A MECHANICAL MODEL OF THE VIBRATION CONVEYOR

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Abstract
Vibrating conveyors as components of assembly lines are used in various technical branches, especially in the automotive industry. They do not only provide mechanical handling of differently shaped and sized parts, but they also allow the transport of loose materials. Hereby the main principle for the transportation of parts is based on the oscillating movements of the carrying element which, at the same time, imparts the vertical and horizontal velocity to the transported item. The vibration conveyors are driven by means of mechanical exciters or electromagnetic components. The mechanical exciters, based on the rotation of an unbalanced object, are either directly attached to the carrying element of the conveyer or are connected to the element that is joined to the conveyor by a flexible linkage. The electromagnetic components induce periodic power or moments between the carrying element and the frame. Generally, it is aimed to tune the system in a way to have the natural frequency according to the main transportation movement matching or nearly matching with the exciter frequency. The transport performance in relation to the energetic requirements of the drive is highest in the resonance zone.

Introduction
Vibrating conveyors have the carrying element often directly connected to the frame by either leaf springs or through an inertia element aiming on reactive dynamic power minimization. The leaf springs form a flexible linkage between the carrying element and either the frame or the inertia mass, and they perform a lead mechanism function simultaneously. From the mechanical point of view, it is basically a two mass model with one of the bodies (either the frame or the inertia mass) connected flexibly to the base or to the frame for reasons of vibroisolation. The second object is connected by a movable and flexible link-up. To provide an optimal vibration conveyor performance, it is necessary to know the influence of different dynamic system parameters on the natural frequency according to the main transportation movement. This is only possible by assembling and analysing its mechanical model. The following article deals with the set up of such a mechanical vibration conveyor model with either a translational motion or a screw motion.

1 Construction of Vibration Conveyors
In general, vibrating conveyors have their carrying elements adjusted to the shape and size of the objects to be handled. Additionally, the carrying surface is modified towards abrasion resistance and, commonly, to achieve a larger friction coefficient. Further, the surface has a significant influence on the emitted conveyor noise. The carrying element is directly connected either to the frame or to the inertia element by use of the springs. Generally, the flexible connection consists of the leaf springs whose stiffness is relatively easy to adapt;
hence, they perform simultaneously the conducting mechanism function. Hereby, the cross stiffness of the spring is significantly lower than the longitudinal stiffness. The leaf springs are to be positioned as a parallelogram system that determines the relation between the horizontal and vertical amplitudes of the oscillating movement of the carrying element. This way it consequently estimates the velocity of the component or material transfer. Vibration conveyers are constructed to provide the translational or screw motions.

1.1 Conveyors with the translational motion
Translationally moved conveyors (Fig. 1) have their tubing or flume-shaped carrying elements connected with the frame by elastic elements. Thus, the transport is handled by both vertically and horizontally directed oscillation movements that perform at the same time. Both movements are bonded with each other by means of a lead mechanism caused by a parallelogram or analogue with the leaf springs. The dynamically forced power is generated via one or two mechanical excites that are connected to the carrying element.

![Fig. 1 Vibration conveyor with translational motion](image)

1.2 Conveyor with the screw motion
Conveyors with the screw motion (Fig. 2) have a cylinder-shaped carrying element on whose cylinder there are placed ground component parts that move via spiral lanes along the inner cylinder wall surface. Their transport is realized simultaneously with both the oscillatory translation movement and the oscillatory rotation movement of the carrying element which is usually connected to the frame by help of the leaf springs. These leaf springs are either placed symmetrically towards the cylinder base and the frame or with respect to the inertia mass. The handled components move then via the spiral lane. The force power will be initiated by one or more central electromagnets placed on the longitudinal axis of the carrying element. It is further possible to place the magnets in a symmetric relation to the perimeter of the carrying element.
2 Mechanical vibrating conveyor model

Summing up, each vibration conveyor has a carrying element that is connected to frame or inertia mass by a movable and elastic linkage. The movable connection is realized with a lead mechanism that defines the mutual relation between the cinematic quantities of horizontal and vertical movement. The elastic connection then consists of e.g. pitched springs or leaf springs that can also carry out a lead mechanism function. To reduce the transmission of dynamic forces, the conveyor frame is elastically connected with the underlay.

Basically, from the mechanical point of view, we analyse a two mass model with one of the bodies performing a general motion, whereas the second body is connected through a lead mechanism having specified its cinematic connection.

With respect to the nature of the conveyor, we can, in the dynamic model, assume one or two symmetrical planes. In the first case, we consider the general plane motion of the frame and the relative movement of the carrying element that is connected via the lead mechanism. In the second case, the movement is traceable in one upright axis and the rotation of both the frame and the carrying element around that axis.

2.1 Conveyors with the translational motion

The conveyor frame is determined by the mass \( m_1 \) and its moment of inertia about the axis passing vertically through its centre of gravity towards the symmetrical axis. To connect with the underlay we apply stiffness to the elastic connection in both the longitudinal direction \( k_{1y} \) and the cross direction \( k_{1x} \) as well as to the damping coefficients \( b_{1y} \) and \( b_{1x} \). The positioning of the spring fixtures at the frame and their distances to the horizontal axis in line with the stiffness coefficients \( k_{1x} \) and \( k_{1y} \) determine the torsion stiffness coefficient \( k_{1\varphi_z} \). (Fig. 3).
The carrying element will be determined by the mass \( m_1 \) and the moment of inertia \( J_{2z} \) about the horizontal axis passing through the centre of gravity. The movable connection with respect to the frame is realized by help of a parallelogram with levers of the length \( l \) and the basic angle \( \beta \) of the levers to the vertical direction. The elastic connection with the frame apply the springs of stiffness \( k_{1x} \) and \( k_{1y} \). Further the fixture distances from the centre of the carrying element gravity also determine the torsion stiffness \( k_{1\phi_z} \) (Fig. 3).

The Lagrange equation provides an equations-of-motion system

\[
\frac{d}{dt} \left( \frac{dE_k}{dq} \right) - \frac{dE_k}{dq} + \frac{dE_d}{dq} + \frac{dE_P}{dq} = 0. \tag{1}
\]

Firstly, it is possible to express the kinetic energy of the system

\[
E_k = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} J_{1z} \dot{\phi}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_2 \dot{y}_2^2 + \frac{1}{2} J_{2z} \phi_{2z}^2, \tag{2}
\]

followed by the potential energy of the system is

\[
E_P = \frac{1}{2} k_{1x} x_1^2 + \frac{1}{2} k_{1y} y_1^2 + \frac{1}{2} k_{1\phi_z} \phi_{1z}^2 + \frac{1}{2} k_{2x} (x_2 - x_1)^2 + \frac{1}{2} k_{2y} (y_2 - y_1)^2 + \frac{1}{2} k_{2\phi_z} (\phi_{2z} - \phi_{1z})^2. \tag{3}
\]

As the potential energy of the formally applied springs with stiffness coefficients \( k_{2x} \), \( k_{2y} \) and \( k_{2\phi_z} \) and connection points \((x_1, y_1)\) and \((x_2, y_2)\) complies with the leaf spring energy of coefficient \( k_2 \), this relation (3) is rewritable as follows:
\[ E_p = \frac{1}{2} k_{1x}x_1^2 + \frac{1}{2} k_{1y}y_1^2 + \frac{1}{2} k_{1\varphi_2} \varphi_1^2 + \frac{1}{2} k_2 (\psi l)^2. \] (4)

It is possible to express similarly the damping energy \( E_d \) dissipated by the dampers in an equivalent way. However, as its application in the following analysis of natural frequencies would have only formal effect, it is overridden in the following relations.

For the considered coordinate data \( x_1, y_1, \varphi_1, x_2, y_2 \) and \( \varphi_2 \) describing the kinematic condition of the vibration conveyor system, the following requirements are to be effective:

\[ x_2 = x_1 - \psi l \cos \beta - \varphi_1 \psi r \cos \gamma, \] (5)

\[ y_2 = y_1 - \psi l \sin \beta - \varphi_1 \psi r \sin \gamma, \] (6)

and

\[ \varphi_1 = \varphi_2. \] (7)

After their substitution into the simplified Lagrange equation:

\[ \frac{d}{dt} \left( \frac{dE_k}{dq} \right) + \frac{dE_p}{dq} = 0 \] (8)

including time \( t \) and the generalized coordinate \( q \) into which it is possible to put \( x_1, y_1, \varphi_1, \psi \) gradually, we receive four motion equations of the system.

As far as for the adjustment there results the following:

\[ (m_1 + m_2) \ddot{x}_1 - m_2 \psi l \cos \beta - m_2 \dot{\varphi}_1 \psi r \cos \gamma + k_{1x} x_1 = 0, \] (9)

\[ (m_1 + m_2) \ddot{y}_1 - m_2 \psi l \sin \beta - m_2 \dot{\varphi}_1 \psi r \sin \gamma + k_{1y} y_1 = 0, \] (10)

\[ (J_{1z} + J_{2z} + m_2 r^2) \ddot{\varphi}_1 - m_2 r (\ddot{x}_1 \cos \gamma + \dot{y}_1 \sin \gamma) + m_2 r \psi l (\cos \beta \cos \gamma + \sin \beta \sin \gamma) + k_{1\varphi_2} \varphi_1 = 0, \] (11)

\[ m_2 \ddot{\psi} l - m_2 (\ddot{x}_1 \cos \beta + \dot{y}_1 \sin \beta) + m_2 r \dot{\varphi}_1 (\cos \beta \cos \gamma + \sin \beta \sin \gamma) + k_2 l \psi = 0. \] (12)

The mentioned differential equations system is possible to apply in respect to natural frequencies and to ensure the influence of the system’s parameters on their values. A correct tuning of the system with respect to the resonance zone enables high vibration performances at relatively low energy exposures.

### 2.2 Conveyors with the screw motion

The conveyor frame is again determined with the mass \( m_1 \) and the moment of inertia \( J_{1y} \) about the vertical axis passing through its centre of gravity. To connect it with the underlay, we apply the stiffness of the elastic supports in the longitudinal direction \( k_{1y} \) as well as in the torsion direction \( k_{1\varphi_y} \). The damping coefficients \( b_{1y} \) and \( b_{1\varphi_y} \) (Fig. 4) are possible to be inducted in an equivalent way.
The carrying element will have determined the mass \( m_1 \) and the moment of inertia \( J_{2y} \) about the vertical axis passing through its centre of gravity. The movable connection in respect to the frame is realized by help of the leaf springs of the length \( l \) and the basic angle \( \beta \) of the levers. The elastic and damping connection with the frame is formed by the springs of stiffness \( k_{2y} \) and \( k_{2\varphi y} \) as well as by the dampers with the damping coefficients \( b_{2y} \) and \( b_{2\varphi y} \).

Again, an equations-of-motion system can be gained using the Lagrange equation. Firstly, it is possible to express the kinetic energy of the system providing the considered movements into a direction and around the vertical axis passing through the centre of gravity of both the carrying element and the frame.

\[
E_k = \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} J_{1y} \dot{\varphi}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 + \frac{1}{2} J_{2y} \dot{\varphi}_2^2, \tag{13}
\]

Subsequently, it is possible to express the system’s potential energy.

\[
E_p = \frac{1}{2} k_{1y} y_1^2 + \frac{1}{2} k_{\varphi 1y} \varphi_1^2 + \frac{1}{2} k_{2y} (y_2 - y_1)^2 + \frac{1}{2} k_{\varphi 2y} (\varphi_2 - \varphi_1)^2. \tag{14}
\]

It is possible to express the damping energy \( E_d \) dissipated by the dampers in an equivalent way. However, for the same reasons as mentioned before, it will not be considered in the following natural frequencies calculation.

Due to the fact that the potential energy of the formally applied springs with the rigidity coefficients \( k_{2y} \) and \( k_{\varphi 2y} \) is identical with the leaf springs energy with stiffness coefficient \( k_2 \), the relation (14) is rewritable as follows:
\[ E_p = \frac{1}{2} k_1 y_1^2 + \frac{1}{2} k_{\varphi_1 y} \varphi_1^2 + \frac{1}{2} k_2 (\nu l)^2. \]  

(15)

For the considered coordinates \( y_1, \varphi_1 y, \) and \( \varphi_2 y \) describing the kinematic condition of the vibration conveyor system with a combined motion, the following requirements are to be effective:

\[ y_2 = y_1 + \psi l \sin \beta \]  

(16)

and

\[ \varphi_2 y = \varphi_1 y + \frac{\psi l}{r} \cos \beta. \]  

(17)

After their substitution into the equation (8) it is possible to gain three motion equations of the system for the coordinates \( y_1, \varphi_1 y \) and \( \psi \). The result of the adjustment will be as follows:

\[ (m_1 + m_2) y_1 + m_2 \psi l \sin \beta + k_{1 y} y_1 = 0, \]  

(18)

\[ (J_{1 y} + J_{2 z}) \varphi_1 y + J_{2 z} \psi l \frac{1}{r} \cos \beta + k_{\varphi_1 y} \varphi_1 y = 0, \]  

(19)

\[ m_2 \dot{y}_1 \sin \beta + m_2 \psi l \sin^2 \beta + J_{2 y} \varphi_1 y \frac{\cos \beta}{r} + J_{2 y} \psi l \frac{\cos^2 \beta}{r^2} + \]  

\[ + k_2 \psi l = 0. \]  

(20)

The referred differential equations system is, similarly to the example mentioned before, possible to be applied in respect to natural frequencies as well as to ensure the influence of the system’s parameters on their values. Again, the correct tuning of system with respect to the resonant zone will enable high vibration performances at relatively low energy exposures.

**Conclusion**

The analysed mechanical vibration conveyor models with both the translational motion and the screw motion together with their related motion equations provide the system tuning that considers the natural frequency of the main vibration movement. It is the only possible approach in order to ensure transportation efficiency performing at the smallest possible mechanical resistance just in the resonant zone. In terms of vibrating conveyors with the translational motion it is possible to obtain the correct tuning through the adjustment of the stiffness of the carrying element or the frame elastic connection. In case of conveyors with the screw movement the mentioned result is possible to be achieved by adjusting the mass parameters of the carrying element.

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Literature

MECHANICKÝ MODEL VIBRAČNÍHO DOPRAVNÍKU

MECHANISCHES MODEL EINES VIBRATIONSTRANSPORTERS

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