COMPRESSION BEHAVIOUR AND ELASTIC RECOVERY OF HIGHLOFT MATERIALS (KELVIN-MAXWELL MODEL)

Jana Přívratská
***Katarina Zelová**

Technical University of Liberec
Faculty of Science, Humanities and Education
Studentská 2, 461 17, Liberec 1, Czech Republic
jana.privratska@tul.cz
*Technical University of Liberec
Faculty of Textile Engineering,
Studentská 2, 461 17, Liberec1, Czech Republic
katarina.zelova@tul.cz

Abstract

The behavior in compression and the elastic recovery of highsloft materials can be described by a Kelvin-Maxwell rheological model. The proposed model is a serial combination of the Kelvin model (parallel connection of Newtonian viscous fluids and elastic materials) and the Maxwell model (serial combination of Newtonian viscous fluids and elastic materials). This combination is able to cover the plastic deformation and relaxation behavior.

In this paper an algorithm for the determination of the input parameters of the proposed rheological model based on experimental data on condition that the load phase is carried out at constant stress for the time t_0 will be presented.

Introduction

It was shown in [1, 2, 3] that the compression resistance and the elastic recovery (*Fig. 1*) of highloft nonwovens (low density fibrous network structures characterised by a high ratio of thickness to weight per unit area)

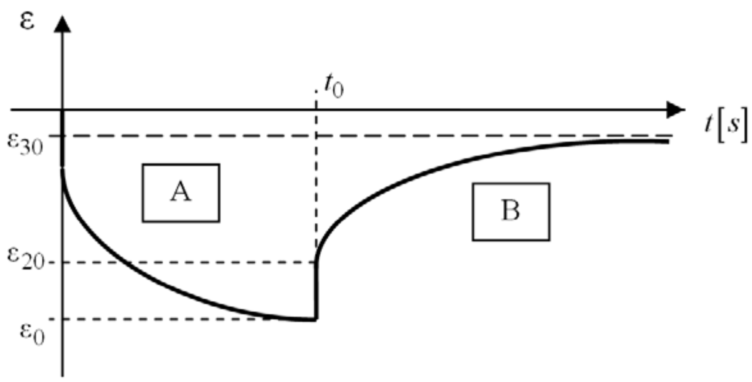

Fig. 1 *Behavior of a highloft material in loading-recovery test*

can be described by a rheological model composed of Kelvin and Maxwell models arranged in series (K-M model), (*Fig. 2*).

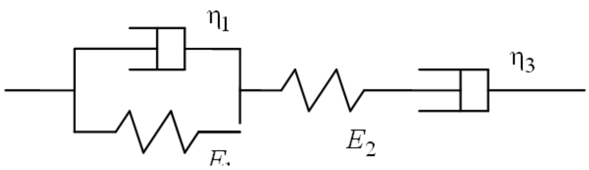

Fig. 2 Kelvin-Maxwell model

1 Model Description

The resulting strain ε of this model is the sum of the strain ε_1 of the Kelvin model and $(\varepsilon_2 + \varepsilon_3)$ of the Maxwell one, where ε_2 describes the strain of its elastic part and ε_3 the strain of its plastic part. Both parts, Maxwell and Kelvin, are under the same stress σ [4]

$$\sigma = E_2\varepsilon_2 = \eta_3 \frac{d\varepsilon_3}{dt} = E_1\varepsilon_1 + \eta_1 \frac{d\varepsilon_1}{dt}, \tag{1}$$

where E_1, E_2 are Young modules of springs (elastic elements) and η_1, η_2 are viscosities of viscosity elements. The stress-strain relation is determined by the differential equation [2]

$$\frac{d^2\sigma}{dt^2} + \left[E_2\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right) + \frac{E_1}{\eta_1}\right]\frac{d\sigma}{dt} + \frac{E_1}{\eta_1}\frac{E_2}{\eta_3}\sigma = E_2\frac{d^2\varepsilon}{dt^2} + \frac{E_1}{\eta_1}E_2\frac{d\varepsilon}{dt}. \tag{2}$$

1.1 Loading Modus

The material is compressed at time $t = 0$ by a constant stress σ_0 and kept for some time t_0.

As $\frac{d^2\sigma}{dt^2} = \frac{d\sigma}{dt} = 0$ for $t < t_0$, equation (2) will be simpler

$$\frac{E_1}{\eta_1}\frac{1}{\eta_3}\sigma_0 = \frac{d^2\varepsilon}{dt^2} + \frac{E_1}{\eta_1}\frac{d\varepsilon}{dt}. \tag{3}$$

Its solution under the initial conditions

$$\varepsilon(t = 0) = \frac{\sigma_0}{E_2} \tag{4}$$

(a jump of strain of the elastic part in Kelvin model) and

$$\frac{d\varepsilon}{dt}(t = 0) = \frac{\sigma_0}{\eta_3} - \frac{\sigma_0}{\eta_1} \tag{5}$$

(velocity of plastic strain of the first and the third part of M-K model) is for $0 \le t \le t_L$

$$\varepsilon(t) = \frac{\sigma_0}{E_2} + \frac{\sigma_0}{E_1}\left(1 - e^{-\frac{E_1}{\eta_1}t}\right) + \frac{\sigma_0}{\eta_3}t \tag{6}$$

where t_L is the time when maximal compression is reached (strain $\varepsilon = -1$).
For $t \geq t_L$ the compression strain is constant, $\varepsilon(t) = -1$.

1.2 Elastic Recovery Regime

At the time $t = t_0$, the stress σ_0 is removed and that is followed by a jump of strain $\varepsilon_0 \to \varepsilon_{20}$. This represents the elastic recovery of the material. The plastic or tenacious strain is equal to ε_{30}. As the stress $\sigma(t) = 0$ for $t \geq t_0$, the left side of equation (2) is zero and we get

$$0 = \frac{d^2\varepsilon}{dt^2} + \frac{E_1}{\eta_1}\frac{d\varepsilon}{dt} \, . \tag{7}$$

The solution of the differential equation (7) for elastic recovery regime is under the initial conditions

$$\frac{d\varepsilon}{dt}(t = t_0) = -\frac{E_1}{\eta_1}(\varepsilon_{20} - \varepsilon_{30}) \text{ and } \frac{d\varepsilon}{dt}(t = t_0) = -\frac{E_1}{\eta_1}(\varepsilon_{20} - \varepsilon_{30}) \, , \tag{8}$$

$$\varepsilon(t) = \varepsilon_{30} + (\varepsilon_{20} - \varepsilon_{30})e^{-\frac{E_1}{\eta_1}(t - t_0)} \, , \quad t_0 \leq t_L \tag{9}$$

$$\varepsilon(t) = \varepsilon_{30} + (\varepsilon_{20} - \varepsilon_{30})e^{-\frac{E_1}{\eta_1}(t - t_L)} \, , \quad t_0 \geq t_L \, . \tag{10}$$

2 Determination of Model Parameters

The analysis of measured stress-strain and recovery curves (*Fig. 1*) makes it possible to find the input parameters E_1, E_2, η_1, η_3 for the M-K model. In experiments we can measure

σ_0, ε_0, ε_{20} and ε_{30}. The parameter E_2 is determined from the equation (1)

$$E_2 = \frac{\sigma_0}{\varepsilon_2} = \frac{\sigma_0}{\varepsilon_0 - \varepsilon_{20}} \, . \tag{11}$$

The parameter η_3 is determined from the equations (1) and (6), as ε_{30} represents residual plastic strain

$$\eta_3 = \frac{\sigma_0}{\varepsilon_{30}} t_0 \, . \tag{12}$$

The rate $x = \dfrac{E_1}{\eta_1}$ can be determined from the elastic recovery curve (10)

$$x = \frac{1}{t_2 - t_1} \ln \frac{\varepsilon(t_1) - \varepsilon_{30}}{\varepsilon(t_2) - \varepsilon_{30}} \, . \tag{13}$$

Comparing the equations (6) and (9) it is possible to find

$$E_1 = \frac{\sigma_0}{\varepsilon_{20} - \varepsilon_{30}}\left(1 - e^{-xt_0}\right) \quad \text{for} \quad t_0 \le t_L \tag{14}$$

or

$$E_1 = \frac{\sigma_0}{\varepsilon_{20} - \varepsilon_{30}}\left(1 - e^{-xt_L}\right) \quad \text{for} \quad t_0 \ge t_L\,. \tag{15}$$

Than

$$\eta_1 = \frac{E_1}{x}\,. \tag{16}$$

Conclusion

The determination of input parameters for the Kelvin-Maxwell model from experiments enables to find a set of constants characterizing highloft materials. With these results it is possible to perform computer simulations of other theoretical experiments in order to propose their optimum design.

Literature

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Prof. RNDr. Jana Přívratská, CSc. Ph.D., Ing. Katarina Zelová

CHOVÁNÍ VYSOCE OBJEMNÝCH MATERIÁLŮ PŘI KOMPRESI
A ELASTICKÉM ZOTAVENÍ
(KELVINŮV-MAXWELLŮV MODEL)

Chování při kompresi a pružné zotavení vysoce objemných materiálů lze popsat pomocí Kelvinova-Maxwellova reologického modelu. Jde o sériové spojení Kelvinova modelu (paralelní spojení newtonovské viskózní kapaliny a hookovské elastické látky) a Maxwellova modelu (sériová kombinace newtonovské viskózní kapaliny a hookovské elastické látky). Tato kombinace je schopna obsáhnout plastickou deformaci i relaxační chování.

Uvádíme algoritmus, jak určit vstupní parametry pro tento reologický model pomocí experimentálních dat v případě, že zátěžová fáze probíhá při konstantním napětí po dobu t_0.

VERHALTEN VON HIGHLOFT MATERIALEN UNTER DRUCKBEANSPRUCHUNG UND DAS RÜCKSTELLVERMÖGEN (KELVIN-MAXWELL-MODELL)

Das Verhalten von hochflorigen (highloft) Materialien unter Druckbelastung und deren elastische Erholung kann durch ein Kelvin-Maxwell Modell beschrieben werden. Es ist dies die Reihenschaltung eines Kelvin-Modells (Parallelschaltung von Newtonschen viskosen Flüssigkeiten und elastischen Materialien) und eines Maxwell-Modells (serielle Kombination von Newtonschen viskosen Flüssigkeiten und elastischen Materialien). Diese Kombination ist in der Lage, die plastische Verformung und das Relaxationsverhalten abzubilden.

Es wird ein Algorithmus vorgestellt, mit dem die relevanten Parameter für das beschriebene rheologische Modell aus experimentellen Daten bestimmt werden können, wenn die Belastungsphase der Zeit t_0 unter konstanter Spannung erfolgt.

ZACHOWANIE WYSOKO OBJĘTOŚCIOWYCH MATERIAŁÓW PRZY KOMPRESJI I ELASTYCZNYM ODKSZTAŁCENIU
(MODEL KELVINA-MAXWELLA)

Zachowanie przy kompresji i elastycznym odkształceniu wysoko objętościowych materiałów można opisać przy wykorzystaniu reologicznego modelu Kelvina-Maxwella. Jest to szeregowe połączenie modelu Kelvina (równoległe połączenie lepkiej cieczy Newtona i materiału elastycznego Hooka) oraz modelu Maxwella (szeregowe połączenie lepkiej cieczy Newtona i materiału elastycznego Hooka). To połączenie jest w stanie objąć zniekształcenie plastyczne oraz odkształcenie (zachowanie "relaksu").

Prezentujemy algorytm w celu określenia parametrów wejściowych dla tego modelu reologicznego przy pomocy danych doświadczalnych, gdy faza obciążenia odbywa się przy stałym napięciu przez czas t_0.