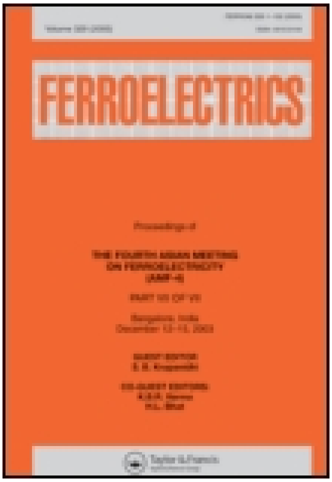

Ferroelectrics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gfer20

PdP152. The influence of the electric stiffening on the resonant frequency temperature dependence of quartz resonators

Jiri Zelenka [a]

[a] Technical University of Liberec , Halkova 6, Liberec, Czechoslovakia

Published online: 10 Feb 2011.

To cite this article: Jiri Zelenka (1992) PdP152. The influence of the electric stiffening on the resonant frequency temperature dependence of quartz resonators, Ferroelectrics, 134:1, 127-131, DOI: 10.1080/00150199208015576

To link to this article: http://dx.doi.org/10.1080/00150199208015576

Ferroelectrics, 1992, Vol. 134, pp. 127–131
Reprints available directly from the publisher
Photocopying permitted by license only

PdP152

THE INFLUENCE OF THE ELECTRIC STIFFENING ON THE RESONANT FREQUENCY TEMPERATURE DEPENDENCE OF QUARTZ RESONATORS

Jiri ZELENKA
Technical University of Liberec, Halkova 6, Liberec
Czechoslovakia

Abstract Holland, Sinha and Tiersten, Lee and Yong proposed a new method for the more precise determination of the temperature dependence of the resonant frequency of quartz resonators in the period from 1976 to 1984. The method is based on the proposition that the small vibrations of the quartz plate are superposed on the large thermally induced deformation. The extension of the Lee and Yong's method is explained in the paper. The piezoelectric properties and the temperature dependence of the piezoelectric constants and permitivities are considered by the description of the modified method.

INTRODUCTION

Lee and Yong presented [1] one set of the first temperature derivatives $c_{pq}^{(1)}$ and one set of the effective second temperature derivatives $c_{pq}^{(2)}$ of quartz. The mentioned sets of the first and second temperature derivatives were calculated from the temperature coefficients of the frequency measured by Bechmann, Ballato and Lukaszek[2]. By the derivation of the temperature derivatives $c_{pq}^{(n)}$ Lee and Yong considered the linear field equations for small vibrations superposed on thermally induced deformations by steady and uniform temperature changes. They derived the deformation caused by the temperature changes from the nonlinear field equations of thermoelasticity in Lagrangian formulation. The inclusion of the nonlinear effects to the expression of the thermally induced deformation makes it possible to describe more precisely the resonant frequency temperature behaviour of the quartz resonators.

When Lee and Yong derived the sets of the temperature derivatives $c_{pq}^{(n)}$ they neglected the influence of the piezoelectric properties of the quartz plates on the resonant frequency. As it was shown by Zelenka and Lee[3] neglecting the piezoelectric properties of the plates and bars caused in some cases a large difference between the calculated and measured values of the resonant-frequency-temperature characteristic of the quartz resonators. To remove the discrepancy the modification of Lee and Yong's procedure is given in this paper.

INCLUSION OF PIEZOELECTRIC PROPERTIES TO THE EQUATIONS OF MOTION FOR SMALL VIBRATIONS SUPERPOSED ON THERMALLY-INDUCED DEFORMATION

We will consider, similarly as Lee and Yong, three states of crystal:

(1) A natural state when the crystal is at rest, free of stress and strain, has a uniform temperature T_0. Let x_i denotes the position of a generic material point, ρ_0 the mass density, C_{ijkl}, C_{ijklmn}, and $C_{ijklmnpq}$ the second, third, and fourth order elastic stiffness of the crystal.

(2) An initial state when the crystal is now subject to a steady and uniform temperature increase from T_0 to T, and is allowed to expand freely. At this state, the position of a material point is moved, due to the thermal expansion from x_i to y_i ($y_i = x_i + U_i$), where U_i denotes initial displacement.

(3) A final state when small-amplitude vibrations are superposed on thermally induced deformations. The position of the material point is moved from y_i to z_i ($u_i = z_i - y_i$), where u_i is the incremental displacement due to vibrations.

The behaviour of the crystal in the initial state can be described by the same set of equations as in Lee and Yong's paper (Eqs.(1) to (8)). The additional stress appears in the crystal caused, due to its piezoelectric properties by the changing of the thermally-induced deformation. But if the temperature changes very slowly, the additional stress will be very small and in the steady state diminished (the electrical charges which caused the additional stress reach zero).

The governing equations in the final state are given as follows:

$$\bar{U}_i = U_i + u_i = z_i - x_i, \quad \bar{\Phi} = \phi + \varphi,$$

$$\bar{E}_{ij} = E_{ij} + e_{ij} = \frac{1}{2}(\bar{U}_{j,i} + \bar{U}_{i,j} + \bar{U}_{k,i}\bar{U}_{k,j}),$$

$$\bar{T}_{ij} = T_{ij} + t_{ij} = C^{\theta}_{ijkl}\bar{E}_{kl} + \frac{1}{2} C^{\theta}_{ijklmn}\bar{E}_{kl}\bar{E}_{mn} + \frac{1}{6} C^{\theta}_{ijklmnpq}\bar{E}_{kl}\bar{E}_{mn}\bar{E}_{pq} + e^{\theta}_{rij}\bar{\Phi}_{,r} + \frac{1}{2} e^{\theta}_{rijkl}\bar{\Phi}_{,r}\bar{E}_{kl} - \lambda^{\theta}_{ij},$$

$$\bar{\Phi}_{,i} = \phi_{,i} + \varphi_{,i},$$

$$(\bar{T}_{ij} + \bar{T}_{ik}\bar{U}_{i,k})_{,j} = \rho_0 \ddot{\bar{U}}_i,$$

$$\bar{P}_i = n_j(\bar{T}_{ij} + \bar{U}_{i,k}) \quad \text{on } S,$$

$$e^{\theta}_{rij}\bar{E}_{ij,r} + \frac{1}{2} e^{\theta}_{rijkl}\bar{E}_{ij}\bar{E}_{kl,r} + \frac{1}{2} e^{\theta}_{rijkl}\bar{E}_{ij,r}\bar{E}_{kl} - \varepsilon^{\theta}_{rj}\bar{\Phi}_{,jr} - \varepsilon^{\theta}_{rjk}\bar{\Phi}_{,j}\bar{\Phi}_{,kr} - \varepsilon^{\theta}_{rjk}\bar{\Phi}_{,jr}\bar{\Phi}_{,k} = 0 \qquad (1)$$

where $\bar{U}_i$, $\bar{E}_{ij}$, $\bar{T}_{ij}$, $\bar{P}_i$ and $\bar{\Phi}$ are total displacement, strain, stress, traction and potential respectively.

The incremantal strain (e_{ij}), stress (t_{ij}), traction (p_i) and potential (φ) give the governing equations for incremental fields:

$$e_{ij} = \frac{1}{2}(u_{j,i} + u_{i,j} + U_{k,j}u_{k,i} + U_{k,i}u_{k,j}),$$

$$t_{ij} = (C^{\theta}_{ijkl} + C^{\theta}_{ijklmn}E_{mn} + \frac{1}{2} C^{\theta}_{ijklmnpq}E_{mn}E_{pq} + \frac{1}{2} e^{\theta}_{rijkl}\phi_{,r})e_{kl} + (e^{\theta}_{rij} + \frac{1}{2} e^{\theta}_{rijkl}E_{kl})\varphi_{,r} + e^{\theta}_{rij}\phi_{,r},$$

$$(t_{ij} + t_{jk}U_{i,k} + T_{jk}u_{i,k})_{,j} = \rho_0\ddot{u}_i,$$

$$p_i = n_j(t_{ij} + t_{jk}U_{i,k} + T_{jk}u_{i,k}) \quad \text{on } S,$$

$$(e^{\theta}_{rij} + e^{\theta}_{rijkl}E_{kl})e_{ij,r} + (e^{\theta}_{rij} + e^{\theta}_{rijkl}e_{kl})E_{ij,r} = (\varepsilon^{\theta}_{rj} + 2\varepsilon^{\theta}_{rjk}\phi_{,k})\varphi_{,jr} + (\varepsilon^{\theta}_{rj} + 2\varepsilon^{\theta}_{rjk}\varphi_{,k})\phi_{,jr} \qquad (2)$$

The material property-temperature relation can be expressed as follows

$$C^{\theta}_{ijkl} = C_{ijkl} + C^{(1)}_{ijkl}\theta + C^{(2)}_{ijkl}\theta^2,$$

$$C^{\theta}_{ijklmn} = C_{ijklmn} + C^{(1)}_{ijklmn}\theta,$$

$$C^{\theta}_{ijklmnpq} = C_{ijklmnpq},$$

$$\alpha^{\theta}_{ij} = \alpha^{(1)}_{ij}\theta + \alpha^{(2)}_{ij}\theta^2,$$

$$e^{\theta}_{ijk} = e_{ijk} + e^{(1)}_{ijk}\theta,$$
$$e^{\theta}_{ijklm} = e_{ijklm},$$
$$\varepsilon^{\theta}_{ij} = \varepsilon_{ij} + \varepsilon^{(1)}_{ij}\theta,$$
$$\varepsilon^{\theta}_{ijk} = \varepsilon_{ijk} \qquad (3)$$

where α^{θ}_{ij} are values of the linear thermal expansion coefficient, e^{θ}_{ijk} and e^{θ}_{ijklm} denote linear and quadratic piezoelectric stress tensor components and $\varepsilon^{\theta}_{ij}$ and $\varepsilon^{\theta}_{ijk}$ are the components of the tensor of linear and quadratic permitivities.

The plate in the initial state is at rest and allowed free expansion, that is

$$U_{j,i} = U_{i,j} = E_{ij} = \alpha^{\theta}_{ij},$$
$$T_{ij} = 0, \quad U_i = 0, \quad \Phi_{,i} = 0. \qquad (4)$$

The substitution from relations (4) into (3) gives the incremental strain-displacement relations

$$e_{ij} = \frac{1}{2}\,(u_{j,i} + u_{i,j} + \alpha^{\theta}_{kj}u_{i,j} + \alpha^{\theta}_{ki}u_{j,i}), \qquad (5)$$

the stress-strain-temperature relations

$$t_{ij} = (C_{ijkl} + D^{(1)}_{ijkl}\theta + D^{(2)}_{ijkl}\theta^2)e_{kl} + (e_{rij} + F^{(1)}_{rij}\theta)\,\varphi_{,r}, \qquad (6)$$

the charge equation of electrostatics

$$(e^{\theta}_{rij} + e^{\theta}_{rijkl}\alpha^{\theta}_{kl})e_{ij,r} - \varepsilon^{\theta}_{rj}\varphi_{,jr} = 0, \qquad (7)$$

and stress equations of motion

$$(t_{ij} + \alpha^{\theta}_{ik}t_{jk})_{,j} = \rho_0\ddot{u}_i, \qquad (8)$$

$$p_i = n_j(t_{ij} + \alpha^{\theta}_{ik}t_{jk}) \quad \text{on S}, \qquad (9)$$

where

$$D^{(1)}_{ijkl} = C^{(1)}_{ijkl} + C_{ijklmn}\alpha^{(1)}_{mn},$$
$$D^{(2)}_{ijkl} = \frac{1}{2}\,\tilde{C}^{(2)}_{ijkl} + C_{ijklmn}\alpha^{(2)}_{mn},$$
$$F^{(1)}_{rij} = e^{(1)}_{rij} + \frac{1}{2}\,e_{rijkl}\alpha^{(1)}_{kl},$$
$$\tilde{C}^{(2)}_{ijkl} = C^{(2)}_{ijkl} + 2C^{(1)}_{ijklmn}\alpha^{(1)}_{mn} + C_{ijklmnpq}\alpha^{(1)}_{mn}\alpha^{(1)}_{pq}, \qquad (10)$$

$\varphi_{,jr}$ can be expressed from Eq. (7)

$$\varphi_{,jr} = \beta^{\theta}_{rj}(e^{\theta}_{rij} + e^{\theta}_{rijkl}\alpha^{\theta}_{kl})\, e_{ij,r}. \tag{11}$$

where β^{θ}_{jr} are the components of the tensor of linear impermeability.

By substituting Eqs. (6), (10) and (11) into Eq. (8) we obtained the incremental displacement equations of motion

$$G_{ijkl}u_{k,jl} = \rho_0 u_i, \tag{12}$$

where

$$G_{ijkl} = C_{ijkl} + \beta_{rj}e^2_{rij} + G^{(1)}_{ijkl}\theta + G^{(2)}_{ijkl}\theta^2,$$

$$G^{(1)}_{ijkl} = (C_{sjkl} + \beta_{rj}e^2_{rij})\,\alpha^{(1)}_{is} + C_{ijsl}\alpha^{(1)}_{ks} + D^{(1)}_{ijkl} +$$

$$+ (\beta_{rj}e^2_{rij})^{(1)} + \frac{3}{2}\beta_{rj}e_{rij}\, e_{rijkl}\alpha^{(1)}_{kl} \tag{13}$$

and $G^{(2)}_{ijkl}$ is the same as in Lee and Yong's paper.

CONCLUSION

The piezoelectric terms in Eqs. (13) are necessary to be considered only when the guided displacement u_i of the vibrations is coupled to the electric field.

REFERENCES

1. P.C.Y. Lee and Y.K. Yong, J. Appl. Phys., 56, 1514 (1984).
2. R. Bechmann, A.D. Ballato and T.J. Lukaszek, Proc. IRE, 50, 1812 (1962).
3. J. Zelenka and P.C.Y. Lee, IEEE Trans. Son. Ultrason., SU-18, 79 (1971).