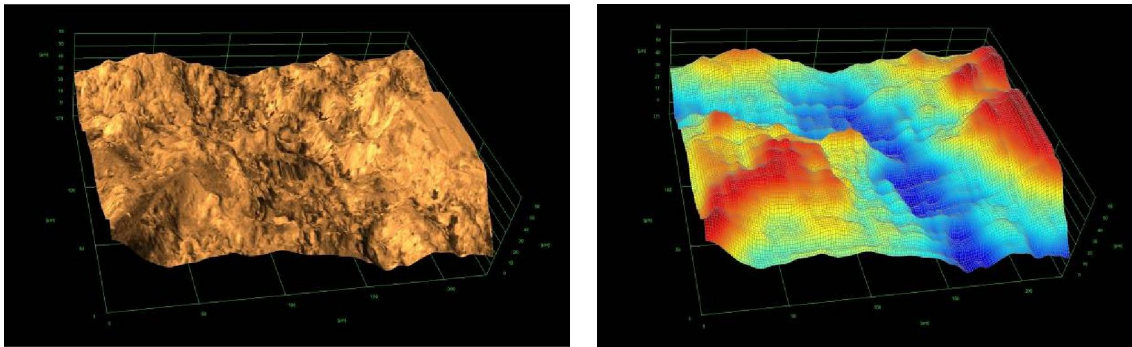


Fig 6.13 Contact plateaus of new pad

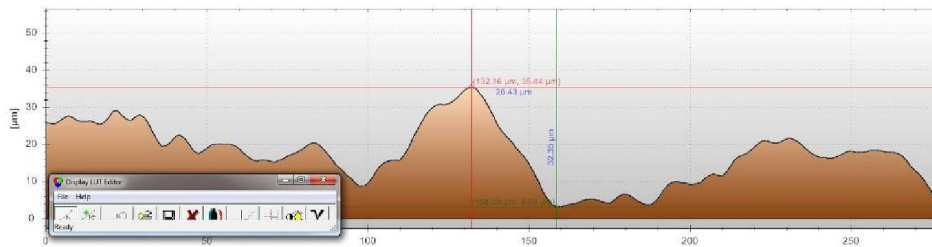


Fig 6.14 Roughness of new pad in example above

During braking the pad surface is against with the disc surface and a contact zone appears between them. This interface depends on the contact surface, material properties and pressure line apply to pads see on figure 6.15. Through these contact areas the friction force will be transmit and keep the disc stop. In this section present contact surface of pad A and pad B. On the figure 6.16 are contact zone of pad A and B at various magnifications by SEM method.

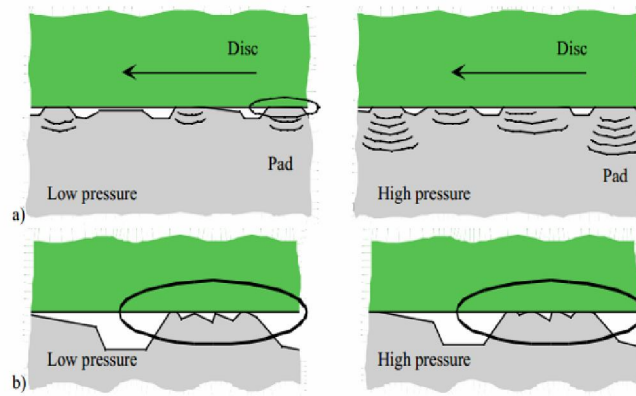
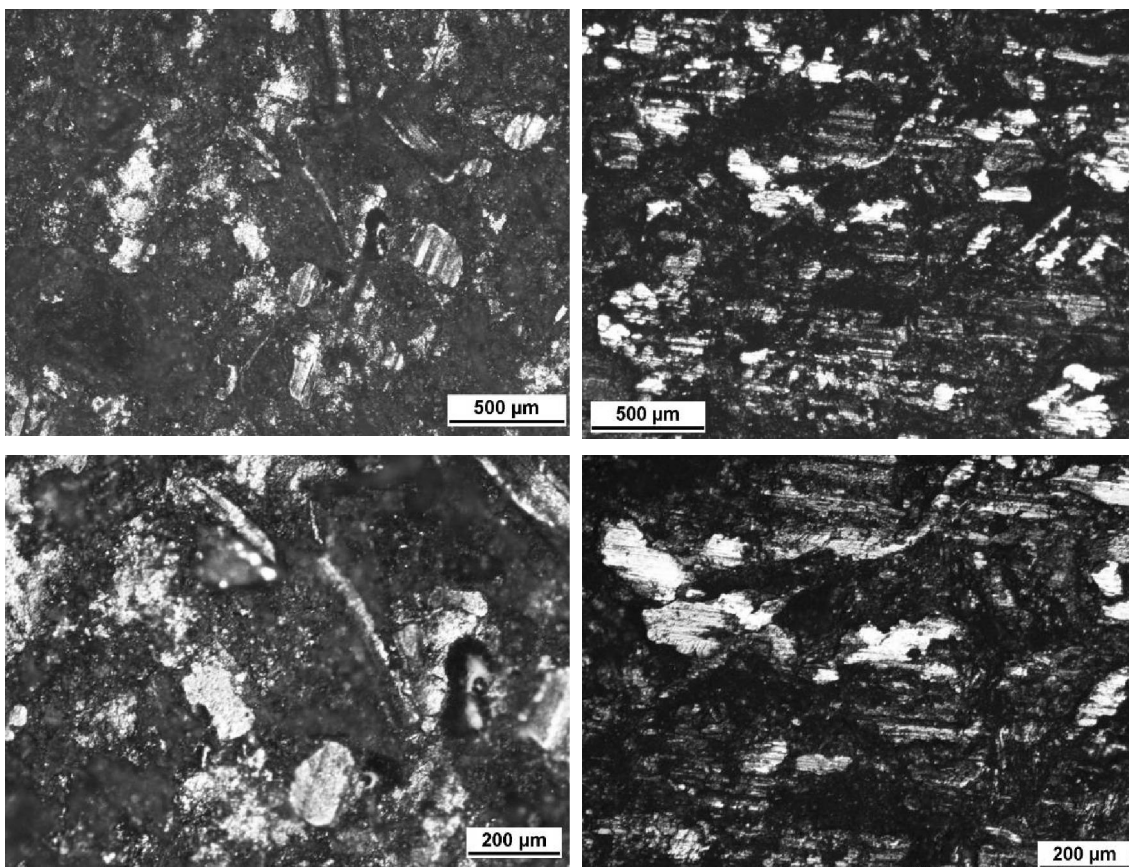


Fig 6.15 Contact between pad and disc with various pressure [111]



a) Pad A - 10% worn

b) Pad B - 70% worn

Fig 6.16 Contact zone of two pads with different worn

Figure 6.16 shows that there are differences in size and number of contact plateaus between two pads. When comparing the contact surfaces between two pads, it can be seen that with the pad of 10% worn the size of the contact zone is larger than that of the pad with 70 % worn but the number of the contact plateaus is less than that of the pad with 70% worn. The paper [109] gives a conclusion that for pad which have more small plateaus will generate more noise than pad with large plateaus. In this experiment it also shows that in 90 stop sequences between two pads A and B, the pad B with high worn was occurred noise more than pad A. In figure 6.5 the percentage of noise occurrence of pad A is 8.8% while for pad B is 11%. Making an analysis of these contact zones, it can be seen that the contact including three types:

- Primary contact is main contact plateaus that is main cause on occurrence noise see in figure 6.17 (right). This contact surface type is often including metal fiber, hard material when they contact to disc material like as two metal surface sliding lead to stick-slip phenomena and occurs noise.

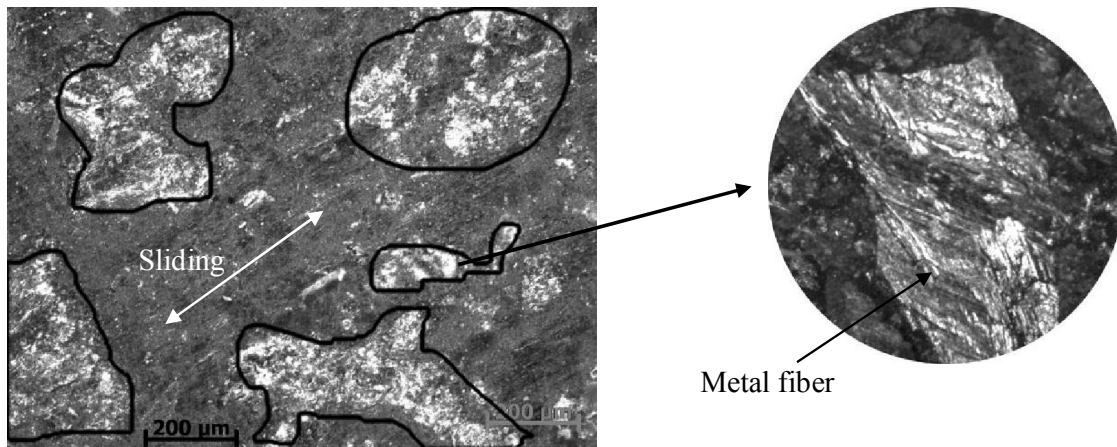


Fig 6.17 Contact zone of pad

- Slave contact is area around the primary contact including filter and binder. This contact plateaus help keep and link material components in pad.
- Matrix contact is normally some kind of phenolic resin. This contact plateaus is softer than primary contact and it may effect by temperature and humidity for creating noise.

Some EDS and SEM analyses of pad B, shown in figures 6.18, 6.19 and 6.20, are examples for these three contact types.

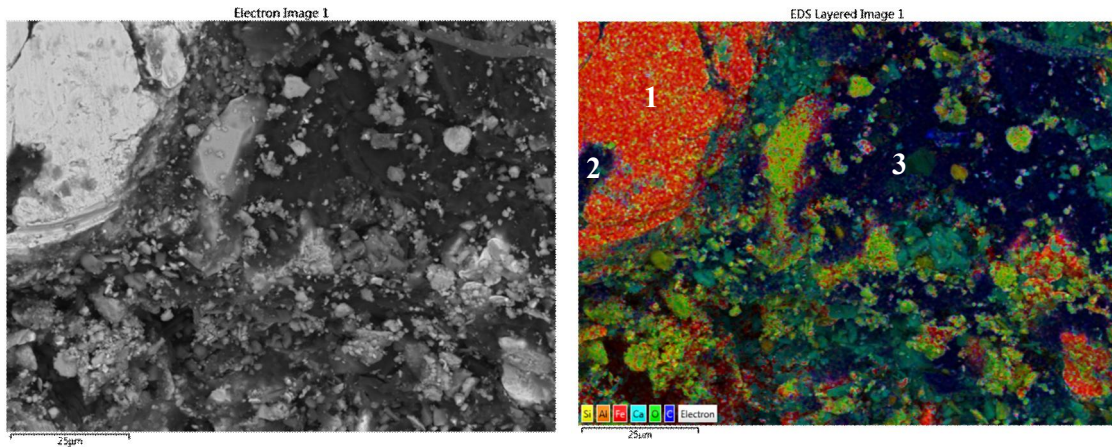


Fig 6.18 X-ray contact area of pad with different contact type

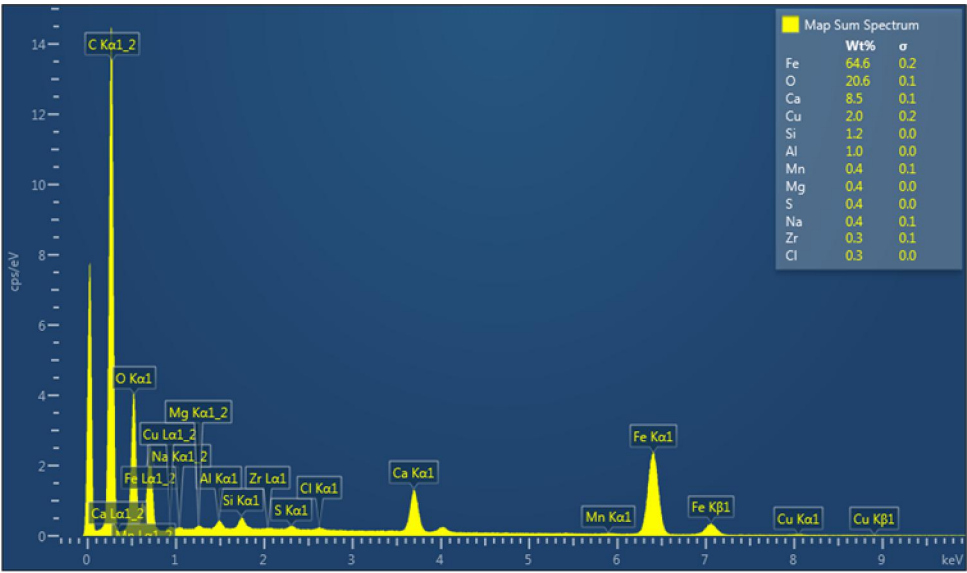


Fig 6.19 Map sum spectrum of chemical components of contact zone

After the braking test about two minutes of 100 km/h with several stop sequences the pad was tested by SEM and EDS method. Figure 6.18 and figure 6.20 shows three contact plateau types, in which the main contact is element Fe - red color or marked by 1, the slave contact plateaus is number 2 and the matrix contact plateaus is marked by number 3. The weight percentage of chemical elements and compounds can be seen in figure fig 6.19 and table 6.2. Some various channels of material are shown in figure 6.21.

Table 6.2 Chemical components of pad at contact zone

Element	Wt%	Wt% Sigma	Standard Label
O	20.59	0.15	SiO ₂
Na	0.35	0.06	Albite
Mg	0.43	0.05	MgO
Al	0.96	0.04	Al ₂ O ₃
Si	1.15	0.04	SiO ₂
S	0.40	0.03	FeS ₂
Cl	0.28	0.03	NaCl
Ca	8.47	0.08	Wollastonite
Mn	0.43	0.08	Mn
Fe	64.62	0.23	Fe
Cu	1.98	0.18	Cu
Zr	0.32	0.10	Zr
Total:	100.00		

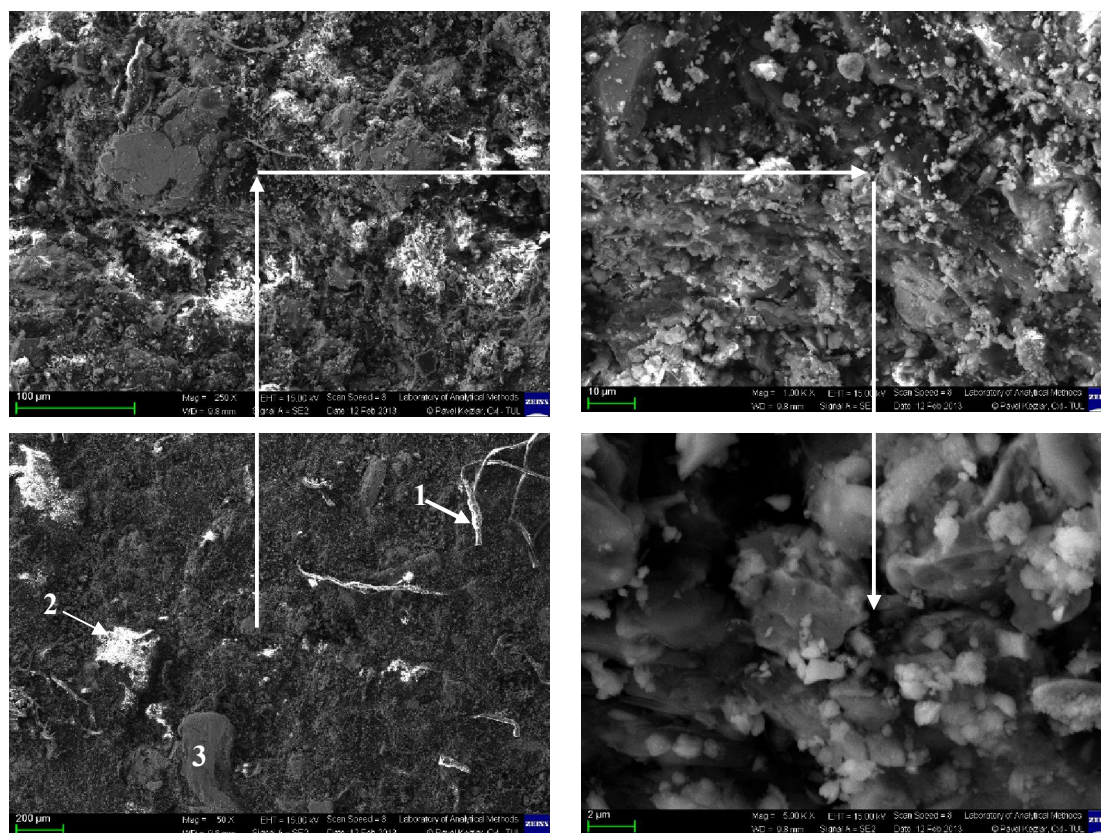


Fig 6.20 SEM of surface of pad with three contact plateau types in various magnifications

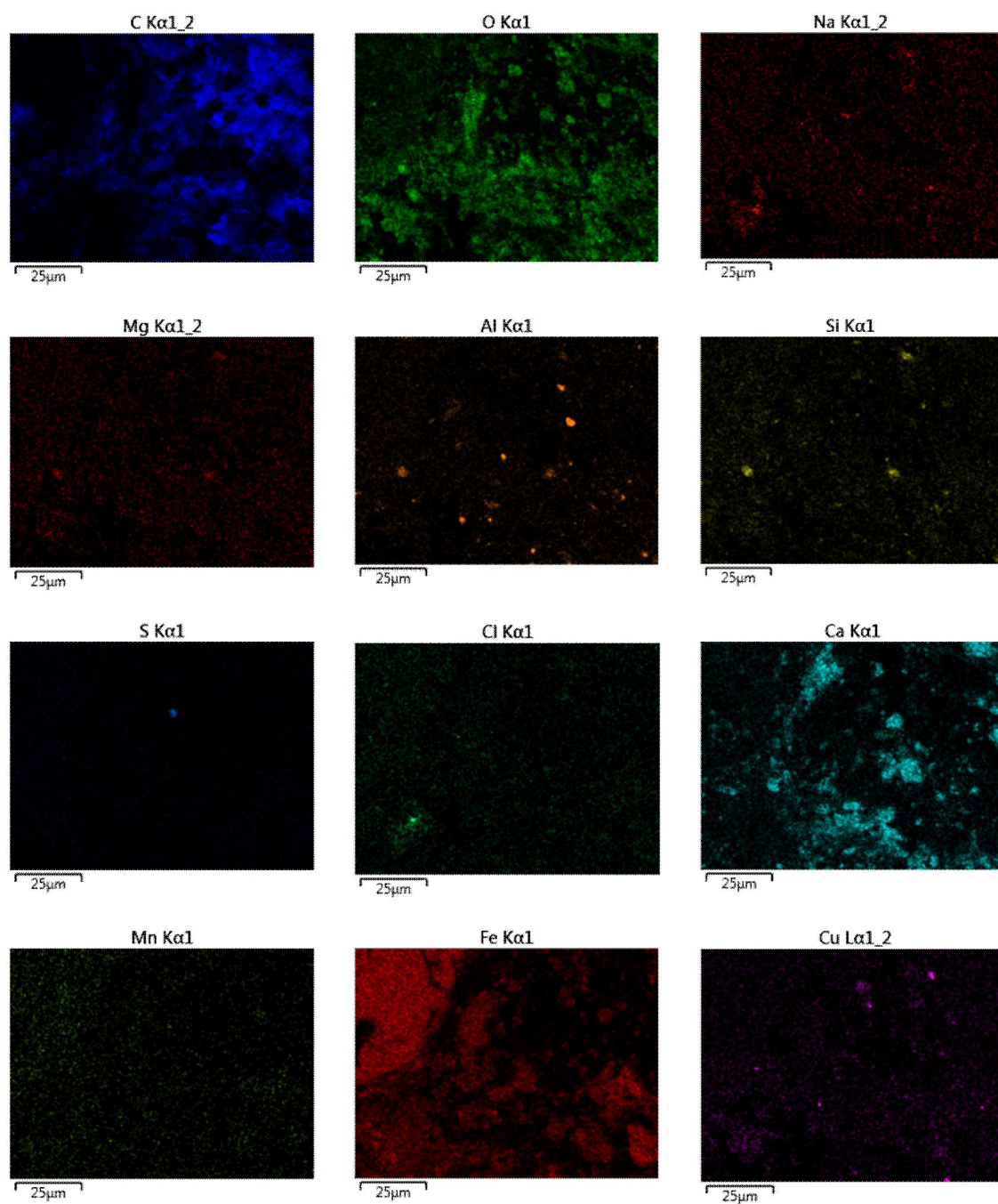


Fig 6.21 Channels of material components detected by EDS method

6.5 EFFECT OF FRICTION DAMPING AND HARDNESS ON BRAKE SQUEAL

6.5.1 Friction damping

Damping effect is one of the factors that effect to brake squeal, it was studied by some authors and they have concluded that if damping adds into the pin (for model pin-on-disc), it will help to reduce the instability [15]. Papers [19, 37, 38, 119] also make conclusions that increasing damping will help to stabilize the system or damping helps to reduce system instability. In chapter 4, the theory and results of simulation show that if the damping ratio is negative, it will lead to the unstable system. The damping ratio of system with various friction coefficients is shown in figure 6.22.

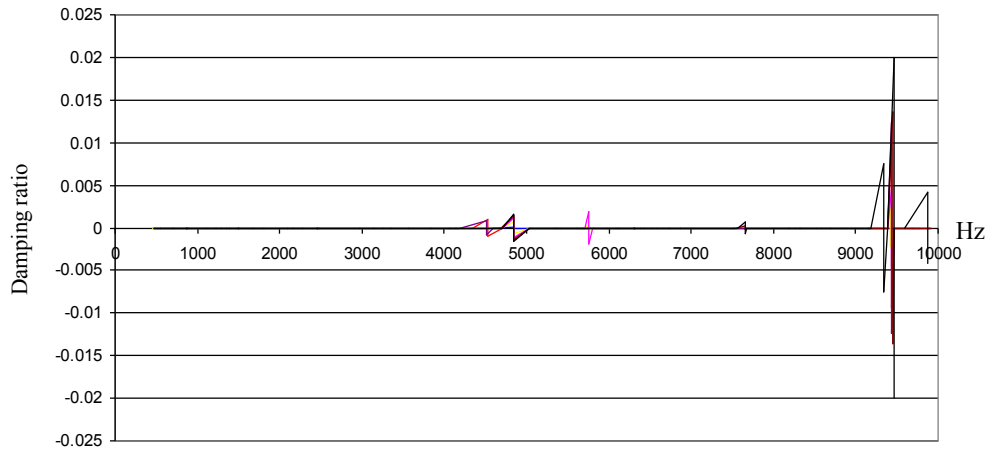


Fig 6.22 Damping ratio of system with different friction coefficient

In this figure, it can be seen that with different friction coefficient the unstable system is different. Detail of unstable frequencies can be seen at the table 4.3. Figure 6.23 presents an effect of friction damping to stable system. This damping due to the friction forces that stabilizing vibration along the contact surface in a direction perpendicular to the slip direction. This result shows the stability of system at the constant of friction coefficient $\mu = 0.35$. When ignoring the effect of friction damping, the system loss stable at three frequencies are 4521.4 Hz, 4837 Hz and 9437.1 Hz. But when considering the effect of friction damping, the stable of system is increasing and only appears two unstable frequencies at 4521.5 Hz and 9438.4 Hz. Thus, it can be recognized that the system is more stable.

The Abaqus automatically detects the contact nodes that are slipping due to velocity differences imposed by the motion of the reference frame or the transport velocity in prior steps. At those nodes, the tangential degrees of freedom will not be constrained and the effect of friction will result in an unsymmetric contribution to the stiffness matrix. At other nodes in contact the tangential degrees of freedom will be constrained [104].

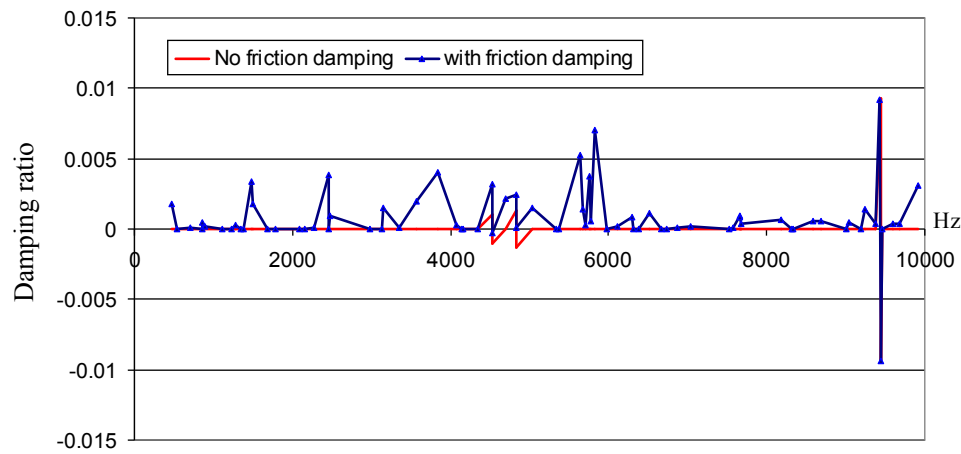


Fig 6.23 Effect of friction damping at $\mu = 0.35$

6.5.2 Hardness of pad and disc surface

During braking process under pressure condition, the pad and disc contact together. The phenomenon of wear occurs at the contact surfaces of pads and disc and the characteristics of the surfaces are changed. The properties of the surface layer change a little bit. One of them is the hardness that effect to the brake squeal. Hardness is the property of a material that enables it to resist plastic deformation, usually by penetration. Hardness is not an intrinsic material property dictated by precise definitions in terms of fundamental units of mass, length and time. A hardness property value is the result of a defined measurement procedure [120].

In this experiment, a method of Leeb rebound hardness test by MH180 Hardness Tester will be considered for surface hardness testing of disc and pads. The results of the test are presented in figure 6.24. It can be seen that the higher worn pad the higher hardness. The hardness value of the new pad, the pad with 10% worn and the pad with 70% worn is 468 HL, 500 HL and 512 HL, respectively. For disc surface, the hardness is 570 HL.

The surface hardness of disc and pad effects directly to damping between two surfaces of pad and disc, thus it leads to differences in percentage of noise occurrence. The relationship between roughness and hardness value is represented in figure 6.25. The roughness of the pad decreases when its hardness increases [121].

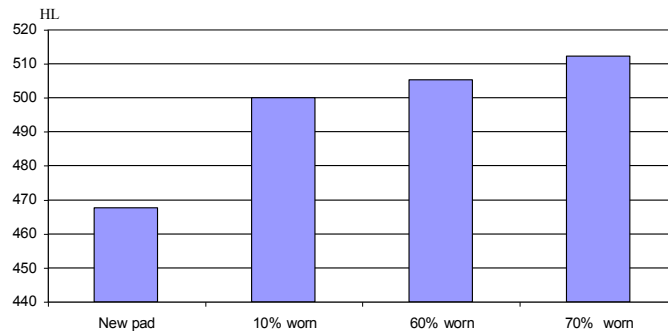


Fig 6.24 Hardness of pads surface with various worn

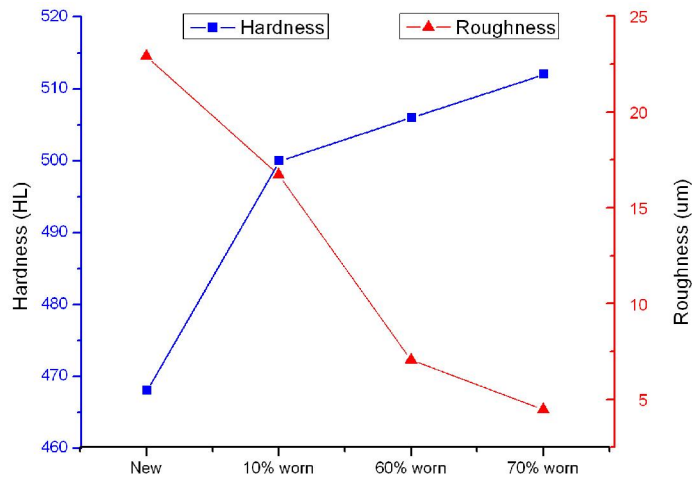


Fig 6.25 Relationship between hardness and roughness of pads with various worn

6.6 CONCLUSION

The brake squeal is a complex problem in identifying and prediction. It depends on factors and the main cause of noise is the friction coefficient between the pad and disc surfaces. Based on the experimental result and the simulation result, it can be seen that the prediction results are missed some frequencies that occur noise in the real conditions. Therefore, this analysis is addressed mainly to study the differences between the contact surfaces of the new pad and the old pad. This will help to explain why frequencies of noise are missed in simulation but they occur in reality.

From the experimental results of the two different worn pads, it can be seen that the percentage of noise occurrence of the higher worn pad is higher than that of the pad with low worn. But the experiment also found that at the speed of 50 km/h the pad with 10% worn occurs noise higher than the pad with 70% worn. Furthermore, the unstable frequencies below 4 kHz are missed in the prediction result of simulation. These can be explained and predicted by analyzing the surfaces characteristic of the disc and the pad. Therefore a quantitative analysis will give some conclusions for this analysis.

The contact zone between pad and disc is including three types, primary contact, slave contact and matrix contact. The primary contact of pad is metal (Fe) that is a cause on creating noise. The pad has many small contact plateaus that have tending occur noise than pads with a few large plateaus.

The roughness of the pad with low worn is higher than the roughness of the pad that has high worn. Another experiment also investigates that the hardness of the pad surface that has high worn is higher than that of the pad with low worn. These factors effect directly to the friction damping. The friction coefficient of the pad with low worn is high and its damping is better than that of the pad with high worn. A simulation shows that when considering the effect of friction damping, the system decreases the unstable frequencies and increases the stability of the system. This can explain why the pad with high worn tends to occur noise rather than the pad with low worn.

CONCLUSION AND RECOMMENDATIONS

7.1 CONCLUSION

Brake squeal is annoying and caused by vibration of the brake components, especially the pads and discs. Besides, brake squeal is subjected by many factors such as temperature, humidity, material properties, etc. For detailed understanding about the brake squeal, therefore, in this dissertation a study of identifying and prediction of vibration and noise of disc brake system was performed to provide solutions for reducing noise of disc brake. The results of dissertation draw out some conclusions that:

Developing the 3D model of the disc brake system with full geometries is necessary for simulation in Abaqus. It describes exactly geometries of components of the disc brake. It helps calculation and simulation that will approach operational cases of disc brake in real condition. Besides, advantage of developing 3D of the disc brake system will support for using methods as FEM and FEA on the computer.

The complex eigenvalue analysis is used for identifying and prediction noise of disc brake that is a advantage method. A complex eigenvalue algorithm is used to solve this eigenvalue problem in order to obtain eigenvalues and eigenvectors in complex values. Eigenvalues with positive real parts are identified as unstable modes, which always appear in complex conjugate pairs. The simulation results are compared with the experimental results in various operation conditions.

Modal analysis is the process of determining the inherent dynamic characteristics of a system in forms of natural frequencies, damping factors and mode shapes, and they are used to formulate the mathematical model for its dynamic behaviour. The results of modal analysis show that the natural frequency of the disc brake components plays an important role in the contribution of vibration and noise. Especially, natural frequency of pad and rotor

is important. If these natural frequencies are close together the resonance phenomena may occur. This leads to high vibration and noise occurrence. The simulation results are nearly the same with the experimental results in range of 10 kHz. Therefore, this analysis will help designer to avoid close natural frequencies when design the disc brake components.

The vibration and noise occur at unstable frequencies, therefore, a stability analysis of the system draw out to predict vibration and noise of the disc brake. Through this analysis, it can be seen that the friction coefficient is the main factor that effect to the stability of the system. The high friction coefficient can increase the unstable frequencies and noise may occur. The FE analyses by complex modes indicate that when two modes close to each other in the frequency range coalesce under the influence of friction and then become coupled thus the system becomes unstable. For each couple modes, a critical friction coefficient is identified. If the friction coefficient exceeds this value, the system become unstable and noise may occur. The prediction result shows that the unstable frequencies occur in range from 4 kHz to 10 kHz. For normal case with $\mu = 0.35$, $P = 3 \text{ bar}$ and $\Omega = 9.7 \text{ rad/s}$ the system has three unstable modes. This result is compared with the experimental result and clearly they are close together but the simulation result misses unstable frequencies below 4 kHz.

The effect of Young's modulus on brake squeal is very large. Considers changing Young's modulus of the pad and the disc, can be given the conclusion: When the Young's modulus of disc increase, the unstable mode is increase. With the pad, the Young's modulus effect to brake squeal less than disc but the characteristic of pad surface is more effect to brake squeal. Similarly, the prediction of pressure shows that the high pressure can lead to high unstable frequency. Therefore, when design pad or disc, it is needed to choose the Young's modulus that is comfortable to limit ability occurrence noise.

The SEM and EDS are tools for analysis very useful for understanding about characteristic of disc and pad surface. Through quantitative analysis and the experimental result shows that percentage occurs noise of pad with high worn will higher than pad that has low worn. This analysis also recommends that effect of roughness and hardness is contributing to the brake noise because it effect directly to damping of system.

The experiment was performed by full suspension of the disc brake system is mounted on the real car. This system is full operation therefore it is suitable for experiment. It helps simulation is more nearly in real operational conditions. Furthermore, the experiment was performed on the chassis dynamometer system. This is good conditions to simulate in various operation of disc brake in actual conditions. The same way, two pads with different worn are used for experiment to predict noise. The experimental result shows that the noise occurrence at the main frequencies 5 kHz and coherent with the signal of accelerometer the amplitude of vibration occur at 2 kHz and 5 kHz. Identifying noise by microphone signal and combine measurement vibration by accelerometer signal is very good for prediction of brake squeal. Beside experiment also identify percentage occur noise of disc brake system in various operation conditions as velocity and pressure.

In summary, the purpose of this work is to present a method for identifying and prediction of vibration and noise of the disc brake by using software Abaqus for simulation and compare with the experiment results in various operational conditions. Through this analysis, the frequencies of noise occurrence of the disc brake can be identified and predicted. The results will help to manufacture and design the disc and brake pad that can reduce the brake squeal.

7.2 RECOMMENDATIONS

Although the work tries to identify and predict vibration and noise of the disc brake but some factors that effect to brake squeal as temperature and humidity have not been considered. Hope in the future some other author will contribute experiment and simulation.

The result will be more exactly if temperature and humidity is not neglect in this work. Because the humidity effect too much on brake squeal in the morning that called “morning sick”.

Modal experiment must be for all components of the disc brake instead only for disc and pad as present. Suggest new equipment as laser for modal test. It will be more exactly.

Frequencies below 1000 Hz must be studied to understand brake squeal phenomena or high frequency until 20 kHz need also to be studied.

The number of stops for each case or each operation case is enough quantity as standard J2521 to obtain result that are more exactly. In this work the number of stops was limited because the cost and time for experiment is too much.

The equipment to capture noise and vibration must have more than two channels because more sensors will make better measurement in various directions. A good result for measurement is including four channels, two for noise measurement and two for vibration measurement in two directions: perpendicular and parallel to the axis of the disc.

Study phenomena of wear of disc and brake pad effect on brake squeal and assessment friction coefficient after each experimental step.

REFERENCES

1. Thompson, J.K., *Brake NVH testing and Measurements*. SAE international 2011.
2. TECHNOLOGY, A.B., *Brake Noise, Vibration and Harshness: Technology Driving Customer Satisfaction*.
3. Bakar, A.R.A., *Modelling and Simulation of Disc Brake Contact Analysis and Squeal*, in *Mechanical Engineering* 2005, University of Liverpool: Liverpool, United Kingdom.
4. Frank Chen, C.A.T., Ronald L. Quaglia, *Disc Brake Squeal: Mechanism, Analysis, Evaluation, and Reduction/Prevention*, 2006, SAE international.
5. K, S., et al., *Brake noise vibration and harshness*, M.t.a. engineering, Editor 2011.
6. Kragelskii I.V, G.N.V., *Frictional Autovibrations*, 1987: Nauka, Moscow.
7. M, G. and M. Nishiwaki, *Study on disc brake groan*. SAE paper no 900007, 1990.
8. J, B., *Mechanisms of brake creep groan*. SAE paper no 973026, 1997.
9. Sergienko, V.P., S.N. Bukharov, and K.A. V., *Noise and Vibration in Brake Systems of Vehicles. Part 1: Experimental procedures*. Friction and Wear, 2008. Vol. 29(No. 3): p. 8.
10. Kinkaid, N.M., O.M. O'Reilly, and P. Papadopoulos, *Automotive disc brake squeal*. Journal of Sound and Vibration, 2003. 267(1): p. 105-166.
11. Papinniemi, A., et al., *Brake squeal: a literature review*. Applied Acoustics, 2002. 63(4): p. 391-400.
12. Lanchester, F.W., *Improvements in the brake mechanism of power-propelled road vehicles*, G.B, Editor 1902: England.
13. Newcomb, T.P. and R.T. Spurr, *A Technical history of the Motor Car* 1989, New York: Adam Hilger.
14. Chung, C.H. and K. Kobayashi, *A new analysis method for brake squeal part 1: Theory for Modal Domain Formulation and Stability Analysis*. Society of Automotive Engineers, Inc, 2000.
15. Jarvis, R.P., *Vibrations induced by dry friction*. proceedings of the institution of mechanical engineers 1963. 178(847-866).
16. Hulten, J.O. and J. Flint, *An assumed modes method approach to disc brake squeal analysis*, SAE, Editor 1999: Warrendale PA, SAE.
17. Ouyang, H. and J.E. Mottershead, *A bounded region of disc-brake vibration instability*. Journal of Vibration and Acoustics-Transactions of the Asme, 2001. 123(4): p. 543-545.
18. Ouyang, H., et al., *A methodology for the determination of dynamic instabilities in a car disc brake*. International Journal of Vehicle Design, 2000. 23(3-4): p. 241-262.
19. Ouyang, J.E., et al., *Dynamic instabilities in a simple model of a car disc brake*, SAE, Editor 1999: Warrendale, PA.
20. Tseng, J.G. and J.A. Wickert, *Nonconservative stability of a friction loaded disk*. Journal of Vibration and Acoustics, 1998. 120(4): p. 7.
21. Baba, H., T. Wada, and T. Takagi, *Study on reduction of brake squeal caused by in-plan vibration on rotor*, in *Technical report*, SAE, Editor 2001: Warrendale PA.
22. Kung, S.W., K.B. Dunlap, and R.S. Ballinger, *Complex eigenvalue analysis for reducing low frequency brake squeal*, in *Technical report 2000-01-0444*, SAE, Editor 2000: Warrendale, PA.
23. Bae, J.C. and J.A. Wickert, *Free vibration of coupled disk-hat structures*. Journal of Sound and Vibration, 2000. 235(1): p. 15.

24. Ono, K., J.S. Chen, and D.B. Bogy, *Stability Analysis for the Head-Disk Interface in a Flexible Disk Drive*. Journal of Applied Mechanics-Transactions of the Asme, 1991. 58(4): p. 1005-1014.
25. Mottershead, D.J. and S.N. Chan, *Vibration- and friction- induced instability in disks*. Shock and Vibration Digest, 1998. 30(1): p. 17.
26. Chang, J.Y. and J.A. Wickert, *Response of modulated doublet modes to travelling wave excitation*. Journal of Sound and Vibration, 2001. 242(1): p. 69-83.
27. Kim, M., J. Moon, and J.A. Wickert, *Spatial modulation of repeated vibration modes in rotationally periodic structures*. Journal of Vibration and Acoustics-Transactions of the Asme, 2000. 122(1): p. 62-68.
28. Raman, A. and C.D. Mote, *Effects of imperfection on the non-linear oscillations of circular plates spinning near critical speed*. International Journal of Non-Linear Mechanics, 2001. 36(2): p. 261-289.
29. Lamb, H. and R.V. Southwell, *The vibrations of a spinning disk*. Royal Society of London Proceedings Series A Mathematics Physics and Engineering Science, 1921. 699(99): p. 8.
30. Milovs, V., *Computer simulation of automotive disc brake noise*, in *Department of Mechanical Engineering*2003, New Jersey Institute of Technology.
31. Spurr, R.T. *A theory of brake squeal*. in *Proceedings of the Automobile Division*. 1961. Institution of Mechanical Engineers
32. Jarvis, R.P. and B. Mills, *Vibrations induced by friction*. Proceedings of the Institution of Mechanical Engineers 1963. 178(32): p. 10.
33. Earles, S.W.E. and M. Badi, *Oscillatory instabilities generated in a double-pin and disc undamped system: a mechanism of disc brake squeal*. Proceedings of Institution of Mechanical Engineers, 1984. 198: p. 6.
34. Earles, S.W.E. and P.W. Chambers, *Disc brake squeal noise generation: Predicting its dependency on system parameters including damping*. International of Vehicle Design 1987. 8: p. 14.
35. Earles, S.W.E. and C. Lee, *Instabilities arising from the frictional interaction of a pin-disk system resulting in noise generation*. Transactions of the American Society of Mechanical Engineers Journal of Engineering for Industry, 1976. 98(1): p. 5.
36. Duour, P., *Noise generation in vehicle brakes*, in *Engineering Department*2002, Cambridge University: Jesus College, Cambridge.
37. North, M.R., *Disc brak squeal in braking of road vehicles*. Automobile division of the institution of Mechanical engineers, 1976: p. 169-176.
38. North, M.R., *Disc brake squeal - A theoretical model*, in *Technical Report 1972, 5*, Editor 1972, Motor Industry Research Association: Warwickshire, England,.
39. Brooks, P.C., et al., *Eigenvalue sensitivity analysis applied to disc brake squeal*. Braking of Road Vehicles, Institution of Mechanical Engineers, 1993: p. 8.
40. Rudolph, M. and K. Popp, *Brake squeal*, in *Final Report of a Joint Research Project Sponsored by the German Federal Ministry of Education and Research*, U.a.A.o.N.D.E.i.E.A. Detection, Editor 2001: Shaker, Aachen. p. 197-225.
41. Rudolph, M. and K. Popp. *Friction induced brake vibrations*. in *CD-ROM Proceedings of DETC'01, DETC2001, VIB-21509*. 2001. Pittsburgh, PA.
42. Ahuja, N., *Some studies on modeling and simulation of a disc brake system for squeal prediction*, in *Graduate Faculty*2005, Auburn University: Auburn, Alabama.
43. Brief, A.T., *Automotive Brake Squeal Analysis Using a Complex Modes Approach*, 2007: Simula, Dassault Systèmes.

44. Kang, J., *Finite element modelling for the investigation of in-plane modes and damping shims in disc brake squeal*. Journal of Sound and Vibration, 2012. 331(9): p. 2190-2202.
45. Lou, G., T.W. Wu, and Z. Bai, *Disk brake squeal prediction using the ABLE algorithm*. Journal of Sound and Vibration, 2004. 272(3-5): p. 731-748.
46. Nouby, M. and K. Srinivasan, *Simulation of the structural modifications of a disc brake system to reduce brake squeal*. Proceedings of the Institution of Mechanical Engineers Part D-Journal of Automobile Engineering, 2011. 225(D5): p. 653-672.
47. Nouby, M., D. Mathivanan, and K. Srinivasan, *A combined approach of complex eigenvalue analysis and design of experiments (DOE) to study disc brake squeal*. International Journal of Engineering, Science and Technology, 2009. Vol. 1, No. 1, 2009, pp. 254-271(No. 1): p. 17.
48. Trichês Júnior, M., S.N.Y. Gerges, and R. Jordan, *Analysis of brake squeal noise using the finite element method: A parametric study*. Applied Acoustics, 2008. 69(2): p. 147-162.
49. Liu, P., et al., *Analysis of disc brake squeal using the complex eigenvalue method*. Applied Acoustics, 2007. 68(6): p. 603-615.
50. HASSAN, M.Z., *Brake Squeal Noise: Finite Element Analysis*, in *Kolej Universiti Teknikal Kebangsaan Malaysia*, F.o.M. Engineering, Editor 2007: Karung Berkunci 1200, Ayer Keroh, Melaka, Malaysia.
51. Guangxiong, C., et al., *Comparative Study on the Complex Eigenvalue Prediction of Brake Squeal by Two Infinite Element Modeling Approaches*. Chinese Journal Of Mechanical Engineering, 2010. 23(1): p. 8.
52. AbuBakar, A.R. and H. Ouyang, *Wear prediction of friction material and brake squeal using the finite element method*. Wear, 2008. 264(11-12): p. 1069-1076.
53. Jung, S.P., et al. *Finite Element Analysis of Thermoelastic Instability of Disc Brakes in World Congress on Engineering 2010*. 2010. London, U.K.
54. Abu Bakar, A.R., H. Ouyang, and D. Titeica, *Modelling And Simulation Of Disc Brake Contact Analysis And Squeal*, in *Seminar on Advances Malaysian Noise, Vibration and Comfort 2005*. p. 1-8.
55. Thai, H.L.H., N. Pavel, and P.T. Nhan, *Identify Source Vibration And Noise Of Disc Brake in The 7th International Conference Research And Development Of Mechanical Elements And Systems*, U.o. Niš, Editor 2011: SERBIA. p. 1-4.
56. Thai, H.L.H. and N. Pavel., *Sensitivity analysis and optimization disc brake by stress constraints*. Applied Mechanics and Materials Online available since, 2011. 52-54: p. 583-588.
57. Cao, Q., et al., *Linear eigenvalue analysis of the disc-brake squeal problem*. International Journal for Numerical Methods in Engineering, 2004. 61(9): p. 1546-1563.
58. AbuBakar, A.R. and H. Ouyang, *Complex eigenvalue analysis and dynamic transient analysis in predicting disc brake squeal*. Vehicle Noise and Vibration, 2006 Vol. 2 (No. 2): p. 13.
59. Ouyang, H., et al., *Predicting Disc Brake Squeal Frequencies Using Two Distinct Approaches* 2011. p. 1-8.
60. Bakar, A.R.A. and H. OUYANG, *Prediction Of Disc Brake Contact Pressure Distributions By Finite Element Analysis*. Jurnal Teknologi, Universiti Teknologi Malaysia, 2005. 43: p. 21-36.
61. Bakar, A.R.A. and H. Ouyang, *Chapter 4: Recent Studies Of Car Disc Brake Squeal*, in *New Research on Acoustics*, B.N. Weiss, Editor 2008, Nova Science Publishers, Inc. p. 159-198.

62. AbuBakar, A.R., et al. *Numerical Analysis of Disc Brake Squeal Considering Temperature Dependent Friction Coefficient*. Universiti Teknologi Malaysia, 81310 UTM Skudai
63. Guillaume, F., et al., *Effects of damping on brake squeal coalescence patterns – application on a finite element model*. Mechanics Research Communications 2007 34 p. 181–190.
64. Felske, A., G. Hoppe, and H. Matth, *Oscillations in squealing disc brakes - analysis of vibration modes by holographic interferometry*, in *Technical Report 780333*, SAE, Editor 1978: Warrendale, PA, 1978.
65. Abendroth, H., *Advances in brake NVH test equipment*. Automotive Engineering International, 1999. 107 (2): p. 60-63.
66. Krupka, R., T. Walz, and A. Ettemeyer, *New techniques and applications for 3D-brake vibration analysis*, in *Technical Report 2000-01-2772*, SAE, Editor 2000: Warrendale, PA, 2000.
67. Giannini, O. and F. Massi, *Characterization of the high-frequency squeal on a laboratory brake setup*. Journal of Sound and Vibration, 2008. 310(1–2): p. 394-408.
68. Giannini, O., A. Akay, and F. Massi, *Experimental analysis of brake squeal noise on a laboratory brake setup*. Journal of Sound and Vibration, 2006. 292(1–2): p. 1-20.
69. Chen, G.X., et al., *Experimental investigation into squeal under reciprocating sliding*. Tribology International, 2003. 36(12): p. 961-971.
70. Cunefare, K.A. and A.J. Graf, *Experimental active control of automotive disc brake rotor squeal using dither*. Journal of Sound and Vibration, 2002. 250(4): p. 579-590.
71. Berndt, P.J. and W. Schweiger, *Experimental and theoretical investigation of brake squeal with disc brakes installed in rail vehicles*. Wear, 1986. 113(1): p. 131-142.
72. Flint, J. and J. Hult  N, *Lining-deformation-induced modal coupling as squeal generator in a distributed parameter disc brake model*. Journal of Sound and Vibration, 2002. 254(1): p. 1-21.
73. Fosberry, R.A.C. and Z. Holubecki, *Disc brake squeal: its mechanism and suppression*, in *Technical Report 1961*, M.I.R. Association, Editor 1961: Warwickshire, England.
74. Fosberry, R.A.C. and Z. Holubecki, *Interim report on disc brake squeal*, in *Technical Report 1959*, M.I.R. Association, Editor 1959: Warwickshire, England.
75. Spurr, R.T. *A theory of brake squeal*. in *Proceedings of the Automobile Division*. 1961.
76. Earles, S.W.E., *A mechanism of disc-brake squeal*, in *Technical Report 770181*, SAE, Editor 1977: Warrendale, PA.
77. Earles, S.W.E. and M. Badi. *Oscillatory instabilities generated in a double-pin and disc undamped system: a mechanism of disc-brake squeal*. in *Proceedings of the Institution of Mechanical Engineers*. 1984.
78. Earles, S.W.E. and P.W. Chambers, *Disc brake squeal noise generation: predicting its dependency on system parameters including damping*. International Journal of Vehicle Design 1987. 8: p. 538-552.
79. North, M.R., *Disc brake squeal - A theoretical model*, in *Technical Report 1972/5*, M.I.R. Association, Editor 1972: Warwickshire, England.
80. Millner, N., *An analysis of disc brake squeal*, in *Technical Report 780332*, SAE, Editor 1978: Warrendale, PA.
81. Murakami, H., N. Tsunada, and T. Kitamura, *A study concerned with a mechanism of disc-brake squeal*, in *Technical Report 841233*, SAE, Editor 1984: Warrendale, PA.
82. Nishiwaki, M., et al., *Study on disc brake squeal*, in *Technical Report 890864*, SAE, Editor 1989: Warrendale, PA.

83. Talbot, C. and J.D. Fieldhouse, *Animations of a disc brake generating noise*, in *Technical Report 2001-01-3126*, SAE, Editor 2001: Warrendale, PA.
84. Triches, J.M., S.N.Y. Gerges, and R. Jordan, *Reduction of Squeal Noise from Disc Brake Systems Using Constrained Layer Damping*. Technical Editor: Atila P. Silva Freire, 2004. Vol. XXVI(No. 3): p. 340-348.
85. Massi, F., Y. Berthier, and L. Baillet, *Contact surface topography and system dynamics of brake squeal*. *Wear*, 2008. 265(11-12): p. 1784-1792.
86. Chen, G.X., et al., *Effect of surface topography on formation of squeal under reciprocating sliding*. *Wear*, 2002. 253(3-4): p. 411-423.
87. Eriksson, M., *Friction and Contact Phenomena of Disc Brakes Related to Squeal*, in *Faculty Of Science And Technology 2000*, ACTA University Upsaliensis Uppsala
88. Eriksson, M. and S. Jacobson, *Tribological surfaces of organic brake pads*. *Tribology International*, 2000. 33(12): p. 817-827.
89. Tworzydło, W.W., E.B. Becker, and J.T. Oden, *Numerical modeling of friction induced vibrations and dynamic instability*. ASME DE, ASME International, New York, 1992. 49: p. 13-32.
90. Tworzydło, W.W., et al. *Experimental and Numerical studies of friction induced oscillations of a Pin on disk apparatus*. in *ASME Design engineering technical conferences*. 1997. New York.
91. analýza, E.m., *Experimentální modální analýza 2009*, VŠB - Technická univerzita Ostrava: Fakulta strojní, Katedra mechaniky
92. Kjer, B., *User Guide Real-time Frequency Analyzer type 2144*, 1991, Bruel & Kjer.
93. MUHAMMAD, Z.H., *Experimental modal analysis of brake squeal noise*, in *Faculty of Mechanical Engineering*, K.U.T.K. Malaysia, Editor 2005: Karung Berkunci 1200, Ayer Keroh, Melaka, Malaysia.
94. He., J. and Z.-F. Fu, *Modal Analysis* 2001, Great Britain: Butterworth-Heinemann, Linacre House, Jordan Hill, Oxford OX2 8DP.
95. Wikipedia.org. http://en.wikipedia.org/wiki/Disc_brake#Calipers.
96. Abu Bakar, A.R. and I.R.H. RAJA, *Disc brake squeal: step-by-step approach using abaqus*, 2010: UTM.
97. Avitabile, P., *Experimental modal analysis (A simple Non-Mathematical Presentation)*, 2000, University of Massachusetts Lowell: Lowell, Massachusetts USA.
98. Bakar, A.R.A., et al., *Stability analysis of disc brake squeal considering temperature effect*. *Jurnal Mekanikal* 2006. No. 22: p. p.26 -38.
99. JOE., Y.G., et al., *Analysis of disc brake instability due to friction induced vibration using a distributed parameter model*. *International Journal of Automotive Technology*, 2008. Vol. 9(No. 2): p. pp. 161-171.
100. Meng., X. and G. Zhou., *Analytical Investigation of the Influence of Friction Coefficient on Brake Noise*, 2009: School of Traffic and Vehicle Engineering. pp. 1-5.
101. Sinou, J.J., *Transient non-linear dynamic analysis of automotive disc brake squeal – On the need to consider both stability and non-linear analysis*. *Mechanics Research Communications*, 2010. 37(1): p. 96-105.
102. Nouby, M.G., S. Mohammed., and A.M. Abd-El-Tawwab., *Understanding Mode-Coupling Mechanism of Brake Squeal Using Finite Element Analysis*. *International Journal of Engineering Research and Applications (IJERA)*, 2012. Vol. 2(Issue 1): p. pp.241-250.
103. Thai, H.L.H., N. Pavel, and P.T. Nhan. *the 10th International Conference on Vibration Problems*. in *Vibration Problems ICOVP 2011*. 2011. Liberec, Czech Republic: Technical University of Liberec.
104. Abaqus, *Abaqus/CAE User's manual, version 10*. V10.

105. Giannini, O., A. Sestieri, and A. Akay, *Prediction of squeal in a laboratory disk brake*. Mechanical Engineering Department, University of Rome "La Sapienza", 2002: p.1-6.
106. Rabia, A.M.M., et al., *An Experimental Study of Automotive Disc Brake Vibrations*. The International Journal of Engineering And Science (IJES), 2013. Vol 2(1): p. 194-200.
107. Kinzel, Q., *Part 6: Test Rig - Exhaust Gas Test Center Sindelfingen*, 2001: QPQ Exhaust Gas Test Center.
108. International SAE, *Disc and Drum Brake Dynamometer Squeal Noise Matrix*, in *Surface vehicle recommended practice*2000.
109. Eriksson, M., F. Bergman, and S. Jacobson, *Surface characterisation of brake pads after running under silent and squealing conditions*. Wear, 1999. 232(2): p. 163-167.
110. Bergman, F., M. Eriksson, and S. Jacobson, *Influence of disc topography on generation of brake squeal*. Wear, 1999. 225: p. 621-628.
111. Eriksson, M., F. Bergman, and S. Jacobson, *On the nature of tribological contact in automotive brakes*. Wear, 2002. 252(1-2): p. 26-36.
112. Bettge, D. and J. Starcevic, *Quantitative description of wear surfaces of disc brakes using interference microscopy*. Wear, 2001. 248(1-2): p. 121-127.
113. Massi, F., Y. Berthier, and L. Baillet, *Contact surface topography and system dynamics of brake squeal*. Wear, 2008. 265(11-12): p. 1784-1792.
114. Hetzler, H. and K. Willner, *On the influence of contact tribology on brake squeal*. Tribology International, 2012. 46(1): p. 237-246.
115. Goldstein, J., *Scanning electron microscopy and x-ray microanalysis*2003, UK: Springer London, Limited.
116. Roubicek, V., et al., *Wear and environmental aspects of composite materials for automotive braking industry*. Wear, 2008. 265(1-2): p. 167-175.
117. Chan, D. and G.W. Stachowiak, *Review of automotive brake friction materials*. Proceedings of the Institution of Mechanical Engineers Part D-Journal of Automobile Engineering, 2004. 218(D9): p. 953-966.
118. Bettge, D. and J. Starcevic, *Quantitative description of wear surfaces of disc brakes using nterference microscopy*. Wear 2001. 248: p. 121-127.
119. Ouyang, H., et al., *Friction-induced parametric resonances in discs: Effect of a negative friction-velocity relationship*. Journal of Sound and Vibration, 1998. 209(2): p. 251-264.
120. England, G., *Hardness test in Independent Metallurgist and Consultant to the Thermal Spray Coating Industry*2008: Kingfishers, Folly Lane North, Upper Hale, Farnham, Surrey,England. p. 1-13.
121. Thai, H.L.H., *Influence of wear on brake squeal*, in *Workshop pro dokrorandy FS a FT TUL T.U.v. Liberci*, Editor 2012, Technická Univerzita v Liberci: Světlanka, Liberec, Czech Republic. p. 177-181.

LIST OF PUBLICATIONS

1. Journals

- 01 **Huynh Le Hong Thai**, Pavel Němeček; *Sensitivity analysis and optimization disc brake by stress constraints*; Journal Applied Mechanics and Materials Vols. 52-54 (2011) pp 583-588.
Doi:10.4028/www.scientific.net/AMM.52-54.583
- 02 **Huynh Le Hong Thai**, Dittrich Aleš and Dráb Ondřej; *Model Predict Vibration and Noise of Disc Brake*; Journal Applied Mechanics and Materials Vol. 232 (2012) pp 461-464, © (2012) Trans Tech Publications
Doi:10.4028/www.scientific.net/AMM.232.461
- 03 **Huynh Le Hong Thai**, Phan Thanh Nhan; *Analysis stability of brake related to squeal by finite element method*; GRANT journal ISSN 1805-062X, 1805-0638 (online),
ETTN 072-11-00002-09-4

2. Conferences

- 01 **Huynh Le Hong Thai**, Pavel Němeček; *Sensitivity analysis and optimization disc brake by stress constraints*; 2011 1st International Conference on Mechanical Engineering, ICME 2011; Phuket; Thailand; 3-4 April 2011.
- 02 **Huynh Le Hong Thai**, Pavel Němeček, Phan Thanh Nhan; *Sensitivity analysis and optimization drum brake by stress constraints*; 16th International Conference "Mechanika - 2011" 7, 8 April 2011 Kaunas, Lithuania.
- 03 **Huynh Le Hong Thai**, Pavel Němeček, Phan Thanh Nhan; *Identify source vibration and noise of disc brake*, the 7th International Scientific Conference research and development of mechanical elements and system IRMES 2011. 27-28, April, 2011 in Zlatibor, Republic of Serbia.
- 04 **Huynh Le Hong Thai**, *Methods analysis vibration and noise of car disc brake*, ICoVP-2011, September 5-8, 2011, PRAGUE, Czech Republic.
Vibration Problems ICOVP 2011: Supplement - The 10th International Conference on Vibration Problems, Technical University of Liberec, 2011.
ISBN: 8073727595, 978-80-7372-7598

- 05 **Huynh Le Hong Thai**, *Simulation and experimental methods to identify vibration characteristic of disc brake squeal*; Workshop pro doktorandy FS & FT , 19-22/09/2011 Světlanka, Czech Republic.
ISBN 978-80-7372-765-9
- 06 **Huynh Le Hong Thai**, Dittrich Aleš and Dráb Ondřej; *Model Predict Vibration and Noise of Disc Brake*; 2012 3rd International Conference on Mechanical and Aerospace Engineering, ICMAE 2012; Paris; France; 7 - 8 July 2012
- 07 **Huynh Le Hong Thai**, Phan Thanh Nhan; *Analysis stability of brake related to squeal by finite element method*; Conference QUAERE 2012, 14-18 kvetna, Hradec Kralove, Czech Republic.
ISBN 978-80-905243-0-9
- 08 **Huynh Le Hong Thai**, *Influence of wear on brake squeal*; Workshop pro Doktorandy – Svetlanka 9-2012, Czech Republic.
ISBN: 798-80-7372-891-5
- 09 **Huynh Le Hong Thai**, *Prediction disc brake squeals by microscopy method Surfaces pads*, Scientific conference with international participation Technologia Europea 2012, 12/2012, Czech Republic.
ISBN: 978-80-905243-4-7