SURFACE ROUGHNESS AND FRACTAL DIMENSION

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ABSTRACT

Surface roughness is one of the main characteristics of fabric responsible for hand feeling. By using of KES the surface height variation (SHV) trace is obtained. For characterization of roughness the mean absolute deviation MAD (denoted by Kawabata. as SMD) is usually used. The main aim of this work is quantitative description of complexity of roughness based on the SHV. The procedure of surface complexity parameters evaluation from SHV traces is described. The core is inspection of power spectral density and variogram (or autocorrelation function) behavior and classification of SHV signal to some groups. For individual groups fractal dimension and surface roughness characteristics are computed by suitable methods. Proposed procedure was checked on the simulated SHV profiles and on the practical examples.

1. INTRODUCTION

Kawabata [1] revealed that surface roughness is one of the main characteristics of fabric responsible for hand feeling. By using of KES the surface height variation (SHV) trace can be obtained. The main part of Kawabata's measuring device is contactor in the form of wire (diameter 0.5 mm). This contactor is moved by constant rate 0,1 cm/sec and SHV is registered on paper sheet. The sample length L=2 cm is used. The SHV corresponds to the surface profile in selected direction (usually in the weft and warp directions are used for SHV creation). Characterization of roughness is based on the mean absolute deviation MAD (the classical descriptive statistical approach). This statistical characteristic is useful for random SHV traces where elements of SHV trace are statistically independent each other. The SHV profile of a lot of fabrics has been identified as irregular and more structured.

The descriptive statistical approach based on the assumptions of independence and normality leads to biased estimators if the SHV have short or long-range correlations [6]. There is therefore necessary to distinguish between standard **white Gauss noise** and more complex models. For description of short range correlations the models based on the **autoregressive moving average** are useful [2]. The long-range correlations are characterized by the **fractal models** [3,5]. The **deterministic chaos** type models are useful for revealing chaotic dynamic in deterministic processes where variation appears to be random but in fact there are predictable [4].

For the selection among above mentioned models the power spectral density (PSD) curve evaluated from experimental SHV can be applied [11].

Especially the fractal models are widely used for rough surface description [7]. For these models the dependence of log (PSD) on the log (frequency) should be linear. Slope of this plot is proportional to **fractal dimension** and intercept to the so-called **topothesy**. For white noise has dependence of log(PSD) on the log(frequency) nearly horizontal plateau for all frequencies (the ordinates of PSD are independent and exponentially distributed with common variance [13]). More complicated rough surfaces as result of grinding can be modeled by the Markov type processes [8]. For these models the dependence of log(PSD) on the log(frequency) has plateau at small

frequencies then bent down and are nearly linear at high frequencies. The fractal type models were criticized by Whitehouse, who concluded that the benefits are more virtual than real [8]. On the other hand the deeper analysis of rough surface should use more complex model than classical descriptive statistics.

The main aim of this work is quantitative description of roughness complexity based on the SHV. The procedure of roughness parameters evaluation from SHV traces is described. The core is inspection of power spectral density and variogram (or autocorrelation function) behavior and classification of SHV signal to some groups. For individual groups the fractal dimension and surface roughness characteristics are computed by using of suitable methods. Proposed procedure is demonstrated on the simulated SHV profiles and on the practical example.

2. FRACTALS

Most of man made objects are geometrically simple and can be classified as composition of regular geometric shapes as lines, curves, planes, circles, spheres etc. Some objects are not be approximated precisely by the regular geometric shapes. One category of these objects is called **fractals**. Benoit Mandelbrot has coined term fractal in the seventies [5]. (From Latin **fractus**, meaning irregular or fragmented). Fractals have two interesting characteristics. First of all, fractals are **self-similar** on multiple scales, in that a small portion of a fractal will often look similar to whole object. Second, fractals have a **fractional dimension**, as opposite to integer dimension of regular geometrical objects. Because fractals are self similar they are constructed by recursion. For **geometrical fractals** is the recursion explicitly visible. Typical example is so called Koch curve shown on fig 1.

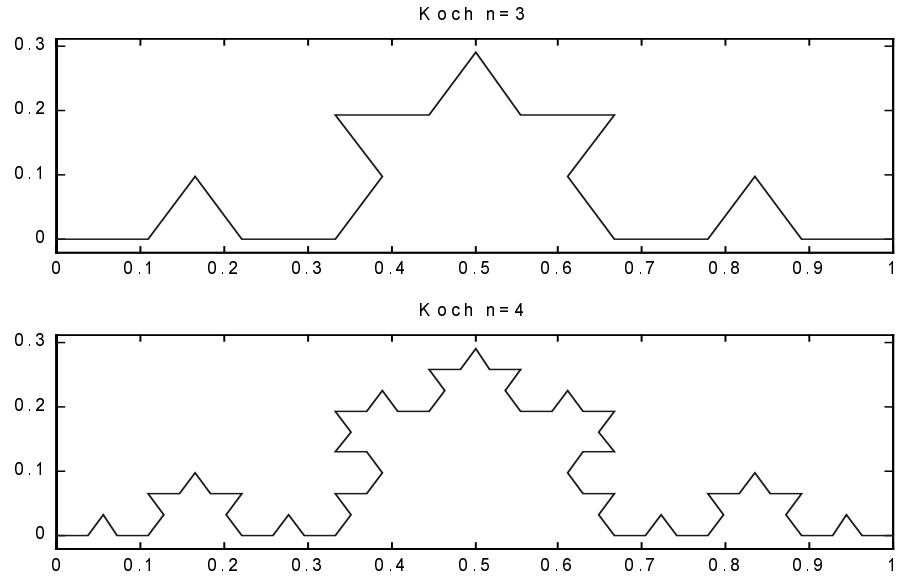

Fig.1 Koch curve (in each step of construction the middle portion of each segment is removed and replaced by two new line segments. First step is line, having length 1).

The interesting facet of the Koch curve is its length LK. In the n-th step is length of segment equal to $LS=1/3^n$ and the curve consists of 4^n segments. Therefore it is $LK=(4/3)^n$.

For **stochastic fractals** or **random fractals** the recursion is more little subtle and may be an artifact of an underlying fractal building process that occurs on multiple spatial scales. The typical generating function is Weistrass Mandelbrot equation, which satisfies to the self-affinity requirement (replaces of self similarity for case of functions). The height *R (d)* of surface in the point *d* or SHV trace is equal to

$$R(d) = G^{D-1} \sum_{i=n_1}^{\infty} \frac{\cos(2\pi g^i d)}{g^{i*(2-D)}} \qquad (1)$$

where $1<D<2$ is fractal dimension, G is characteristic length scale of surface and g^i determines the frequency spectrum of surface roughness. The suitable value of this parameter is $g=1.5$. The simulated stochastic fractal generated by eqn. (1) for D = 0.8 is shown on the fig 2.

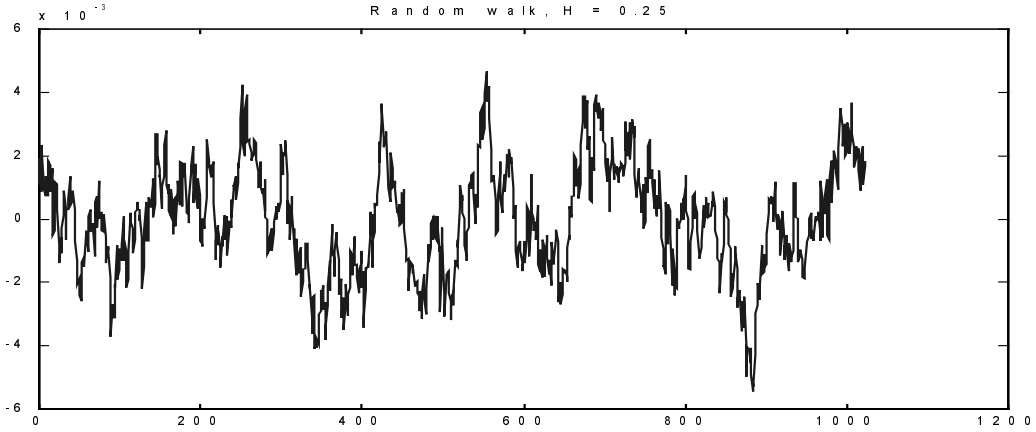

Fig 2. Stochastic fractal generated by eqn. (1) for D = 0.8

This curve corresponds to the nonstationary random process and describes fractional Brownian motion fB. The lowest frequency is then related to the sample length L according to relation $g^{n_1} = 1/L$.

The evaluation of D and G from random fractals is based on the power spectral density $P(\omega)$ function, which has for eqn. (1) the power law form

$$P(\omega) = C * \omega^{-B} \quad \text{where } C = \frac{G^{2(D-1)}}{2\ln(g)} \text{ and } B = 5 - 2D \quad \text{for } 1/L \le \omega \le \infty \qquad (2)$$

The power law form is typical feature of fractals. It is typical for wider class of so called processes having long memory characteristics [16]. Long memory processes can be defined in terms of power scaling in the length domain i.e. the power type function is valid for autocorrelation function and variogram as well.

In some cases are fractals stationary random processes like fractional Gaussian noise fG. The stationary fG process can be simply obtained as successive differences of fB process. Stationary fG process corresponding to fB from fig2. is shown on the fig.3.

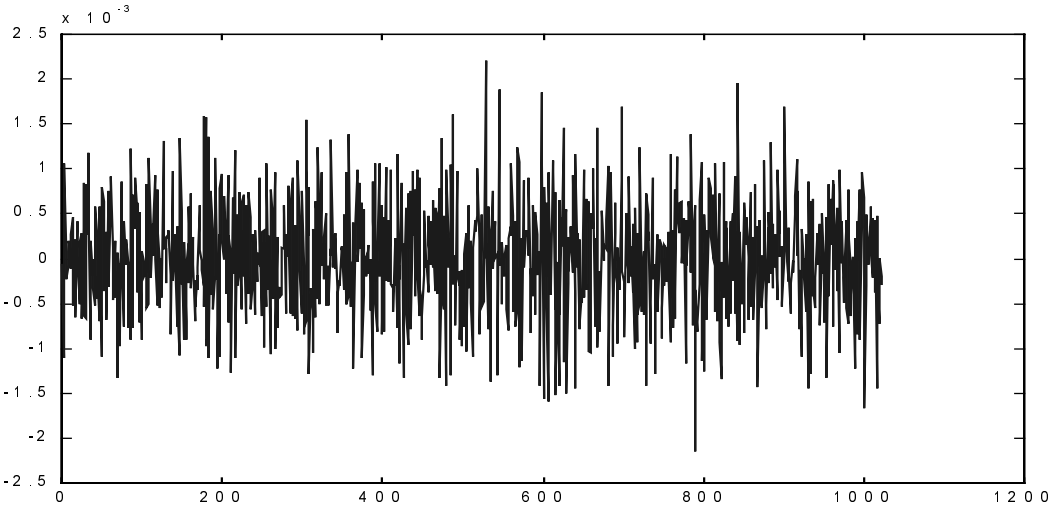

Fig 3. Fractional Gaussian noise created from fractional Brownian motion - fig.2.

Fractals in the form of fG are fully characterized by the mean and autocorrelation function C(x) in the form

$$C(x) = \frac{1}{2}[(x+1)^{2H} - 2x^{2H} + (x-1)^{2H}]$$

where H is Hurst coefficient. For .5<H<1 is fG process long memory type. Hurst coefficient is directly connected with so called fractal dimension *D*.

The main characteristic of both fractals types is fractal dimension. The problem with dimension is well known in measurement e.g. coastal length. English meteorologist L. Richardson found hat the apparent length of a coastline seemed to increase whenever the length of the measuring stick was reduced. When the log of measuring stick was plotted against the logarithm of the total length of coastline, the points tended to lie on the straight line. The slope of this line is measure of roughness or meandering of coastline. Mandelbrot has introduced the same approach for characterization of fractals by fractal dimension. Let we have measuring device for evaluation of some characteristics (for the one dimensional measure it is *length*, for two dimension it is *area* and in three dimensional case it is *volume*) having measure Q. The characteristic of whole object is defined as $N(Q)*Q$, where $N(Q)$ is number of Q units for measurement of the whole object (e.g. number of stick placements to measure of coastal length). Notice that whenever we measure an object, we always have the relation $N(Q)=(1/Q)^D$, where D is dimension of object. For fractals the D is not integer but can be estimated as limit

$$D = -\lim_{Q \to 0} \left(\frac{\log(N(Q))}{\log(1/Q)} \right) \qquad (3)$$

For the Koch curve is in the n th stage $Q= LS=1/3^n$ and $N(Q)=LK/LS=4^n$. Then $D=log(4)/log(3)=1.2619$. The fractal dimension of Koch curve is therefore slightly complex in comparison with line. For random fractals is simpler to use power spectral density or related functions. Some techniques for fractal dimension computations are summarized e.g., in [10]. The methods for computation oh Hurst coefficient is described in [17].

3. SURFACE ROUGHNESS DESCRIPTION

There are two reasons for measuring surface roughness. First, is to control manufacture and is to help to ensure that the products perform well [2]. In the textile branch the former is the case of special finishing but the later is connected with comfort appearance and hand.

From a general point of view, the rough surface display process which have two basic geometrical features:

(1) Random aspect: the rough surface can vary considerably in space in a random manner, and subsequently there is no spatial function being able to describe the geometrical form,
(2) Structural aspect: the variances of roughness are not completely independent with respect to their spatial positions, but their correlation depends on the distance. Especially surface of weaves is characterized by nearly repeating patterns and therefore some periodicities are often identified.

From the SHV trace is possible to evaluate a lot of roughness parameters. Classical roughness parameters are based on the set of points $R(d_i)$ $i=1..M$ defined in the sample length interval L. The measurement points d_j are obviously selected as equidistant and then $R(d_j)$ can be replaced by the variable R_j . For identification of positions in length scale is sufficient to know sampling distance $ds = d_j - d_{j-1} = L/M$ for $j>1$.The standard roughness parameters used frequently in practice are [19]:

(i) Mean Absolute Deviation *MAD*. This parameter is equal to the mean absolute difference of surface heights from average value (R_a). For a surface profile this is given by

$$MAD = \frac{1}{M}\sum_j |R_i - R_a| \tag{4}$$

This parameter is often useful for quality control. However, it does not distinguish between profiles of different shapes. Its properties are known for the case when R_j*'s* are independent identically distributed (.i.i.d.) random variables

(ii) Standard Deviation (Root Mean Square) Value *SD*. This is given by

$$SD = \sqrt{\frac{1}{M}\sum_j (R_i - R_a)^2} \tag{5}$$

Its properties are known for the case when R_j*'s* are independent identically distributed (.i.i.d.) random variables. One advantage of SD over MAD is that for normally distributed data can be simple to derive confidence interval and to realize statistical tests. SD is always higher than MAD and for normal data is $SD = 1.25*MAD$. It does not distinguish between profiles of different shapes as well. The parameter *SD* is less suitable than MAD for monitoring certain surfaces having large deviations (corresponding distribution has heavy tail).

(iii) Mean Height of Peaks *MP*. This is calculated as the average of the profile deviations above the reference value R (often is $R = R_a$). It is given as mean value of peaks P_i, $i = Np$ where

$P_i = R_i - R$ for $R_i - R > 0$ and $P_i = 0$ elsewhere

(iv) Mean Height of Valleys *MV*. This is calculated as the average of the profile deviations below the reference value R (often is $R = R_a$.). It is given as mean value of valleys V_i, $i = Nv$ where

$$V_i = R - R_i \quad \text{for} \quad R_i - R < 0 \quad \text{and} \quad V_i = 0 \quad \text{elsewhere}$$

The parameters *MP* and *MV* give information on the profile complexity. Exceptional peaks or valleys are not considered but are useful in tribological applications.

(v) The Standard Deviation of Profile Slope *PS*. This is given by

$$PS = \sqrt{\frac{1}{M}\sum_j \left(\frac{dR(x)}{dx}\right)_j^2} \tag{6}$$

(vi) The Standard Deviation of Profile Curvature *CP*. This quantity called often as waviness is defined by the similar way

$$PC = \sqrt{\frac{1}{M}\sum_j \left(\frac{d^2R(x)}{dx^2}\right)_j^2} \tag{7}$$

The slope and curvature are characteristics of a profile shape. The *PS* parameter is useful in tribological applications. The lower the slope the smaller will be the friction and wear. Also, the reflectance property of a surface increases in the case of small *PS* or *PD*.

(vii) Mean Slope of the Profile *MS*. This is given by

$$MS = \frac{1}{M}\sum_j \left|\frac{dR(x)}{dx}\right|_j \tag{8}$$

Mean slope is an important parameter in several applications such as in the estimation of sliding friction and in the study of the reflectance of light from surfaces.

(viii) Ten Point Average *TP*. This characteristic is defined as the average difference between the five highest peaks and five deepest valleys within a surface profile. The parameter *TP* is sensitive to the presence of high peaks or deep scratches in the surface and is preferred for quality control purposes.

These parameters are useful in the case of functional surfaces or for characterizing surface bearing and fluid retention and other relevant properties. For the characterization of hand will be probably best to use waviness *PC*. The characteristics of slope and curvature can be computed for the case of fractal surfaces from power spectral density, autocorrelation function or variogram.

A set of parameters for profile and surface characterization are collected in [8]. These parameters are divided to the following groups:

- Statistical characteristics of height distribution (variance, skewness, kurtosis)
- Spatial characteristics as autocorrelation or variogram (denoted in engineering as structural function)

- Functional characteristics (connected with fluid retention or flow properties)

There exists a vast number of empirical profile or surface roughness characteristics suitable often in very special situations. Some of them are closely connected with characteristics computed from fractal models (fractal dimension and topothesy). Greenwood [9] proposed a general theory for description of surface roughness based on the distribution of heights.

General surface topography is usually broken down to the three components according to wavelength (or frequency) [8]. The long wavelength (low frequency) range variation is denoted as ***form.*** This form component is removed by using of polynomial models or models based on the form shape. The low wavelength (high frequency) range variation is denoted as ***roughness*** and medium wavelength range variation separates ***waviness.*** The most common way to separate roughness and waviness is spectral analysis. This analysis is based on the Fourier transformation from space domain d to the frequency domain $\omega = 2\pi / d$.

4. SPECTRAL ANALYSIS

The primary tool for evaluation of periodicities is expressing of signal *R(d)* by the Fourier series of sine and cosine wave

$$R(d) = \frac{a_0}{2} + \sum_k (a_k \cos(2\pi * k * d) + b_k \sin(2\pi * k * d)) \qquad (9)$$

Quantity *d* is often time or distance from origin and *k=1,2,3,4….* The first two terms have period 1, the second two terms have period 1/2, the third two terms have period 1/3 etc. One consequence of this is that the different pairs of terms are orthogonal (integral of their product is zero). This fact facilitates fitting of Fourier series to experimental data. The term $a_o/2$ can be made zero by centralization (i.e. subtracting of mean value). By using of Euler formula *exp(ia)=cos(a)+isin(a),* where *i* is imaginary unit the Fourier series may be written in the compact form

$$R(d) = \sum_k c_k \, exp(-2\pi * k * d)$$

The complex coefficients c_k have real and imaginary part a_k and ib_k In Fourier series only the terms up to *k=M/2* contain any useful information. After this bound are real coefficients repeated symmetrically and imaginary coefficients repeated antisymmetrically. The Fourier Transform is conversion of data from series according to *d* to the series of frequencies $\omega = 2\pi * k / (M * T)$, for k=1, 2, 3…

$$RF(\omega) = \sum R(d) * exp(-i * \omega * d) \qquad (10)$$

Function *RF* is symmetric about frequency $\omega = \pi / T$. For discrete data the fast Fourier Transform (FFT) leads to transformed complex vector *DRF.* Vector *DRF* may be used for creation of power spectral density. *P( )*

$$P(\omega) = DRF * conj(DRF) / T^2 = abs(DRF)^2 / T^2 \qquad (11)$$

where conj(.) denotes conjugate vector. The *P( )* is estimator of spectral density function and contains values corresponding to contribution of each frequency to the total variance of *R(d)*. Frequency of global maxim on *P( )* is corresponding to the length of repeated pattern and height corresponds to the nonuniformity of this pattern. Spectral density function is therefore generally useful for evaluation of hidden periodicities. For continuous case has the Fourier spectrum form

$$P(\omega) = \frac{1}{L} \int_{-\infty}^{\infty} R(x) * \exp(-2\pi * i * \omega * x) dx \qquad (12)$$

where *R(x)* is height amplitude function estimated as SVH trace *R(d), L* is sample length and *i* is imaginary unit.

The estimation of the spectral density function *P(ω)* is relatively straightforward in theory but in practice situation is more difficult since data are only available in discrete samples of limited extent. For finite sample lengths if is necessary to use windowing (avoiding leakage) de-trending (avoiding non stationarity of mean) and filtration of parasite frequencies [14]. Several methods of estimating the spectral density function are available. More precise estimates can be obtained by using of sophisticated procedures as averaged periodogram of overlapped windowed signals (Welch method) or multiple signal classification (MUSIC). The maximum entropy spectral analysis (MEM) provides smoother and higher resolution spectra for red-noise processes, which therefore would appear to be more suitable for good estimation. The method of MEM spectral estimation use of the Fourier transform between *P(ω)* and the autocorrelation function [16]. It is necessary to specify before computation the order of AR model. *P(ω)* is then selected to maximize entropy such that the inverse Fourier transform of *P(ω)* yields the autocorrelation function. These spectral estimators are available in Signal Processing toolbox of MATLAB system [18]. For the white noise (independent standard normal random numbers) are the estimators of spectral density shown on the fig 4.

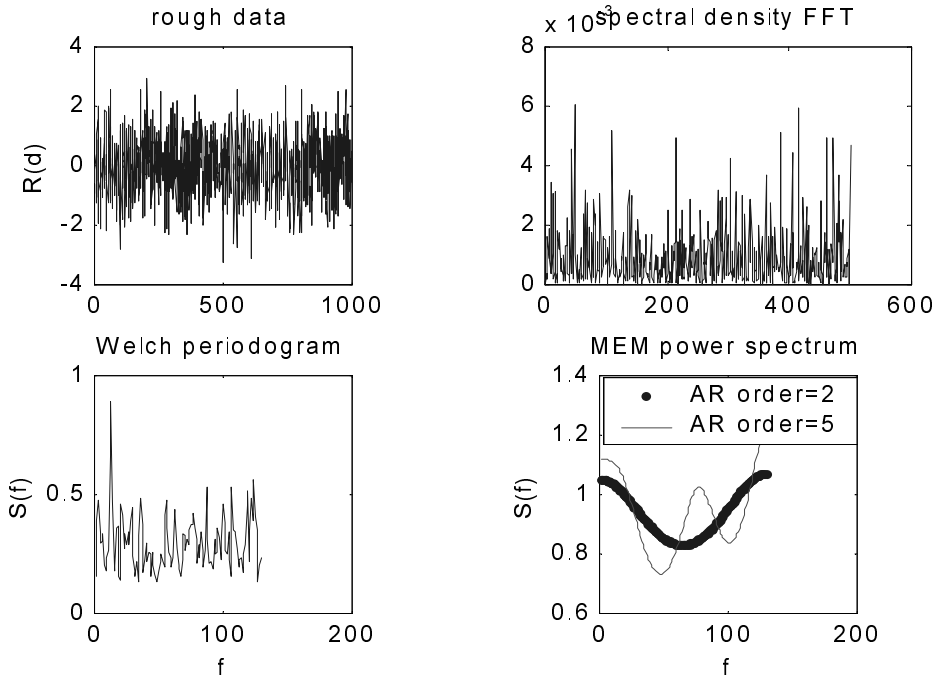

Fig 4. Raw data (white noise) and estimators of power spectral density

It is clear that the rough FFT based estimator shows the random fluctuations. Both more sophisticated estimators show on the first sight the false one or more periodicities. By using of statistical testing are these local peaks found to be insignificant.
For simulation of these estimators behavior for periodic structure with added random noise *N(0,1)* the function

$$R(d) = 3 * \sin(2*\pi*10*t) + 4 * \sin(2*\pi*4*t) + N(0,1)$$

was generated.
The estimators of spectral density and raw data are given on the fig 5.

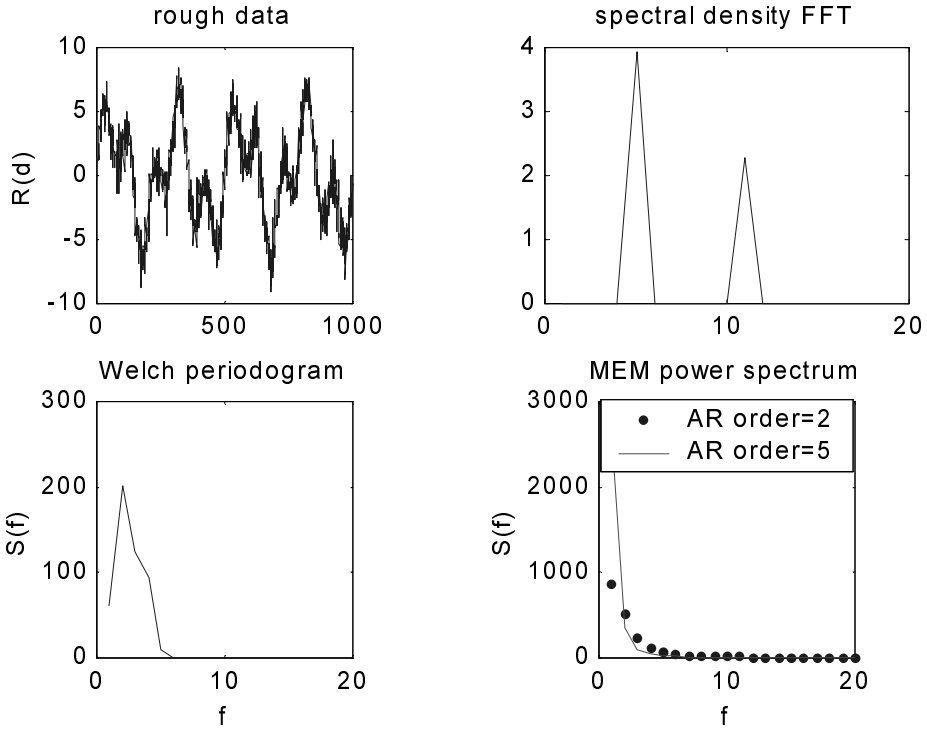

Fig 5. Spectral densities for periodic function with added whiter noise

For very high level of noise N(0,400) are these estimators non effective see fig 6.

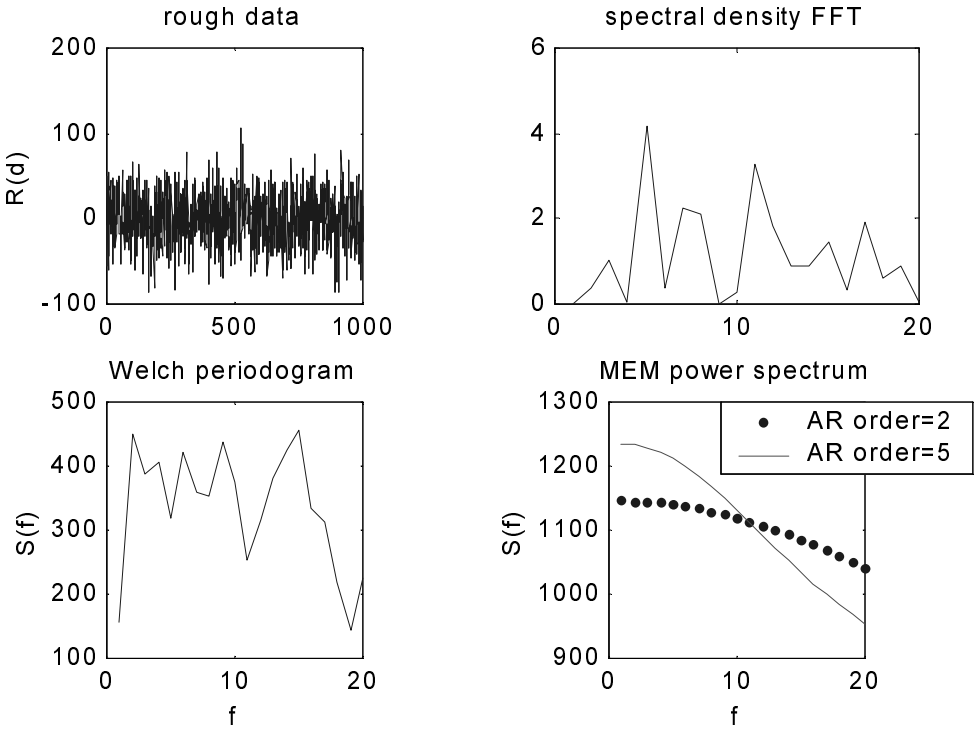

Fig 6. Spectral estimators for data from fig 2 with high level of noise *N (0,400)*

The spectral estimators for finite data length corrupted by random errors could be inaccurate. The more sophisticated procedures are very sensitive to the tuning parameters. For estimation of fractal dimension is therefore the best way to use simple FFT based method with proper data pretreatment (detrending, windowing) [17].

4. STATISTICAL ANALYSIS

A basic statistical feature of *R(d)* is autocorrelation between distances. Autocorrelation depends on the lag *h* (i.e. selected distances between places of thickness evaluation). The main characteristics of autocorrelation is covariance function *C(h)*

$$C(h) = \mathrm{cov}(R(d), R(d+h)) = E((R(d) - E(R(d)) * (R(d+h) - E(R(d))))$$

and autocorrelation function ACF(h) defined as normalized version of *C(h)*

$$ACF(h) = \frac{C(h)}{C(0)} \tag{13}$$

ACF is one of main characteristics for detection of short and long-range dependencies in dynamic (time) series. It could be used for preliminary inspection of data. The computation of sample autocorrelation directly from definition is for large data tedious. The technique of *ACF* creation based on the FFT is contained in Signal Processing toolbox of MATLAB (procedure xcorr.m) [18]. The spectral density is the Fourier transform of autocorrelation function *C(h)*

$$P(\omega) = \frac{1}{2\pi} \int_0^{\infty} C(t) * exp(-i * \omega * t)\, dt \tag{14}$$

. The ACF is inverse Fourier transform of spectral density

$$C(h) = \int_0^{\infty} P(\omega) * exp(i * \omega * h) d\omega \tag{15}$$

These relations show that characteristics in the space and frequency domain are interchangeable.

In spatial statistics is more frequent variogram, (called often as structure function) which is defined as one half variance of differences *(R(d) - R(d+h))*

$$\Gamma(h) = 0.5 * D[R(d) - R(d+h)] \tag{16}$$

or

$$\Gamma(h) = 0.5 * [E(R(d) - R(d+h))^2] \tag{17}$$

For stationary random process is mean value independent on lag *h* i.e. *E(R(h)) = m* and then

$$\Gamma(h) = 0.5 * E(R(d) - R(d+h))^2 \tag{18}$$

For random processes having stationarity of second order is valid

$$C(h) = E[R(d) * R(d+h)] - m^2 \tag{19}$$

Variance is then equal to

$$D(R(d)) = C(h=0) = C(0) \tag{20}$$

and variogram is directly related to covariance

$$\Gamma(h) = C(0) - C(h) \tag{21}$$

The variogram is relatively simpler to calculate and assumes a weaker model of statistical stationarity, than the power spectrum. Several estimators have been suggested for the variogram. The traditional estimator is

$$G(h) = \frac{1}{2M(h)} \sum_{j=1}^{M(h)} (R(d_j) - R(d_{j+h})^2 \tag{22}$$

where *M(h)* is the number of pairs of observations separated by lag *h*. Problems of bias in this estimate when the stationarity hypothesis becomes locally invalid have led to the proposal of more robust estimators. One such estimator has been created by Cressie and Hawkins [14]. Another estimator has been suggested by Isaaks and Srivastava [15].
The sample covariance function (ACF is normalized by variance) is defined as

$$Ck = \frac{1}{M-k} \sum_{i=1}^{M-k} (R(d_i) - Ra) * (R(d_{i+k}) - Ra) \tag{23}$$

where *Ra* is sample mean. Corresponding sample spectral density is

$$Pk = \frac{1}{M}\sum_{z=0}^{M-1} Cz \exp(-\frac{2*\pi*i*z*k}{M}) \tag{24}$$

and for variogram is valid

$$Gk = \frac{2}{M}\sum_{z=0}^{M-1} Pz*[1-\exp(\frac{2*\pi*i*z*k}{M})] \tag{25}$$

For the white noise (independent standard normal random numbers) are the estimators of ACF, variogram and covariance function on the fig 7.

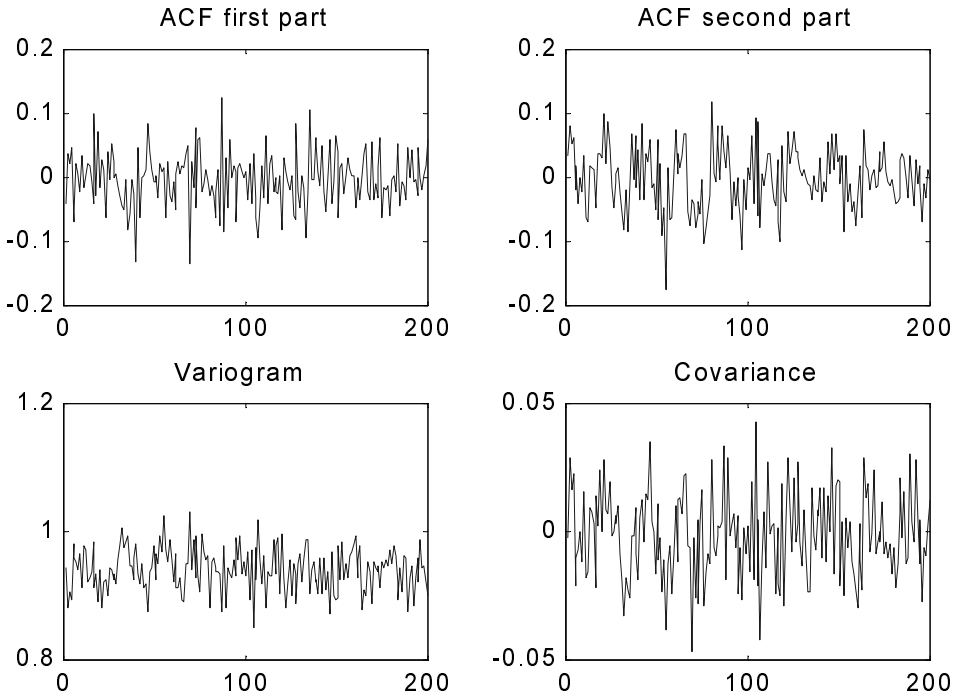

Fig. 7 Estimators of autocorrelation function, variogram and covariance function for white noise

The same estimators are given for the case of periodic function with added white noise. (see fig.5.) on the fig 8.

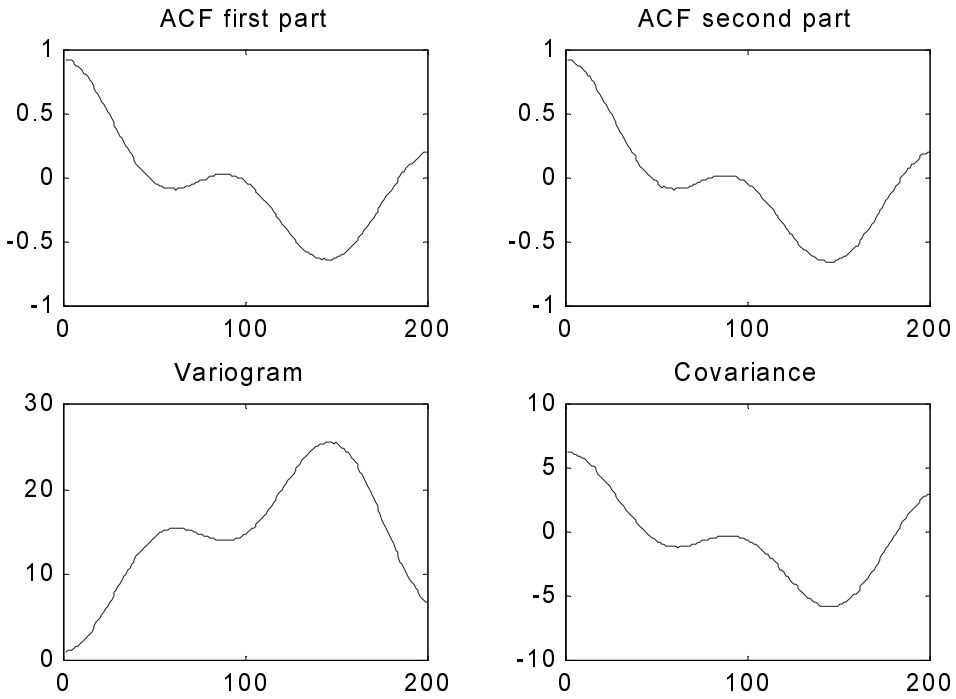

Fig 8. The ACF, variogram and covariance function for periodic function with added white noise

For very high level of noise N(0, 400) are periodicities hidden and not detected by these estimators. This level of noise is in practice not the result of measurement errors but result of random fluctuations of surface profile

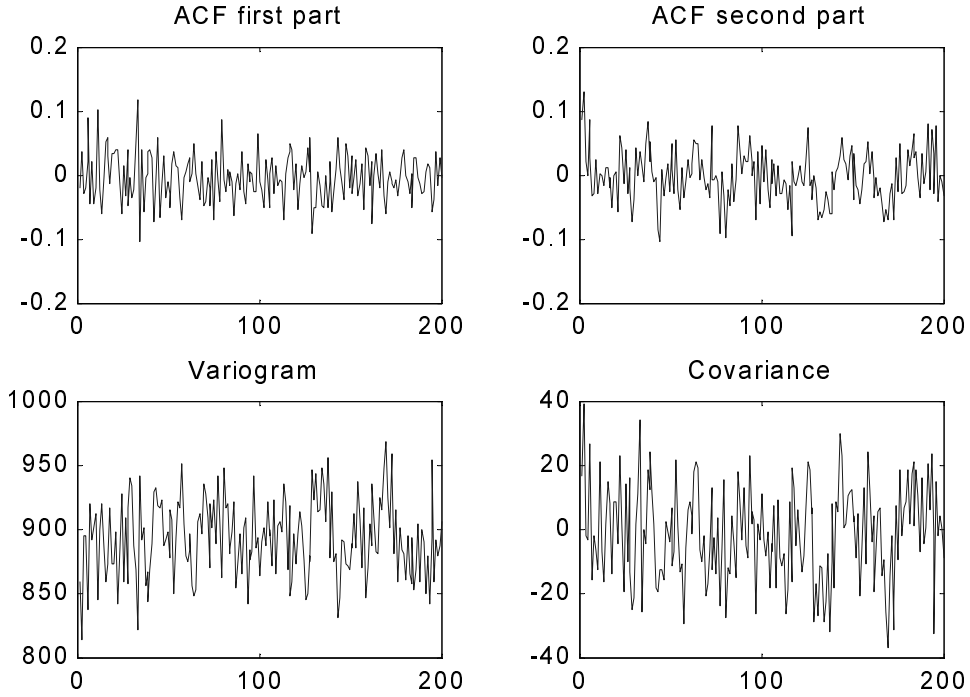

Fig. 9. ACF, variogram and covariance for periodic data embedded in high level noise N(0,400)

It can be summarized that simple statistical characteristics are able to identify the periodicities in data but the reconstruction of "clean" dependence is more complicated. The variogram is often sufficient for characterization of surface profiles.

6. NATURE OF SURFACE PROFILES

There exists a lot of surface profiles models. For illustration of individual characteristics describing their complexity the main types of these models were generated.

I. Fractal surface fB

The simulated stochastic fractal generated by eqn. (1) for D = 0.25 is shown on the fig 10 and for D=0.75 on the fig. 11. On the same figures are power spectral densities in log-log scale, variograms in log-log scale and amplitude histogram. The practically perfect linearity of log vartiograms and scattered linearity of log power spectral densities are typical for self-affine curves. Histograms of amplitudes R_i indicate multimodality or skewed distribution,

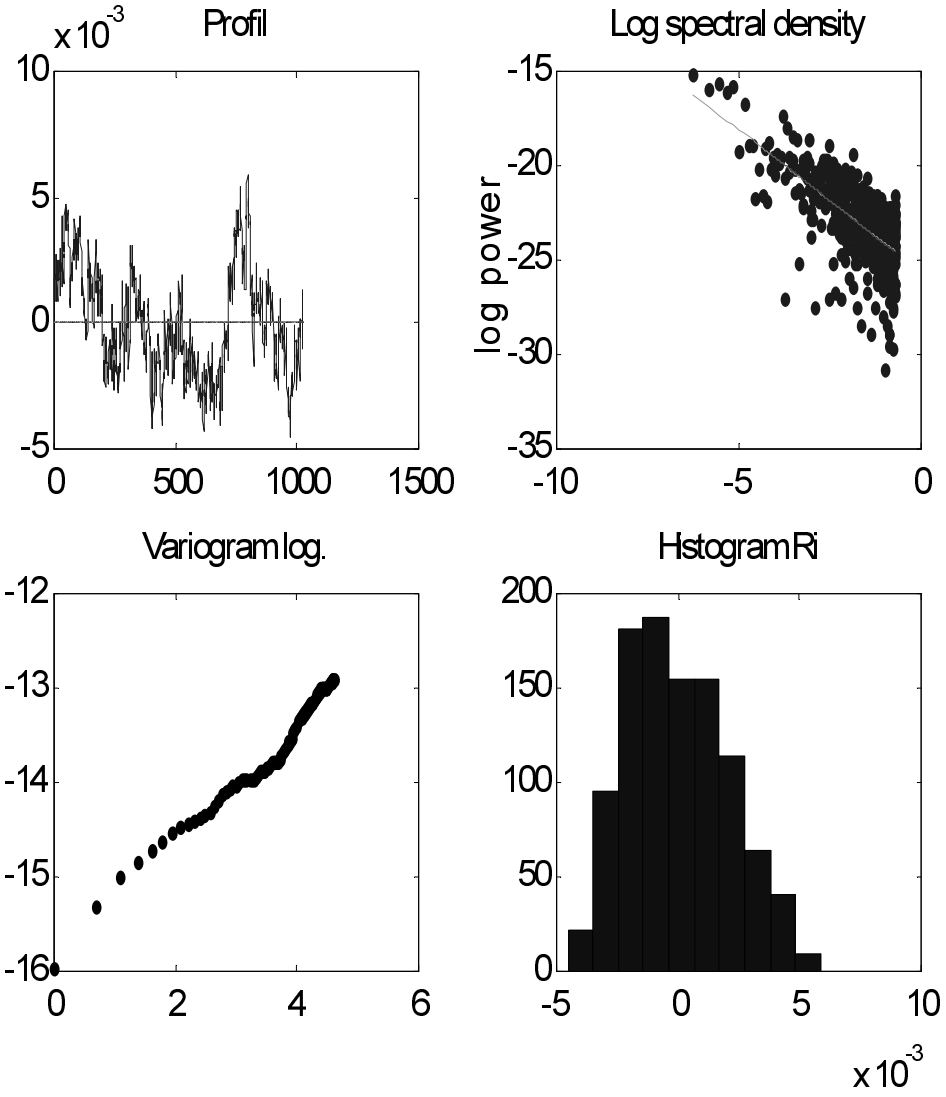

Fig 10. Stochastic fractal generated by eqn. (1) for D = 0.25

These curves correspond to the nonstationary random process and describe fractional Brownian motion fB.

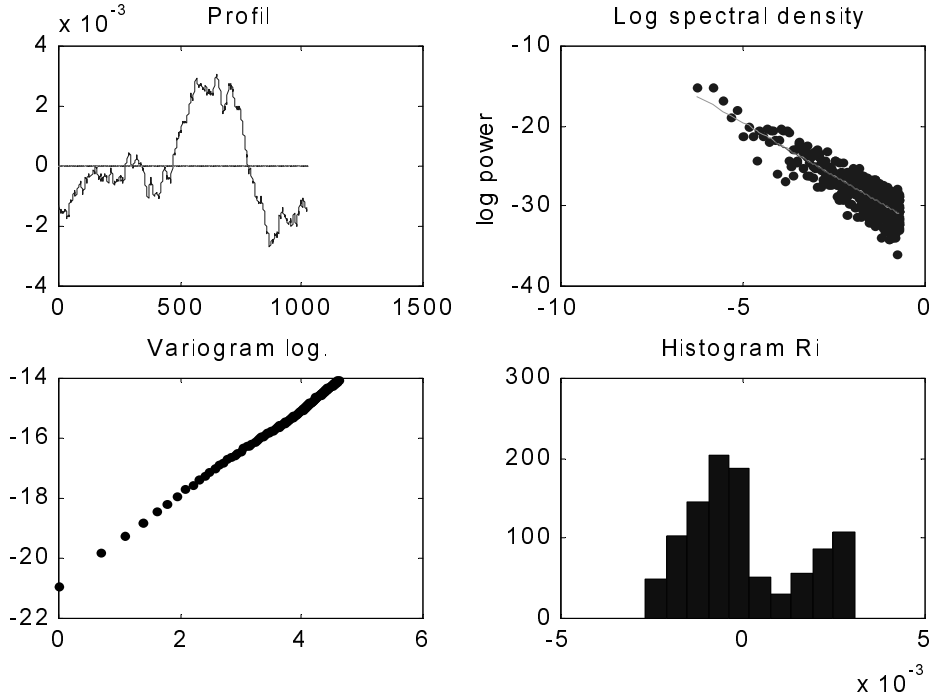

Fig 11. Stochastic fractal generated by eqn. (1) for D = 0.75

II.Fractional Gaussian noise fG.

Stationary fG process corresponding to fB from fig 11. is shown on the fig.12.

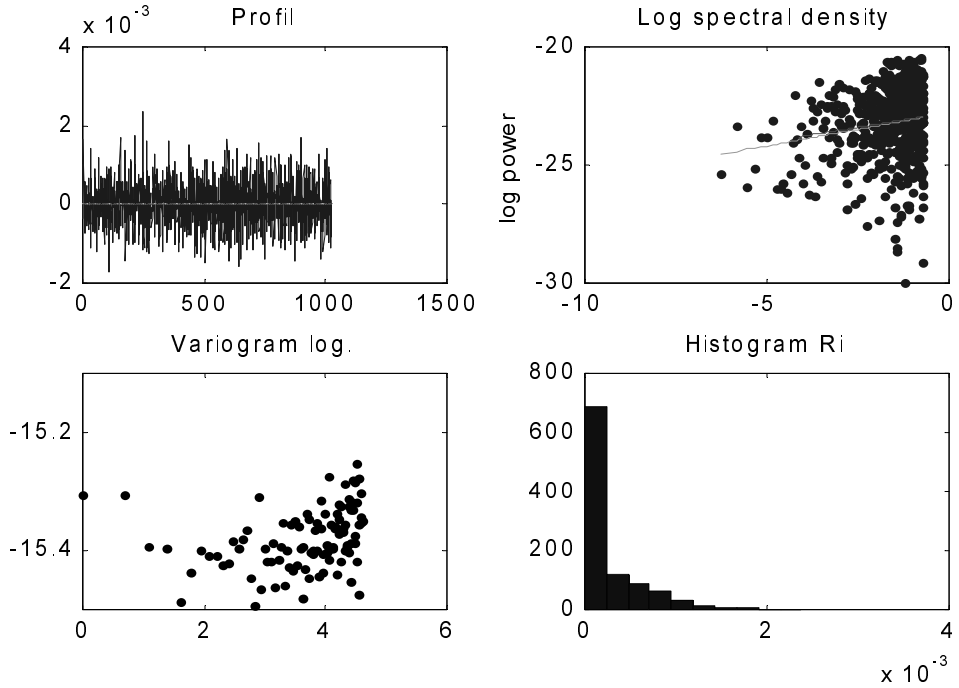

Fig 12. Fractional noise corresponding to first difference of the fractional Brownian motion from fig 11.

III. Markov model

A lot of engineering surfaces obey fractal like behavior for high frequencies. For small frequencies the log-log power spectral density exhibits nearly constant portion. The Markov type models can express this behavior. Simplest one has form

$$R_{i+1} = r * R_i + (1-r) * \sigma_m * N(0,1) \tag{26}$$

where r is autocorrelation coefficient. For this process is variance C(0) equal to

$$C(0) = \frac{1+r}{1-r}\sigma_m^2$$

The ACF has very simple form

$$ACF(h) = \sigma_m^2 r^h \tag{27}$$

and for power spectral density is valid

$$P(\omega) = \frac{b\sigma_m^2}{\pi(1+b^2\omega_m^2)} \tag{28}$$

where b is constant connected with *r*.

Markov type profile for r=0.8 is shown on the fig. 13

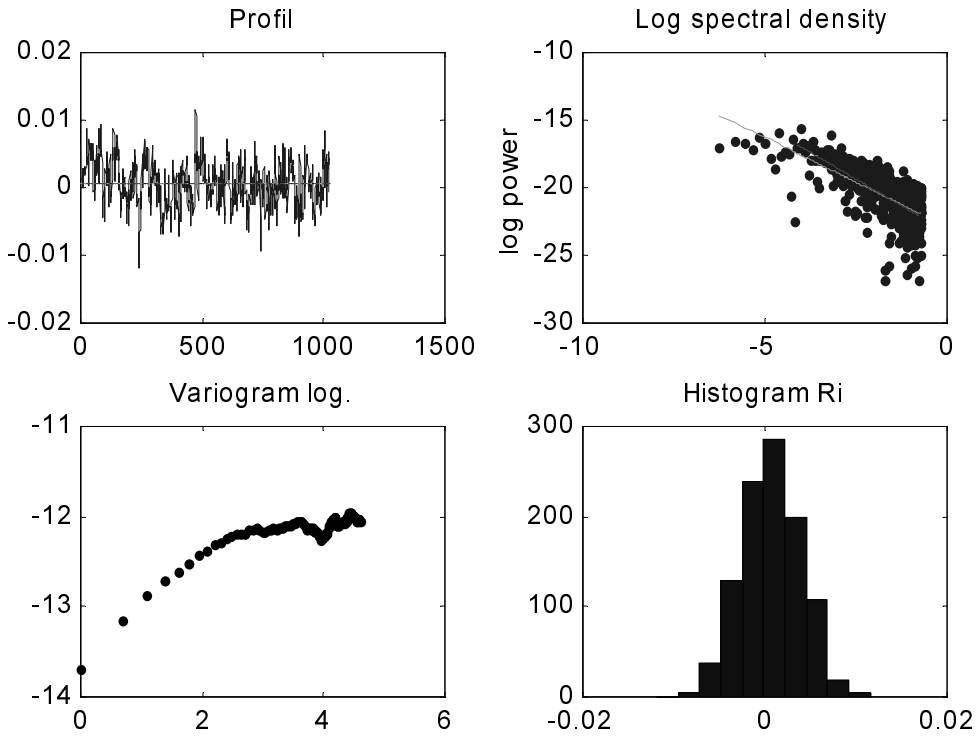

Fig. 13 Markov type surface profile generated by the eqn (26)

It is evident that the degree of linearity for small lengths (corresponds to the high frequencies) is still very high for log-log variogram.

Based on these and other extended simulations we can conclude that:

- Power spectral density of Gauss noise has a lot of local extremes. Variogram has random fluctuations in small scale
- Power spectral density of composite sine waves embedded in high-level noise exhibit no right pattern. The same is valid for variogram.
- Power spectral density of fractal type (fBm) surface profiles exhibits scattered linear trend according to the theory. The variogram exhibits more strict linearity with relative small scatter. Variogram is here typical power function of *h*.
- Fractional Gaussian surface profiles are typical by random fluctuations of variogram.
- Markov like surface profiles have linear portion on variogram at smaller lags *h*. For higher lags the plateau is visible.

The very simply calculated variogram can replace the spectral power density. For periodic surface profiles having small noise level enables power spectral density the identification of periodicities.

7. ESTIMATION OF THE FRACTAL DIMENSION

A convenient way of characterizing the smoothness of an isotropic surfaces is Hausdorf or fractal dimension. If the surface is very smooth is fractal dimension equal to $D_p = 2$. For extremely rough surfaces is fractal dimension approaching to limit value $D_p = 3$. General definition of fractal dimension is based on the capacity principle [12] In measurement of surface profile (thickness variation $R(h)$) the data are available through one dimensional line transect surface. Such data represents curve in plane. Two dimensional fractal dimension D *is then number between 1* (for smooth curve) and *2* (for rough curve). If a surface may be modeled by a stationary, isotropic Gaussian field then the relation

$$Dp = D + 1 \tag{29}$$

The expected variance of the increment of Brownian motion can be expressed using a value of the Hurst exponent H, where $H = 0.5$ [17]

$$E(R(d) - R(d+h))^2 \approx |h|^{2H} \tag{30}$$

For fractional Brownian motion *fB* is H in the interval (0,1). Where $H = 0$ this denotes a surface of extreme irregularity and $H = 1$ denotes a smooth surface. Exponents H and fractal dimension D are in fact related

$$D = D_T + 1 - H \tag{31}$$

where D_T is the topological dimension such that D is in interval (2,3) for a surface and (1, 2) for a cut across a surface. Note that fractional Brownian motion can be expressed in terms of a power law variogram

$$\Gamma(h) \approx c|h|^H \tag{32}$$

where c is a constant. Similarly, for $P(\omega)$ is valid

$$S(\omega) = c_1 * |\omega|^{-(1+2H)} \tag{33}$$

where exponent (1+2H) lies in the interval (1,3). Fractal dimension is conventionally obtained through estimating the parameter from a LSE linear regression of the log-log transformation of Equations (32) and (33). The same results can be obtained assuming that thickness variation $R(d)$ is stationary Gaussian process and covariance function $C(h)$ is sufficiently smooth[12, 13]. The behavior of this function near the origin can be described by power type model

$$C(0) - C(h) \approx c|h|^{\alpha} \tag{34}$$

These formulas may also be verified for another processes related to Gaussian one. In practice is behavior expressed by eqn. (32) valid near origin and by enq (33) in a neighborhood of infinity. In general it is D computed from this relation denoted as effective fractal dimension.

There are several problems with estimating fractal dimension in this fashion. First, elevation points, points on the variogram and the error term in the LSE regression are likely to be autocorrelated. Second, data points in log-log space are unequally spaced and, third, decisions concerning an acceptable cutoff for goodness of fit (R^2) of the linear function are of an arbitrary a priori nature. Since the aim of the line fitting exercise in estimating fractal dimension is the description of the relationship rather than prediction, the bias introduced by the first problem is not critical. A solution to the second is to re-sample the data using a geometric progression, but at a cost of a dramatic reduction in the number of points used in the line fitting exercise. An alternative to the third is to estimate the standard error SE around the slope of a regression line. Based on these equations the program TLOU in MATLAB for estimation of fractal dimension from variogram and power spectral density has been constructed. Based on the preliminary testing the results of computation from variogram were more stable and reliable. From power spectral density the d corresponding to the global maximum of $P(\omega)$ can be evaluated as well

8.ROUGHNESS AND FRACTAL DIMENSION

It can be shown that most of classical roughness characteristics as RMS roughness, the density of summits and the mean absolute surface slope are functions of fractal dimension and cut-off frequencies only. These roughness parameters are not intrinsic properties of a surface, and vary with the conditions of measurement. Self-similar fractal curves and surfaces are described completely by a single parameter, the fractal

dimension D, which is an intrinsic property of the surface and does not change with the scale of measurement.

Nayak {12] showed that the statistical geometry of an isotropic random Gaussian surface could be expressed in the terms of the moment of power spectral function

$$m_k = \int_{\omega_H}^{\omega_L} \omega^k P(\omega) d\omega \tag{35}$$

The roughness Rq (standard deviation) is simply $Rq = \sqrt{m_0}$ and the density of summits is

$$DS = \frac{m_4}{m_2 * 6 * \pi * \sqrt{3}} \tag{36}$$

After substituting from eqn. (33) to the eqn (35) and integration for the case when $\omega_L >> \omega_H$ the following approximations are valid

$$m_0 = \frac{-c_1}{\alpha+2}\omega_H^{\alpha+2} \quad m_2 = \frac{c_1}{\alpha+4}\omega_L^{\alpha+4} \quad m_4 = \frac{c_1}{\alpha+6}\omega_L^{\alpha+6} \tag{37}$$

In other words, none of the roughness parameters defined by the moment equations is an intrinsic property of the surface. The roughnesses *Rq* is independent of the sampling interval but depends on the high/pass cut-off, while summit densities are independent of the high/pass cut-off but depend on the sampling interval. Furthermore, eqn. (37) require that $3 \; -3 < \alpha + 1 < -1$, that is to say those amplitudes of surface wavelengths must fall off quite sharply, but not too sharply, as the wavelengths get smaller. Fortunately this condition is usually satisfied for real surfaces.

More simple equations can be derived by using of variogram *G(h)*. The standard deviation of profile slope is given by [17, 20]

$$PS = \sqrt{\frac{G(ds)}{ds^2}} = \sqrt{c * ds^{2-2D}}$$

and the standard deviation of profile curvature is

$$PD = \sqrt{\frac{4G(ds) - G(2ds)}{ds^4}} = \sqrt{(4 - 2^{4-2D}) * c * ds^{-2D}}$$

In these equations is D fractal dimension estimate obtained from variogram, c is intercept in log log variogram plot (see eqn (32)) and ds is sampling distance.

9. SHV TRACE EVALUATION

The processing of SHV traces from Kawabata device can be divided to the two phases. In the first phase the following tasks are solved:

- Digitalization of trace picture by image analysis system
- Removing parasite objects (grid, axes, base line etc)
- Creation of PSD and variogram

. First of all the low ω_L and high ω_H surface frequency bands have to be specified. These cut-off frequencies are related to the wavelength limits l_L and l_H i.e. $l_L = 2\pi / \omega_L$

and $l_H = 2\pi / \omega_H$. The low pass cut-off is related to Nyquist criterion i.e. $l_L = ds / 2$ and the high pass cut-off is dependent on the maximum interesting wavelength. For nonregular SHV $l_H = L$ has to be selected. The results of digitalization and parasite object removing is set of "clean" heights $R(d_i)$ of fabric in places $0<d_i<L$ (L is maximum investigated sample length and i = 1…M is number of places). The distance between places $ds = d_{i+1} - d_i$ is constant. For the case of Kawabata device is L=2 cm and dp=2/(M-1) cm.

The core of **pretreatment phase** is creation of PSD. Rough estimator is based on the FFT and it is vector having as elements the squared spectral amplitudes abs(Pk)^2. The FFT has its own drawbacks and limitations [14]. It has problems with leakage and resolution and non-stationary signals. The removing mean value and linear trends is then necessary. For reduction of leakage effect the windowing i.e. multiplying by suitable spectral window is useful. In program **TLOU** the simple parabolic window or hamming window are selected. Clean data and data after windowing for weave used as example are shown on the fig. 14

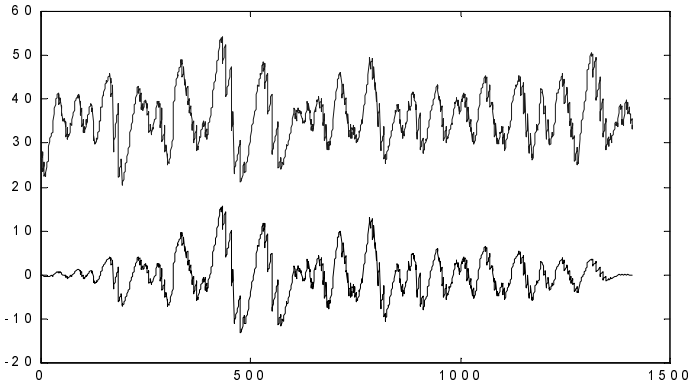

Fig 14. Clean data (upper curve) and data after removing of trend, centering and windowing.

The resultant spectral "leakage" distorts the actual signal but reduces effect of finite length. The dependence of log(P( )) on spatial frequency for pretreated signal from fig. 14. is shown on the fig. 15.

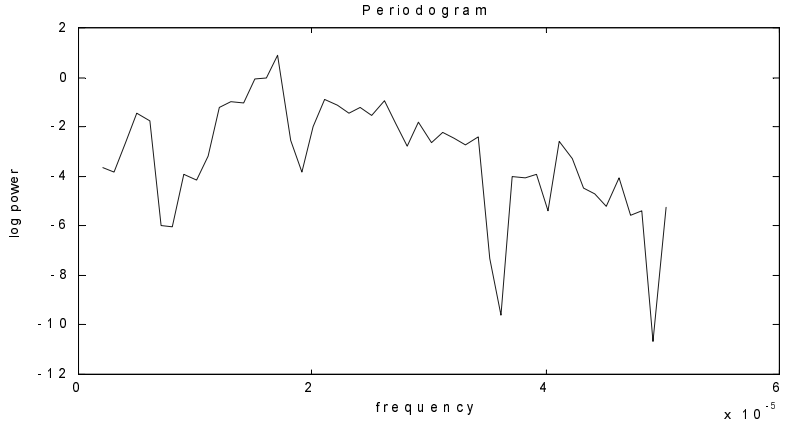

Fig 15 Power spectral density in semi log scale

In the second phase the signal is classified according to the slope $S = (\alpha + 1)$ of log(P( ))) on the log(frequency) dependence. The following categories have been proposed (see [11]):

1. **Fractional Gaussian noise fG** for range.*–1<S<0.38*. In this case the fractal dimension from power spectrum can be used but variogram is not suitable.
2. **Fractional Brownian motion fB** for range 1.04<S<3. In this case can be used the variogram for estimation of fractal dimension as well.
3. **Transition case** for range of *S* between *0.38* and *1.04***.** For this case the cumulative sum of SHV should be created (transformation to the case 2)
4. **No fractal behavior** for cases when the power law model is invalid (in two decade range). For this case the **chaotic models** (broad bands) or **ARIMA models** (narrow peaks) has to be used

Special techniques for estimation of fractal dimension for the above-mentioned cases are presented in [11]. For realization of this computations and evaluation of fractal dimension based on the definition (see eqn. (3)) the MATLAB program **TLOU** has been created. This program uses the cleaned and pre-treated $R(d_j)$ for creation of variogram and P( )) function (based on FFT). Fractal dimensions (*Dp* from P( )) and *Dv* from variogram) are obtained through estimating the parameter from a LSE linear regression of the corresponding models. The fractal dimension *Dd* based on the definition is computed by using the code Divider2 described in [15]

9. EXAMPLE

The computation of surface roughness parameters based on the concept of fractality is critically dependent on the estimation of P( )). Our procedures in the program **TLOU** has been selected based on the simulated fB and fG fractal curves by using of code Mwfractal described in [15]. As an example of practical application of **TLOU** the surface roughness profile for one weave has been measured on the Kawabata instrument. Weave was three end twill (wef sett 35 and warp sett 16) created from ply polyester yarns 25x2 tex. The clean SHV trace is shown on the fig.14 and P( )) curve on the fig.15. The log log P( )) trace and regression lines are on the fig 16.

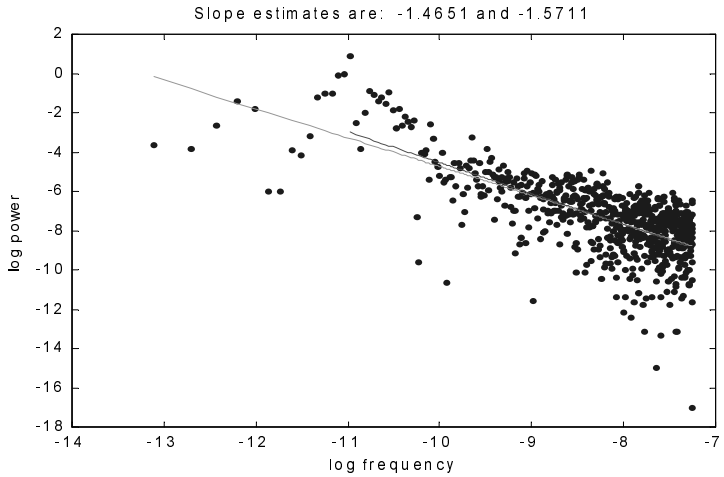

Fig 16. The test of fractality (log log PSD trace). The least squares lines correspond to the full data and portion of data having high frequencies.

The slope *S* about 1.48 classifies the SHV signal to the second group. The plot of variogram in log log scale is on the fig. 17

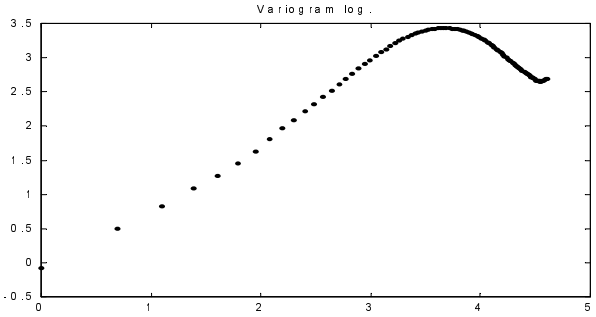

Fig. 17 Variogram in log log scale

It is visible that the linearity is acceptable up to the 3 and half decades. The computed fractal dimensions are ***Dv = 1.629***, ***Dp =1.714*** and ***Dd = 1.536***. Characteristic from Kawabata device is for this case ***SMD = 4.931***.

10. CONCLUSION

The analysis of SHV based on the TLOU program is in reality more complex. The classical roughness characteristics and topothesy are computed as well and many other techniques of fractal dimension calculation are included. In the future the analysis will be extended to the chaotic models and autoregressive models. The roughness parameters PC and PS will be correlated wit subjective roughness meaning. With some modifications it will be possible to use this approach for characterization of the surface profiles obtained from another techniques or longitudinal variability of linear (yarns) or plane textile structures (fabric).

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9. REFERENCES

[1] Kawabata S.: Text. Mach. Soc. Japan 1980
[2] Quinn B.G., Hannan E. J.: The estimation and Tracking of Frequency, Cambridge University Press 2001
[3] Constantine A. G,. Hall P.: J. Roy. Stat. Soc. **B56**, 97, (1994)
[4] Ott E,. Sauer T., Yorke J.A. Eds: Copying with chaos, J. Wiley 1994
[5] Mandelbrot B.B., Van Ness J.W.: SIAM Review 10, 442 (1968)
[6] Meloun M, Militký J., Forina M,: Chemometrics for Analytical Chemistry, vol1 PC Aided Statistical Data Analysis, Ellis Horwood, Chichester 1992
[7] Whitehouse D. J.: Wear **249**,345 (2001)
[8] Sacerdotti F., Griffiths B. J., Butler C. and Benati F.: Proc. Inst. Mech. Engfs. **214B**, 811, (2000)
[9] Greenwood J.A.: Proc. Roy. Soc. London **A393**, 133 (1984)
[10] Mannelqvist A., Groth M.R.: Appl. Phys. **A69**, (2001) in press
[11] Eke A. et all : Eur. J. Physiol. **439**, 403 (2000)
[12] Nayak P. R.: Trans. ASME : J. Lub. Tech. **93F**, 398 (1971)
[13] Bloomfield P.: Fourier analysis of time series, J. Wiley. New York 2000
[14] Muthuswamy J. , Thakor N.V.: J. of Neurosci. Methods **83,**1 (1998)

[15] Middleton G. V.: Data analysis in the earth science using MATLAB, Prentice Hall NJ, 2000
[16] Kendziorski C. M. et. all. : Physica A, 273,439 (1999)
[17] Wu J.: Wear 230,194 (1999)
[18] Anonym: Signal Analysis Toolbox, Mathworks Inc., 2000
[19] Wu J.: Wear **239**, 36 (2000)